

Stopping Cross Section of Solids for Protons, 50–600 keV*

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The stopping cross sections of Li, Cu, LiF, CaF₂, Pb, and Au for protons have been measured over the energy range $E_p=50-600$ keV; the stopping cross section of Be for protons has been measured for $E_p=50-2600$ keV. An electrostatic analyzer and magnetic spectrometer were used to measure the energy loss of protons in a thin film of the stopping material. The density of the stopping film was determined by weighing, or by quantitative chemical analysis of a known area. The ratio of the stopping cross section of Al, Cu, Mn, Ta, and Pb to that of Au was measured by observing the yield of elastically scattered protons from these targets for $E_p=200-600$ keV. The ratio of stopping cross section of Ca, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn, to that of Mn was measured for $E_p=200-600$ keV by the same method. In the region $Z=23-29$, the stopping cross section decreases as the atomic number of the stopping material increases. The probable error of absolute stopping cross section measurements in this experiment is 3%.

I. INTRODUCTION

THE precision with which nuclear reaction cross sections can be determined is often limited by the poor accuracy of the available values of the stopping cross section of the target material. The experiment described in this paper was undertaken to supply reliable values of the stopping cross section of Li, Be, LiF, and CaF₂, frequently used target materials. As a test of our experimental method, the measurements were extended to include Cu, Au, and Pb, materials whose stopping properties have been extensively studied. The wide discrepancy between our results and other recent measurements of these materials, especially gold, led us to test the internal consistency of our results by an independent method: measuring the ratio of the stopping cross section of two different materials by comparing the yield of protons elastically scattered from targets of the two materials. This is a simplification of a method used earlier to determine the absolute stopping cross section of ice.¹ By this same scattering method, ratios of stopping cross sections of materials of different Z were measured in the region $23 \leq Z \leq 30$ to confirm a report by Cooper² of anomalous Z dependence of the stopping cross section in this neighborhood.

II. METHOD

Monoenergetic protons were obtained from the 600-keV Van de Graaff generator with electrostatic beam energy analyzer. Since the direct proton beam may break the delicate foils used in the energy loss measurement, the analyzed proton beam was scattered from a thick gold target to reduce the intensity of the beam passing through the foils. A double-focusing charged-particle spectrometer accepted protons scattered at 90° by the gold target and measured their energy. With the spectrometer set to detect particles of a given energy,

for example 277 keV in the illustration in Fig. 1, the counting rate of the spectrometer was measured as a function of beam energy incident on the gold target: (1) with the scattered beam entering the spectrometer directly; (2) with a thin aluminum foil interposed in the scattered beam; and (3) after a layer of the stopping material had been deposited on the aluminum foil by vacuum evaporation. The energy loss in the condensed layer of stopping material is simply the difference in the energy of the steps (2) and (3), with a 1% correction to allow for the energy imparted to the recoil gold nucleus. The proton energy at the step is taken to be that energy at which the counting rate is one-half the counting rate at the top of the step.

To determine the atomic stopping cross section, $\epsilon = -(1/N)dE/dX$, where N is the number of atoms per unit volume of the stopping material and dE is the energy lost by the protons in traversing a distance dX through the stopping material, it is necessary to measure NdX , the number of stopping atoms per unit area, or the areal density of the stopping layer. This latter quantity was measured in two different ways. The thin aluminum foils were supported in the scattered proton

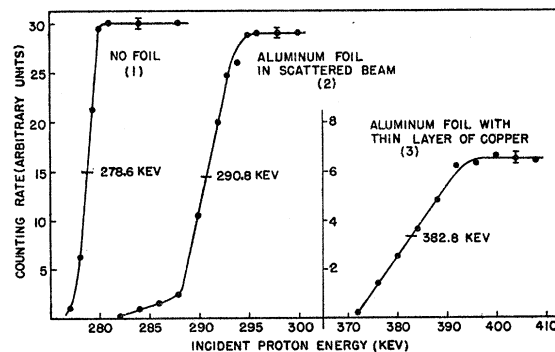


Fig. 1. Counting rate in the magnetic spectrometer as a function of energy of protons incident on the gold target. In curve 1, the scattered protons enter the spectrometer directly. In curve 2, a thin Al foil has been placed between the target and the spectrometer. In curve 3, a layer of Cu has been deposited on the Al foil. Spectrometer is set to detect protons of 277 keV.

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¹ W. A. Wenzel and W. Whaling, *Phys. Rev.* **87**, 499 (1952).

