Consequences of Charge Independence for Strange-Particle Reactions*

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The implications of isotopic-spin conservation for fast strange-particle reactions, especially the production processes as well as the K-particle interactions with nuclei, have been investigated. The possibility of distinguishing the Gell-Mann, Pais, and Salam-Polkinghorne theories of strange particles is also discussed.

I.

GELL-MANN[1,2] has recently proposed a theory of strange particles which accounts for their stability and copious production in terms of a new concept, *viz.*, the conservation of strangeness. A brief statement of this proposal, expressed in terms that can be readily adapted to our subsequent discussion, can be given in the following way.

We introduce two charge spaces, labeled by I_1 and I_2, respectively, of which the first is the usual isotopic-spin space. We then postulate that the various elementary particles (hyperons and mesons) carry intrinsic angular momenta $\mathbf{I}_1$ and $\mathbf{I}_2$ in these spaces and so can be characterized by the eigenvalues of the operators I_1^2, I_{1z}, I_2^2, I_{2z}, provided the latter quantities are conserved in the presence of the *strong* interactions (those couplings responsible for the production of π mesons and strange particles).

In Gell-Mann's scheme, it is supposed that the strong interactions are invariant with respect to arbitrary rotations in I_1 space, but only with respect to rotations about the z-axis in I_2 space; accordingly, for this case, I_1, I_{1z}, and I_{2z} are good quantum numbers. The conservation of I_{1z} and I_{2z}, which takes place even in the presence of *electromagnetic* interactions, implies that charge is conserved, since the charge Q of a particle is given by the relation

$$Q=I_{1z}+I_{2z}+\tfrac{1}{2}t, \tag{1}$$

where $t=1$, 0, and -1 for fermions, bosons, and antifermions, respectively. The "strangeness" quantum number S is directly related to I_{2z} by the equality $S=2I_{2z}$. Upon introduction of the *weak* interactions, which account for the instability of the strange particles, I_{1z} and I_{2z} (and hence S) are no longer separately conserved.

A trivial variation of the Gell-Mann scheme can now be obtained by using, in place of Eq. (1), the following definition for the charge:

$$Q=I_{1z}+I_{2z}', \tag{2}$$

i.e., I_{2z} and I_{2z}' are related by the equation

$$I_{2z}'=I_{2z}+\tfrac{1}{2}t. \tag{3}$$

Upon introducing a new strangeness quantum number $S'=2I_{2z}'$, we notice that, whereas the "ordinary" particles [nucleons ($\mathfrak{N}$) and π mesons] are characterized in the original Gell-Mann formulation by strangeness $S=0$, we now have $S'=1$ and 0 for $\mathfrak{N}$ and π, respectively. In Table I, we have listed the assignments of I_1, I_{2z}, and I_{2z}' for the various types of elementary particles.

II.

A generalization of Gell-Mann's scheme can be obtained by postulating invariance of the strong interactions under arbitrary rotations in both I spaces, independently. This is essentially the result contained in Pais's theory.[2,3] However, there are two ways of proceeding with this generalization, depending upon which of the two definitions of charge [(1) or (2)] is adopted, and the distinction between the resultant classifications of the elementary particles is no longer trivial. Pais's original theory is based on Eq. (1); the variation of this theory that is obtained from Eq. (2) was given recently by Salam and Polkinghorne (referred to henceforth as SP).[4]

The magnitude of the intrinsic spin in I_2 space will now be a good quantum number. If we denote this quantity by I_2 or I_2', according to whether the charge is defined by Eq. (1) or Eq. (2), and assume that I_2 and I_2' take on the minimal values allowed by Table I, we obtain the assignments of quantum numbers given in Table II.[5] It will be noticed that, whereas the Gell-Mann scheme deals with displaced charge multiplets,

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1 M. Gell-Mann, Phys. Rev. **92**, 833 (1953); also a paper entitled "The Interpretation of the New Particles as Displaced Charge Multiplets" (to be published).

2 M. Gell-Mann and A. Pais, Proceedings of the 1954 Glasgow Conference (Pergamon Press, London, 1955), p. 342.

3 A. Pais, Proc. Nat. Acad. Sci. **40**, 484 (1954); *Proceedings of the Fifth Annual Rochester Conference*, 1955 (Interscience Publishers, Inc., New York, 1955), p. 131.