² Green, Cooper, and Harris, *Phys. Rev.* **98**, 466 (1955).

beam by means of a quartz fiber cantilever balance, with which the weight of the clean aluminum foil could be determined. After the energy loss in the foil was measured, a shield with a circular opening of 0.4375-in. diameter was placed directly in front of the foil, between the foil and the furnace from which the stopping material was evaporated. Evaporated material passing through the opening in the shield condensed on the aluminum foil, covering an area given by the area of the opening in the shield. The weight of the deposited layer was then measured with the cantilever balance to determine the mass per unit area of the deposited layer. The complete operation of evaporation, weighing, and energy loss measurement could thus be performed in the vacuum; this is essential in handling a chemically active material such as lithium. Because of the relatively high vapor pressure of lead at room temperatures and the consequent difficulty of carrying out a clean, line-of-sight evaporation of this material, the lead evaporation was made in a separate vacuum system with more elaborate shielding and trapping.

The beryllium foils used in the energy loss measurements were self-supporting and could be treated in a more direct manner.³ As in the procedure described for the aluminum foils above, the energy lost by protons in the beryllium foil was determined at several positions over the area of the foil to make sure that the thickness was uniform. The foil was removed from the vacuum chamber and a circular area of 0.704 ± 0.001 -inch diameter was punched out of the center of the foil and weighed with the quartz balance.

The density of the lithium deposit was also measured by an independent method. A thin Al foil was placed over a hole in a tantalum plate in such a position that the scattered protons passed through the Al foil before entering the magnetic spectrometer. As the lithium from the evaporation furnace was deposited on the Al foil, it was also collected on the much larger area, eight square inches, of the tantalum plate. After the energy loss in the lithium deposit had been measured as before, the total amount of lithium on the tantalum plate was measured by a quantitative acid-base titration and the density of lithium over the hole in the center of the plate was calculated. Since the exit aperture of the furnace was located only five inches in front of the plate, there was a variation in the density of the deposit over the area of the plate. The variation calculated from the geometry was confirmed experimentally by measuring, again by quantitative analysis, the amount of lithium deposited on smaller areas of the plate away from evaporation axis. Since the lithium metal readily combines with residual oxygen or water vapor in the vacuum system, it is desirable to know the composition of the deposited layer. This was determined by collecting on the back of the gold target some of the lithium evaporated from the furnace and analyzing the

composition of this lithium deposit by elastic scattering of protons. Only negligible surface layers of oxygen and carbon, and a trace of sodium, were present; since the carbon layer increased during the analysis, it seems likely that most of the carbon was deposited by the proton beam during the course of the analysis and hence would not be present on the Li layer used in the energy loss measurement.

Since the stopping cross section ϵ is a function of proton energy, the ΔE determined in this experiment actually determines only an average value of ϵ over this energy interval. However, this average value is equal to the value of ϵ at a particular energy E_p within the interval which can be calculated if the energy dependence of ϵ is known.⁴ If the energy dependence of $\epsilon(E)$ is given by $\epsilon(E) = KE^{-\gamma}$, where K and γ are constants, then $E_p^{-\gamma} = (E_i^{\gamma+1} - E_f^{\gamma+1}) / [(\gamma+1)(E_i - E_f)]^{-1}$, where E_i is the energy of the protons entering the stopping layer and E_f is the energy of the protons emerging from the layer. The value $\gamma = \frac{1}{2}$ was assumed in evaluating E_p . The stopping material was placed on that side of the aluminum foil nearer the target; consequently the energy of the protons passing through the aluminum was the same for steps (2) and (3) in our procedure above, and a knowledge of the variations in thickness of the aluminum foil with proton energy was not necessary.

With thick stopping layers, the steps are broadened by straggling and a precise location of the midpoint energy becomes difficult. Very thin stopping layers introduce uncertainty in the measurement of the density of the layer, so that some compromise is necessary. The absolute measurements of stopping cross section described above were made with layers of such thickness that protons of mean energy near 400 keV would lose 100–250 keV. To extend the measurements to other proton energies, it is sufficient to determine only the variation of ϵ with proton energy, a measurement which can be made with much thinner layers since the density of the layer need not be determined. If a thin layer of stopping material is evaporated on the gold target, the gold step (1) in Fig. 1 will be displaced to higher energy, since more energy must be supplied to the incident proton to make up for the energy loss in penetrating the layer of stopping material. The relative measurement of ϵ then consists of measuring the variation of the step displacement with incident proton energy, always using the same stopping layer. The energy dependence of the stopping cross section of Li, Be, LiF, CaF₂, and Cu were determined in this way with layers as thin as 10 keV for 100 keV protons. The energy dependence of $\epsilon(\text{Pb})$ was measured by observing the three steps of Fig. 1 at different settings of the magnetic spectrometer but with the same foil and stopping layer of Pb, thus determining the thickness of a given layer as a function of proton energy. For Au, the energy dependence was determined from the variation with E_p of the width of the step

³ We are indebted to Professor Hugh Bradner of the University of California who supplied the Be foils used in this experiment.

⁴ Chilton, Cooper, and Harris, Phys. Rev. **93**, 413 (1954).

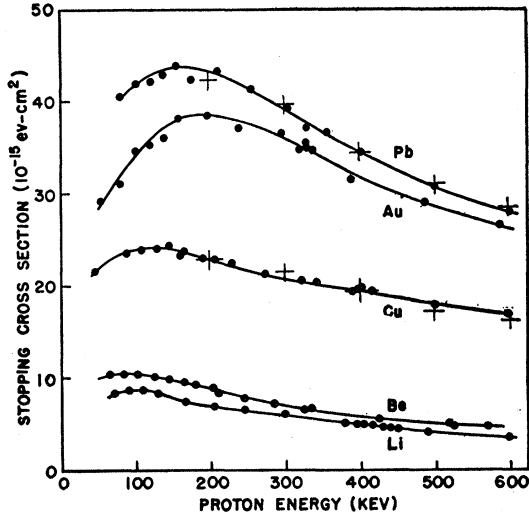


FIG. 2. Experimental values of the stopping cross section as a function of proton energy. The points plotted as crosses are from measurements of the ratio of the stopping cross section relative to gold.

spectrum observed when protons are scattered elastically from a thin layer of Au deposited on an Al backing. The gold measurements were also checked by placing a commercial gold leaf foil in the scattered beam and measuring the energy lost by the transmitted protons as a function of proton energy; measurements of the gold foil and of the evaporated layer agree well. The energy dependence of $\epsilon(\text{Be})$ was also measured in the range $E_p = 200\text{--}600$ keV with an unsupported Be foil in the scattered beam.

Ratios of stopping cross sections of different materials have been measured by means of the relation between ϵ_{eff} , the effective stopping cross section,⁵ and the counting rate N/Q at the top of step (1) in Fig. 1:

$$\begin{aligned} \epsilon_{\text{eff}} &= \epsilon(E_1) \frac{E_2}{E_1} + \epsilon(E_2) \frac{\cos\theta_1}{\cos\theta_2} \\ &= \left(\frac{d\sigma}{d\Omega} \right)_\theta \frac{2E_2 Q \Omega}{RN}, \end{aligned} \quad (1)$$

where $(d\sigma/d\Omega)_\theta$ is the differential scattering cross section at an angle θ in the center-of-mass system, Ω is the solid angle subtended by the aperture of the spectrometer at the target (c.m. system); R is the momentum resolution, $P/\Delta P$, of the spectrometer; N/Q is the number of scattered particles of energy E_2 counted in the spectrometer per Q protons incident on the target, and N is to be evaluated for particles scattered from the surface of the target; θ_1 and θ_2 are the angles between the normal to the target and the direction of the incident and scattered beams, and the subscripts 1 and 2 refer to incident and scattered particles, respectively. The use of

⁵ Snyder, Rubin, Fowler, and Lauritsen, Rev. Sci. Instr. 21, 852 (1950).

Eq. (1) to determine ϵ_{eff} experimentally is subject to uncertainties in the measurement of R , Q , Ω_{lab} , and the counting efficiency. In a measurement of the ratio of the stopping cross sections of two different materials, these instrumental parameters cancel out and one has simply

$$\epsilon_{\text{eff}}/\epsilon_{\text{eff}}' = (d\sigma/d\Omega)_\theta E_2 N' \Omega / (d\sigma/d\Omega)_{\theta'} E_2' N' \Omega', \quad (2)$$

where (Ω/Ω') is a function of the mass of the scattering nucleus and the angle of observation but is independent of the solid angle of the instrument. For $(d\sigma/d\Omega)_\theta$, we have used the Rutherford scattering cross section multiplied by a factor $(1 - 34Z^{7/5}E^{-1} \text{ ev})$ to allow for the electron screening¹; this screening correction amounts to 6% for 200-keV protons scattered from gold, the lowest energy at which this method can be employed with confidence because of the increasing and uncertain neutral component of the scattered beam at low energies. Large deviations from the Rutherford scattering cross section may be expected for low- Z nuclei so that this method has not been used for materials lighter than aluminum.