4 A. Salam and J. C. Polkinghorne, Nuovo cimento **2**, 685 (1955).

5 SP (reference 4) distinguish between the θ and τ meson by making the assignments $I_1=\frac{1}{2}$, $I_2'=\frac{1}{2}$ and $I_1=0$, $I_2'=1$ for these two particles, respectively. However, the observation of the production mechanism $\mathfrak{N}+\mathfrak{N}\to\mathfrak{N}+\Sigma+\tau$ by P. S. Goel and K. A. Noelakantan [see the Report on the Pisa Conference by R. E. Marshak, Atomic Energy Commission Report NYO-7138 (unpublished)] would seem to preclude this assignment for the τ meson. Accordingly, following Gell-Mann and Pais, we treat the θ and τ meson on an equal footing.

the essential feature of Pais's theory is that one now has double multiplets.

III.

As is well known, the assumption that the $\mathfrak{N}$-$\mathfrak{N}$ and π-$\mathfrak{N}$ interactions are charge-independent (or, equivalently, conserve isotopic spin) leads to important relations for reactions involving pions and nucleons.[6–8] In the theories of strange particles, we may expect analogous restrictions to appear when only strong interactions are involved.

The situation is now somewhat more complicated, however. So far as the implications of the conservation of ordinary isotopic spin are concerned—we also refer to this symmetry property as charge independence of the first kind or CI_1—, the results will clearly be the same for all three schemes—Gell-Mann, Pais, SP. In the Pais and SP theories, we have also to deal with the conservation of isotopic spin in I_2 space, i.e., with charge independence of the second kind (CI_2). One has therefore, in principle, a means of distinguishing all three classification schemes from one another.

TABLE I. Classification of elementary particles according to the Gell-Mann scheme.

Particle		I_1	I_{2z}	I_{2z}'
Hyperons:	$\mathfrak{N}$	$\frac{1}{2}$	0	$\frac{1}{2}$
	Λ	0	$-\frac{1}{2}$	0
	Σ	1	$-\frac{1}{2}$	0
	Ξ	$\frac{1}{2}$	-1	$-\frac{1}{2}$
Mesons:	π	1	0	0
	K	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	$\bar{K}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

An essential question which needs to be considered at this point, however, is this: How is one to reconcile CI_2 with the facts that Pais's formulation contains too many particles, some of which are multiply charged, and that SP assign $\mathfrak{N}$ and Ξ, with widely differing masses, to the same double multiplet? To answer this question, we assume, first, with Pais, that the superfluous particles that appear in his scheme are sufficiently massive so that they decay quickly (in other words, the mass degeneracy with respect to I_{2z} is supposed to be lifted). Secondly, we will conjecture that the interactions that give rise to the mass differences that now appear in both the Pais and SP theories do not alter appreciably the charge-independent character of the interparticle forces at the energies under consideration (these interactions may be with fields of very heavy quanta, say).

Hence, it will be assumed that CI_2 is still applicable, in the usual way, to reactions involving strange particles, except that one must take notice of the fact that the matrix elements depend explicitly on the masses of the incoming and outgoing particles. A comparison of a set of reactions that are related by CI_2 but whose outgoing products, say, do not have corresponding masses is then impossible (except at energies that are sufficiently high so as to render the mass differences negligible) unless one knows the functional form of the

[6] K. M. Watson, Phys. Rev. **85**, 852 (1952).
[7] Van Hove, Marshak, and Pais, Phys. Rev. **88**, 1211 (1952); L. Van Hove, Atomic Energy Commission Report NYO-3704, 1952 (unpublished).
[8] D. Feldman, Phys. Rev. **89**, 1159 (1953).

TABLE II. Classification of elementary particles according to Pais and Salam-Polkinghorne.

Particle		I_1	I_2	I_2'
Hyperons:	$\mathfrak{N}$	$\frac{1}{2}$	0	$\frac{1}{2}$[a]
	Λ	0	$\frac{1}{2}$	0
	Σ	1	$\frac{1}{2}$	0
	Ξ	$\frac{1}{2}$	1	$\frac{1}{2}$[a]
Mesons:	π	1	0	0
	$K, \bar{K}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

[a] Since $\mathfrak{N}$ and Ξ bear the same quantum numbers in the SP theory, they are assigned to the same double multiplet.

TABLE III. Implications of CI_1 for π-$\mathfrak{N}$, π-d and $\mathfrak{N}$-$\mathfrak{N}$, $\mathfrak{N}$-d reactions.