The experimental procedure involves only the observation of the counting rate at the top of the step for protons scattered from one material, then the target is shifted slightly to expose an evaporated layer of a second material, and the step height for scattering from the second material is measured. The determination of $\epsilon(E_1)$ from ϵ_{eff} follows from

$$\frac{\epsilon_{\text{eff}}}{\epsilon(E_1)} = \frac{\partial E_2}{\partial E_1} \frac{\epsilon(E_2) \cos\theta_1}{\epsilon(E_1) \cos\theta_2} = \frac{\partial E_2}{\partial E_1} + \left(\frac{E_1}{E_2} \right)^{\frac{1}{2}} \frac{\cos\theta_1}{\cos\theta_2}, \quad (3)$$

since $\epsilon(E)$ varies approximately as $E^{-\frac{1}{2}}$ in the energy range, 200–600 keV, in which this method was employed.

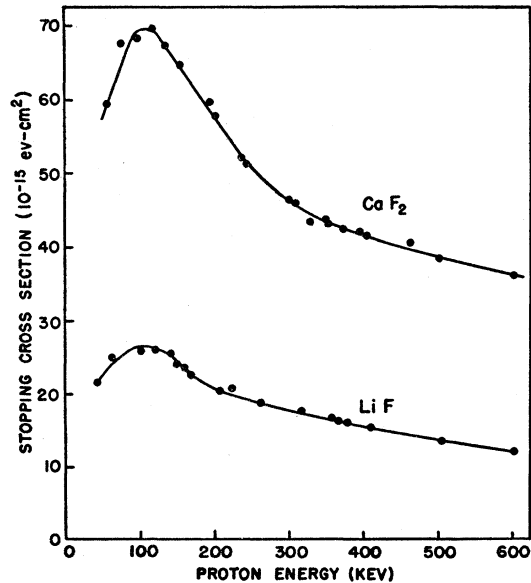


FIG. 3. Experimental values of the molecular stopping cross section as a function of proton energy.

Since E_1/E_2 is very nearly unity for protons scattered from heavy elements, this assumed energy variation of $\epsilon(E)$ cannot introduce a significant error. To insure very smooth scattering surfaces, microscope cover plates were used as the backing for the evaporated layers. Examination of the spectrum of elastically scattered protons revealed no significant impurity in the evaporated layers.

III. RESULTS

The experimental results are plotted in Figs. 2-4. Values taken from the smooth curves drawn through the experimental points are listed in Tables I and II.

(a) Lithium

Absolute measurements of the stopping cross section of lithium were made at $E_p=382, 430, 441,$ and 450 kev by the quantitative chemical analysis of layer thickness, and at 404 and 416 kev by weighing the layers; both types of measurement were made with layers in which the protons lost approximately 250 kev of energy. The energy dependence of $\epsilon(\text{Li})$ was determined for proton energies between 71 and 596 kev with layers 22 kev thick for protons of 100 kev.

There have been no other direct determinations of the stopping cross section of lithium for protons. Haworth and King,⁶ and Warters *et al.*⁷ have measured the relative energy dependence of $\epsilon(\text{Li})$ for protons between 40 and 1300 kev. Their results are in good agreement with the energy dependence found in this experiment.⁸⁻¹⁰

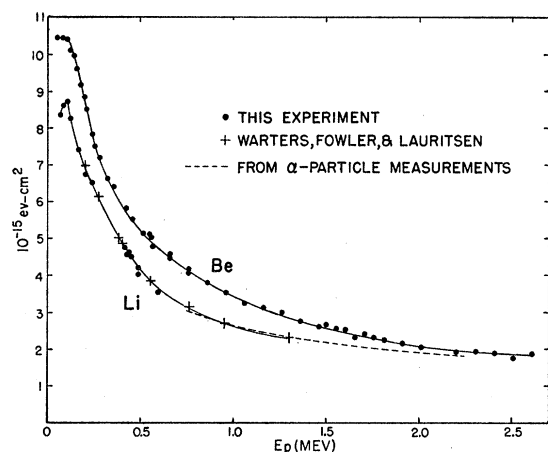


FIG. 4. Experimental values of the atomic stopping cross section of Li and Be for protons. The crosses are measurements by Warters⁷ of the energy dependence of $\epsilon(\text{Li})$ normalized to the results of this experiment at 400 kev. The dashed line is computed from the stopping cross section of lithium for alpha particles.

⁶ L. J. Haworth and L. P. D. King, *Phys. Rev.* **54**, 48 (1938).

⁷ Warters, Fowler, and Lauritsen, *Phys. Rev.* **91**, 917 (1953).

⁸ Both Haworth and King and Warters *et al.* normalized their relative measurements by calculating the stopping cross section with a mean ionization potential for lithium of 32 ev, a value derived by Mano⁹ from Rosenblum's¹⁰ alpha-particle measurements. Mano did not consider the reduced contribution by the K-electrons in lithium to the stopping of low-energy ions, and

TABLE I. Values of the stopping cross section (in units of 10^{-15} ev cm^2), taken from the smooth curves through the experimental points.

Proton energy (kev)	Li	Be	Cu	Au	Pb	LiF	CaF ₂	LiF-Li
50		10.3	21.5	28.2		23.0		
75	8.45	10.5	23.0	31.7	39.4	25.7	66.8	
100	8.70	10.45	24.0	34.5	41.6	26.4	69.4	
125	8.35	10.2	24.3	36.6	43.0	26.0	69.0	
150	7.72	9.80	24.0	37.9	43.7	24.2	66.0	
175	7.25	9.27	23.4	38.6	43.9	22.4	61.4	
200	6.90	8.73	22.9	38.9	43.6	21.0	57.2	
250	6.30	7.75	21.7	37.9	41.7	19.2	50.2	
300	5.75	7.03	21.0	36.2	39.0	17.9	46.2	12.2
350	5.28	6.44	20.2	34.0	36.5	16.7	43.6	11.4
400	4.88	5.98	19.5	31.8	34.4	15.6	41.6	10.7
450	4.50	5.58	18.8	30.0	32.3	14.4	39.6	9.9
500	4.15	5.25	18.1	28.4	30.6	13.5	38.0	9.3
550	3.90	4.96	17.5	27.3	29.2	12.7	36.7	8.8
600	3.65	4.75	16.9	26.3	28.0	12.2	35.6	8.6

TABLE II. Values of the stopping cross section (in units of 10^{-15} ev cm^2), taken from the smooth curves through the experimental points.

E_p (Mev)	Be	Li
0.7	4.33	3.29
0.8	4.00	3.03
0.9	3.70	2.79
1.0	3.43	2.62
1.2	3.02	2.40
1.4	2.70	2.25
1.6	2.46	
1.8	2.27	
2.0	2.08	
2.3	1.92	
2.6	1.85	

Rosenblum¹⁰ has measured the velocity of alpha particles as a function of their range in lithium and we have calculated from his results the stopping cross section of lithium for alpha particles. Using the relation that the stopping cross section for alpha particles is four times that for protons of the same velocity, the stopping cross section determined from Rosenblum's alpha-particle measurements have also been plotted in Fig. 4. The measurements of Warters *et al.*,⁷ which were made in this laboratory by the same method used in this experiment, have been normalized to the results of the present experiment at 400 kev and are plotted in Fig. 4. The agreement between the proton and alpha measurements is very good.

(b) Beryllium

Absolute values for the stopping cross section of beryllium were measured at 323 and 518 kev with foils approximately 100 kev thick for 300 -kev protons. The

consequently, the I value calculated by his method is not independent of the ion energy. Applying Mano's method of calculation to Rosenblum's data, we find that for $E_\alpha=9$ Mev, $I=27$ ev; for $E_\alpha=3$ Mev, $I=64$ ev. The value $I=32$ ev used to normalize the earlier relative measurements gave a result for $\epsilon(\text{Li})$ about 10 percent greater than the values measured in this experiment.

⁹ G. Mano, *Ann. phys.* **1**, 407 (1934).

¹⁰ S. Rosenblum, *Ann. phys.* **10**, 408 (1928).

energy dependence of $\epsilon(\text{Be})$ was determined for proton energies between 65 and 569 keV with a layer approximately 10 keV thick for 100-keV protons; with the 3-MeV electrostatic generator and the 16-in. spectrometer, these measurements have been extended to 2.6 MeV. The stopping cross section of beryllium in this energy range has been measured by Warshaw¹¹ by Madsen and Venkateswarlu,¹² and by Kahn.¹³ The results of this experiment agree well with the measurements of Madsen and Venkateswarlu, and of Kahn, but are some 20 percent higher than the measurements of Warshaw. To be certain that our higher value was not due to a small amount of contaminant of high Z in the beryllium, the composition of the foil was determined by observing the spectrum of elastically scattered protons. Only negligible surface layers of carbon and oxygen were present.

(c) Copper

Absolute measurements of the stopping cross section of copper were made for protons of energy 344, 380, and 401 keV with layers of Cu in which the protons lost approximately 100 keV. The energy dependence was measured for E_p between 50 keV and 600 keV with a layer of 17-keV thickness at 100 keV. The stopping cross section of Cu has been measured by Warshaw,¹¹ Kahn,¹³ and by Green, Cooper, and Harris²; the results from this experiment agree well with the earlier measurements. The measurement of Madsen¹⁴ at 380 keV is 15 percent lower than the other measurements. This discrepancy is probably due to his use of commercial rolled foils of Cu.¹⁵

(d) Gold

Absolute measurements of the stopping cross section of gold were made with protons of energy 321, 327, 330, 335 keV with layers of gold approximately 100 keV thick. The energy dependence of $\epsilon(\text{Au})$ was determined for protons of energy between 57 and 587 keV with a layer 16 keV thick for 100-keV protons.