	Reaction	Relation
1*a*	$\pi^+p \to \Sigma^+K^+$	
b	$\pi^-p \to \Sigma^0K^0$	$\Delta(a,2b,c) \geqslant 0$[a]
c	$\to \Sigma^-K^+$	
2*a*	$\pi^+p \to \Lambda^0K^+\pi^+$	
b	$\pi^-p \to \Lambda^0K^0\pi^0$	$\Delta(a,2b,c) \geqslant 0$
c	$\to \Lambda^0K^+\pi^-$	
3*a*	$\pi^+p \to \Sigma^+K^+\pi^0$	
b	$\to \Sigma^+K^0\pi^+$	
c	$\to \Sigma^0K^+\pi^+$	
d	$\pi^-p \to \Sigma^+K^0\pi^-$	$\Delta(2a,b,2c) \geqslant 0$[b]
e	$\to \Sigma^0K^+\pi^-$	
f	$\to \Sigma^0K^0\pi^0$	
g	$\to \Sigma^-K^+\pi^0$	
h	$\to \Sigma^-K^0\pi^+$	
4*a*	$\pi^+p \to pK^+\bar{K}^0$	
b	$\pi^-p \to pK^0K^-$	$(2c+2d)^{\frac{1}{2}} \geqslant \lvert a^{\frac{1}{2}}-b^{\frac{1}{2}}\rvert$
c	$\to nK^+K^-$	
d	$\to nK^0\bar{K}^0$	
5*a*	$\pi^+p \to \Xi^0K^+K^+$, etc.	Same as 4
6*a*	$\pi^+d \to p\Sigma^+K^0$	
b	$\to p\Sigma^0K^+$	$\Delta(a,2b,c) \geqslant 0$
c	$\to n\Sigma^+K^+$	
7*a*	$pp \to p\Lambda^0K^+$	
b	$np \to p\Lambda^0K^0$	$\Delta(a,b,c) \geqslant 0$
c	$\to n\Lambda^0K^+$	
8*a*	$pp \to p\Sigma^+K^0$	
b	$\to p\Sigma^0K^+$	
c	$\to n\Sigma^+K^+$	
d	$np \to p\Sigma^0K^0$	$a+c+e+f=2(b+d+g)$[c]
e	$\to p\Sigma^-K^+$	
f	$\to n\Sigma^+K^0$	
g	$\to n\Sigma^0K^+$	
9*a*	$pp \to dK^+\bar{K}^0$	
b	$np \to dK^0\bar{K}^0$	$\Delta(a,b,c) \geqslant 0$
c	$\to dK^+K^-$	
10*a*	$pd \to d\Sigma^+K^0$	$a=2b$
b	$\to d\Sigma^0K^+$	

[a] We use the notation $\Delta(a,b,c) \geqslant 0$ to denote the three triangular inequalities $a^{\frac{1}{2}}+b^{\frac{1}{2}}-c^{\frac{1}{2}} \geqslant 0$, $b^{\frac{1}{2}}+c^{\frac{1}{2}}-a^{\frac{1}{2}} \geqslant 0$, $c^{\frac{1}{2}}+a^{\frac{1}{2}}-b^{\frac{1}{2}} \geqslant 0$.
[b] There is also an equality relating the eight cross sections, which is of the form of a phase relationship (reference 7). This equality is lost, however, when one deals with unpolarized particles (reference 8).
[c] There are also inequalities for this case, e.g., $\Delta(a,2b,c) \geqslant 0$, etc.

TABLE IV. Implications of CI_1 for K-$\mathfrak{N}$ and K-d reactions.