A comparison of the different measurements of the stopping cross section of gold indicates that some of the experimental factors affecting the measurements are not yet thoroughly understood. In the past seven years, $\epsilon(\text{Au})$ has been measured in this energy range by several investigators: Wilcox,¹⁶ Huus and Madsen,¹⁷ Warshaw,¹¹ Madsen,¹⁴ and Cooper *et al.*² The first five measurements listed above agree reasonably well with each other, but their value is as much as 25% below the result of the sixth measurement from Ohio State. In the region where

¹¹ S. D. Warshaw, *Phys. Rev.* **76**, 1759 (1949).

¹² C. B. Madsen and P. Venkateswarlu, *Phys. Rev.* **74**, 648 (1948).

¹³ D. Kahn, *Phys. Rev.* **90**, 503 (1953).

¹⁴ C. B. Madsen, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* **27**, No. 13 (1953).

¹⁵ S. K. Allison and S. D. Warshaw, *Revs. Modern Phys.* **25**, 779 (1953).

¹⁶ H. A. Wilcox, *Phys. Rev.* **74**, 1743 (1948).

¹⁷ T. Huus and C. B. Madsen, *Phys. Rev.* **76**, 323 (1949).

the two experiments overlap, 400–600 keV, the results of this experiment agree within 1.5% with the measurements from Ohio State. Although a critical analysis of the lower values might question one or two details, such as the use of beaten foils in the Danish measurements and the fact that Wilcox seemed to observe a difference between the stopping of deuterons and protons of the same velocity, it has not been possible to discover any feature common to all of these experiments that might explain the lower values. Our higher value is consistent with our measurement of the ratio $\epsilon(\text{Au})/\epsilon(\text{Cu})$. Our value of this ratio determined by Rutherford scattering is 1.63 at 400 keV; the ratio of our absolute values at 400 keV is 1.64.

(e) Lead

Absolute measurements of the stopping cross section of lead were made for protons of 329 and 353 keV, with layers of lead approximately 70 keV thick. The energy dependence of $\epsilon(\text{Pb})$ was measured for proton energies between 80 and 598 keV with a layer of lead 15 keV thick at 100 keV. The stopping cross section of lead has been measured previously only between 400 and 1000 keV by Green, Cooper, and Harris²; in the region where the two experiments overlap the agreement is within 1%. The stopping cross section of lead for alpha particles has been calculated from Rosenblum's⁹ measurements and agrees well with the results of these two measurements for protons. The ratio $\epsilon(\text{Pb})/\epsilon(\text{Au})$ agrees with the value determined by Rutherford scattering, the maximum deviation is 3% at 200 keV.

(f) LiF and CaF₂

Absolute values of the stopping cross section of LiF were measured for protons of 358, 367, and 379 keV; with layers of LiF in which the protons lost approximately 175 keV; the energy dependence of $\epsilon(\text{LiF})$ was determined for proton energies between 46 and 605 keV with a layer 20 keV thick for 100-keV protons. Absolute values of the stopping cross section of CaF₂ were measured at 331, 352, and 374 keV with layers of CaF₂ approximately 100 keV thick; relative values of $\epsilon(\text{CaF}_2)$ were measured for proton energy between 56 and 602 keV with a layer of CaF₂ 16 keV thick for 100-keV protons. There have been no earlier measurements of the stopping cross section of these materials, which are of interest because of their frequent use as F and Li targets. If the contribution of the lithium atom is subtracted from the molecular stopping cross section of LiF, one obtains a reasonable value for the stopping cross section of fluorine that lies between the measured¹⁸ stopping cross sections of oxygen and neon.

(g) Al, Mn, Ta

The ratios $\epsilon(\text{Al})/\epsilon(\text{Au})$, $\epsilon(\text{Mn})/\epsilon(\text{Au})$, $\epsilon(\text{Ta})/\epsilon(\text{Au})$ were measured for protons of energy between 200 and

¹⁸ Reynolds, Dunbar, Wenzel, and Whaling, *Phys. Rev.* **92**, 742 (1953).

600 kev. Values of $\epsilon(\text{Al})$, $\epsilon(\text{Mn})$, and $\epsilon(\text{Ta})$ computed from our measured value of $\epsilon(\text{Au})$ are listed in Table III. The value of $\epsilon(\text{Al})$ obtained in this way agrees well with the values measured by previous workers.^{11,13,14,16,19} The value of $\epsilon(\text{Mn})$ determined from our ratio agree within 2.5% with the directly measured values of the Ohio State group.² In the tantalum measurements, a clean surface of commercial rolled Ta sheet was used for the scattering targets. There have been no earlier measurements of the stopping cross sections of Ta for protons in this energy range. As a check on this method of measuring stopping cross sections, Pb and Cu were also determined relative to gold; the results are listed in Table III and are plotted as crosses in Fig. 2 to demonstrate the good agreement between these two methods of measurement.

(h) Ca, V, Cr, Fe, Co, Ni, Cu, Zn

For each of these materials, the ratio $\epsilon(X)/\epsilon(\text{Mn})$ was measured for E_p between 200 and 600 kev by the Rutherford scattering yield method. From the values for $\epsilon(\text{Mn})$ in Table III, values of $\epsilon(X)$ have been calculated and are listed in Table IV. Since all of

TABLE III. Values of the stopping cross section (in units of 10^{-16} ev cm^2) for protons, computed from the ratio $\epsilon(X)/\epsilon(\text{Au})$. The values for $\epsilon(\text{Au})$ used in this computation are the values listed in Table I.

Energy (kev)	Al	Mn	Cu	Pb	Ta
200	17.5	26.6	22.8	42.4	36.9
300	14.7	23.7	21.6	39.9	33.8
400	12.5	20.3	19.4	34.6	29.8
500	11.5	17.5	17.2	31.2	26.7
600	10.8	16.4	16.3	28.4	24.6

the values in Table IV contain the common factor $[\epsilon(\text{Mn})/\epsilon(\text{Au})]\epsilon(\text{Au})$, the absolute values are subject to a possible systematic error. However, an error in this factor will not impair the accuracy with which this data describes the relative variation of ϵ with Z in the neighborhood of Mn.

Cooper and his associates² have observed that for protons of 500 kev, $\epsilon(Z)$ shows a periodic variation with atomic number Z superimposed on the theoretical $Z(\log Z^{-1})$ dependence: the stopping cross section increases with Z more rapidly than $Z(\log Z^{-1})$ for those elements in which the last added electron is in an s or p shell. The first region in the periodic table in which d electrons appear begins at $Z=21$, and the materials we have studied illustrate the Z dependence of $\epsilon(Z)$ in this region.

In Fig. 5 is shown the variation of ϵ with Z for protons of 500 kev. In addition to the results obtained in this experiment, the figure includes earlier measurements in this laboratory of the stopping cross sections of gases,

¹⁹ Parkinson, Herb, Bellamy, and Hudson, Phys. Rev. 52, 75 (1937).

TABLE IV. Values of the stopping cross section (in units of 10^{-16} ev cm^2) for protons computed from the ratio $\epsilon(X)/\epsilon(\text{Mn})$. The values for $\epsilon(\text{Mn})$ used in this computation are the values listed in Table III.

Energy (kev)	Ca	V	Cr	Fe	Co	Ni	Cu	Zn
200	25.9	29.9	29.2	27.4	25.7	24.3	22.9	24.3
300	21.3	25.3	24.9	24.4	22.7	23.0	22.1	23.0
400	17.6	21.4	21.4	21.1	19.7	20.0	19.8	20.4
500	15.3	18.5	18.4	18.1	17.2	17.6	17.0	18.6
600	14.3	16.6	17.1	16.8	16.7	16.7	16.5	17.6

and values for metals in the recent paper by Green, Cooper, and Harris at Ohio State²; the fairly consistent 1.5% difference between our measurements and those at Ohio State are well within the limit of error of either measurement. For the elements with $Z < 20$, ϵ increases uniformly with Z ; the lower values at $Z=10$ and $Z=18$ may be accounted for by the relatively tighter binding of electrons in closed shells. Between $Z=23$ and $Z=29$, the over-all variation is a decrease of ϵ with increasing Z . At $Z=30$, ϵ again increases as the second $4s$ electron is added to form Zn.

The explanation of this anomalous behavior is not known. The transition metals also have unusually large values of the electronic specific heat. The large specific heat is interpreted²⁰ in terms of an effective mass for the valence electrons in these metals of several times the electronic rest mass, $18m_0$ in the case of Ni, for example. If these valence electrons manifest an equally large mass in their interaction with the incident proton, the energy loss to these electrons would be reduced. In Cu, which marks the lowest value of ϵ in this series of elements, the effective mass of the valence electrons is only slightly greater than m_0 . However, in Cu the $3d$ shell is completed, and the low energy loss to closed