	Reaction	Relation
1a	$K^-p \to \Lambda^0\pi^0$	
b	$K^-n \to \Lambda^0\pi^-$	$2a=b$[a]
2a	$K^-p \to \Sigma^+\pi^-$	
b	$\to \Sigma^0\pi^0$	
c	$\to \Sigma^-\pi^+$	$a+c+e=2(b+d)$
d	$K^-n \to \Sigma^0\pi^-$	$a+c+d=2(b+e)$[b]
e	$\to \Sigma^-\pi^0$	
3a	$K^-p \to \Lambda^0\pi^+\pi^-$	
b	$\to \Lambda^0\pi^0\pi^0$	
c	$\to \Lambda^0\pi^-\pi^+$	Same as 2
d	$K^-n \to \Lambda^0\pi^0\pi^-$	
e	$\to \Lambda^0\pi^-\pi^0$	
4a	$K^-n \to K^-n$	
b	$K^-p \to K^-p$	$\Delta(a,b,c) \geqslant 0$[c]
c	$\to \bar{K}^0n$	
5a	$K^-p \to \Sigma^+\pi^0\pi^-$	
b	$\to \Sigma^+\pi^-\pi^0$	
c	$\to \Sigma^0\pi^+\pi^-$	$a+b+f+g+h+k+l+m$ $=2(c+d+e+i+j)$
d	$\to \Sigma^0\pi^0\pi^0$	
e	$\to \Sigma^0\pi^-\pi^+$	$b+c+e+f+h+j+k+m$ $=2(a+d+g+i+l)$
f	$\to \Sigma^-\pi^+\pi^0$	
g	$\to \Sigma^-\pi^0\pi^+$	$a+c+e+g+h+i+k+m$ $=2(b+d+f+j+l)$
h	$K^-n \to \Sigma^+\pi^-\pi^-$	
i	$\to \Sigma^0\pi^0\pi^-$	
j	$\to \Sigma^0\pi^-\pi^0$	
k	$\to \Sigma^-\pi^+\pi^-$	
l	$\to \Sigma^-\pi^0\pi^0$	
m	$\to \Sigma^-\pi^-\pi^+$	
6a	$K^-n \to n\pi^-\bar{K}^0$, etc.	Same as 8, Table III[c]
7a	$K^-n \to K^0\Xi^-$, etc.	Same as 4
8a	$K^-n \to \Xi^-\pi^-K^+$, etc.	Same as 8, Table III
9a	$K^-p \to \Lambda^0K^+K^-$	
b	$\to \Lambda^0K^0\bar{K}^0$	$\Delta(a,b,c) \geqslant 0$
c	$K^-n \to \Lambda^0K^0K^-$	
10a	$K^+n \to \Lambda^0K^0K^+$, etc.	Same as 9
11a	$K^-n \to K^0\Sigma^-\bar{K}^0$, etc.	Same as 8, Table III
12a	$K^+p \to K^+\Sigma^+K^0$, etc.	Same as 8, Table III
13a	$K^-d \to \Sigma^0n$	
b	$\to \Sigma^-p$	$2a=b$[d]
14a	$K^-d \to \Lambda^0n\pi^0$	$2a=b$[a]
b	$\to \Lambda^0p\pi^-$	
15a	$K^-d \to \Sigma^+n\pi^-$	
b	$\to \Sigma^0p\pi^-$	$a+d+e=2(b+c)$
c	$\to \Sigma^0n\pi^0$	
d	$\to \Sigma^-p\pi^0$	$a+b+e=2(c+d)$[e]
e	$\to \Sigma^-n\pi^+$	
16a	$K^-d \to pnK^-$	
b	$\to npK^-$	$\Delta(a,b,c) \geqslant 0$[c]
c	$\to nn\bar{K}^0$	
17a	$K^-d \to dK^-\pi^0$	$2a=b$[c]
b	$\to d\bar{K}^0\pi^-$	
18a	$K^-d \to p\Xi^-K^0$, etc.	Same as 16
19a	$K^-d \to \Lambda^0\Sigma^0K^0$	$2a=b$
b	$\to \Lambda^0\Sigma^-K^+$	
20a	$K^-d \to \Sigma^+K^0\Sigma^-$, etc.	Same as 15

[a] See reference 1.
[b] There also exist inequalities for this case, e.g., $\Delta(a,4b,c) \geqslant 0$ (see reference 11).
[c] Similar relations are obtained when K^+ particles are used as projectiles.
[d] See reference 11.
[e] These relations imply the equality $b=d$, which has been given by Lee (reference 9); one has also some inequalities, e.g., $\Delta(a,4c,e) \geqslant 0$.

matrix elements for the various isotopic-spin eigenstates involved.

The arguments of the preceding paragraphs are, of course, highly speculative and have clearly been introduced so as to provide some plausibility for the retention of I_2 as a good quantum number in the Pais and SP proposals. For, otherwise, the generalizations inherent in the latter theories do not constitute any improvement over the Gell-Mann scheme.

IV.