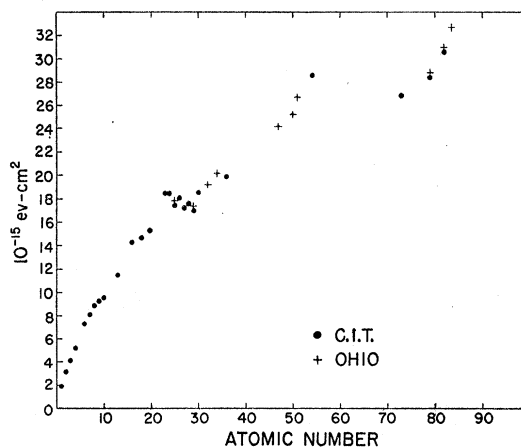


FIG. 5. The variation of atomic stopping cross section with atomic number. $E_p=500$ kev. The crosses are measurements at Ohio State by Green, Cooper, and Harris.²

²⁰ F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, New York, 1940).

TABLE V. Sources of experimental error are tabulated with the percent error in the stopping cross section contributed by each.

A. Energy measurements	
(1)	Calibration of the electrostatic beam analyzer, $\pm 0.5\%$.
(2)	Drift of magnetic spectrometer, $\pm 1\%$.
(3)	Location of midpoint of step, $\pm 2\%$.
B. Density measurements	
(1)	Weight of deposit: $\pm 0.9\%$ with balance; $\pm 1.7\%$ with chemical analysis.
(2)	Area of deposit, $\pm 1\%$.
(3)	Geometry, $\pm 0.1\%$.
(4)	Purity of sample, $\pm 0.5\%$.

shells, which can be seen at atomic numbers 10 and 18, may lower the stopping cross section of Cu.

IV. ACCURACY

The experimental uncertainties which limit the accuracy of our measurements are listed in Table V together with an estimate of the maximum percentage error in ϵ contributed by each. The largest uncertainty arises from the straggling which spreads out the step (3) in Fig. 1 and makes the location of the midpoint of the step uncertain. Although the straggling decreases as the proton energy is increased, the displacement ΔE also decreases so that the percentage uncertainty in ΔE remains fairly constant at about 2%. Mass measurements with the quartz fiber cantilever balance were reproducible to ± 1.5 micrograms, and the weight of the deposited layer was always greater than 400 micrograms. The determination of mass density of the lithium layer by chemical analysis is less accurate than the balance measurement because of the uncertainty in our estimate of the distribution of the lithium deposit over the area of the collector plate and the transmission foil. A possible source of error in these Li measurements would be the failure of the Li to deposit at equal rates on the very thin Al foil and on the 0.005-inch thick Ta plate. It was this uncertainty that led to the development of the balance method to measure the weight of the layer actually used in the energy loss measurements. Since the two methods agree, this source of error has been neglected in computing the probable error in ϵ .

For those materials that are visible against an aluminum background, the area of the layer was

measured to within a few tenths of one percent by a travelling microscope measurement of the diameter of the circular deposit. Geometrical errors, which arise from failure to hold the aluminum foil flat and normal to the proton beam, were held to less than 0.1%. The purity of all samples used in preparing the evaporated layers was 99% or better, by weight, according to the manufacturer. A further test of the purity of the evaporated layers is provided by the energy spectrum of the protons scattered from the evaporated surface. This test is very sensitive in detecting heavy contaminants which might introduce significant error in the measurement of the stopping cross section of light elements. If these various errors are compounded as independent random errors, a total probable error of ± 3 percent is found for the stopping cross sections of Li, Be, Cu, Pb, Au, LiF, and CaF_2 .

The error in the stopping cross-section ratio measurements for different materials is more difficult to estimate. Uncertainty in θ_1 and θ_2 , counting statistics, and the uncertainty in E_1 and E_2 , lead to a probable error in $\epsilon(A)/\epsilon(B)$ of ± 1 percent. However, repeated measurements of the same ratio $\epsilon(A)/\epsilon(B)$ with different samples of A and B show a spread of several percent which we believe to be caused by variations in the condition of the scattering surface. We have found that the yield of protons scattered from a surface is reduced by surface scratches which have a depth of the order of magnitude of the depth of penetration of the protons into the surface before they are scattered. Thin layers of metal condensed on optically flat glass or quartz surfaces were found to be the most satisfactory way of minimizing spurious results from this source. In the Ta measurements a 0.005-inch commercial sheet was used for the target and consequently our value of $\epsilon(\text{Ta})$ may be too high since any surface irregularities in the sheet would lead to a high value of the stopping cross section. For the values in Tables III and IV other than Ta, we believe that $\pm 5\%$ is a conservative estimate of the probable error. The values of $\epsilon(\text{Cu})$ and $\epsilon(\text{Pb})$ determined by the ratio method in Table III differ from the directly measured values in Table I by 5 percent at most.

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