We have examined the implications of CI_1 and CI_2 for all possible strange-particle reactions in which one has an incident pion, nucleon, or K particle impinging on a target nucleon or deuteron, subject only to the limitation that no more than three particles shall emerge in any reaction. The results are tabulated in Tables III to VI. The consequences of CI_1 for some of the reactions listed have been studied previously by several authors.[1,9–11]

The constraints imposed by charge independence take the form of equalities and inequalities which relate the differential cross sections for reactions that differ from one another solely in the assignment of I_{1z} (or I_{2z}) to the particles involved. Clearly, reactions for which one can deduce relations in the form of equalities are to be preferred as a test of charge independence. For such reactions, one can generally also derive weaker relations in the form of inequalities, some of which have been noted in the footnotes to the tables. All the relations given in the tables are applicable to reactions involving either polarized or unpolarized particles. They are also valid when applied to total cross sections except that some relations may be lost (one can then, for example, no longer distinguish $K^-+p\to\Lambda^0+\pi^++\pi^-$ from $K^-+p\to\Lambda^0+\pi^-+\pi^+$); one must also exercise the customary care in defining total cross sections when identical particles are emitted.

The symmetry property that we have denoted by CI_1 also implies, as a special case, invariance under rotations through 180° about the x-axis, say, in I_1 space (charge symmetry of the first kind or CS_1). For the sake of brevity, we have not listed reactions that are related by CS_1 only; these are quite familiar and are readily recognized. On the other hand, we have not ignored the implications of CS_2; indeed, most of the relations deduced for Pais's theory and ascribed to CI_2

TABLE V. Implications of CI_2 for π-$\mathfrak{N}$, π-d, and $\mathfrak{N}$-$\mathfrak{N}$, $\mathfrak{N}$-d reactions.

	Reaction	Relation: Pais	Relation: SP
1a	$\pi\mathfrak{N} \to \mathfrak{N}K\bar{K}$		
b	$\to \mathfrak{N}\bar{K}K$	$a=b$	$\Delta(a,b,c) \geqslant 0$
c	$\to \Xi KK$		
2a	$\pi d \to dK\bar{K}$	$a=b$	
b	$\to d\bar{K}K$		
3a	$\mathfrak{N}\mathfrak{N} \to dK\bar{K}$	$a=b$	
b	$\to d\bar{K}K$		

[9] T. D. Lee, Phys. Rev. **99**, 337 (1955).
[10] S. Gasiorowicz, University of California Radiation Laboratory Report No. UCRL-3074, 1955 (unpublished).
[11] Case, Karplus, and Yang, Phys. Rev. **101**, 358 (1956).

are equally correct under the less stringent requirement of CS_2.

V.

We conclude this note with several miscellaneous remarks.

(1). The implications of CI_1 and CI_2 for strange-particle reactions are valid even if the K-particle beams contain τ mesons. The only precaution that needs to be observed is that the proportion of such τ mesons to be found among incident K particles must be kept constant for a series of comparable reactions.

(2). The equalities that are implied by CI_1 all have the form of Watson's relation,[6] which may be phrased in the following way. Suppose we have a proton or charged K particle incident on a target nucleon which is, with equal probability, a neutron or a proton,[12] and consider a set of reactions that are identical with one another except for the I_{1z} assignment of the target and outgoing particles. Then, denoting by ν_+, ν_-, and ν_0 the number of positive, negative, and neutral π mesons (or Σ particles[13]) emitted into a given solid angle, we have

$$\nu_+ + \nu_- = 2\nu_0. \tag{4}$$

Although the utility of these relations with respect to Σ particles may well be limited (because of the difficulty in distinguishing Σ^0 from Λ^0), this is not the case for π mesons. A test of the validity of CI_1 for strange particles can thus be made by counting the charged and neutral π mesons emitted in the absorption of K^- particles that have been brought to rest in d.[14]

(3). Although we have assumed specifically that the target nucleus is $\mathfrak{N}$ or d, our results dealing with the implications of CI_1 can be readily generalized so as to be applicable to the use of other light nuclei as targets. Thus, by way of example, in every reaction in which we have a deuteron target and a nucleon or deuteron appearing as one of the reaction products, we can make the replacement $d \to He^4$, $p \to He^3$, $n \to H^3$; the utility of k^--He^4 reactions as a test of CI_1 has recently been noted by Lee.[9]

TABLE VI. Implications of CI_2 for K-$\mathfrak{N}$ and K-d reactions.

	Reaction	Relation: Pais	Relation: SP
1a	$K\mathfrak{N} \to K\mathfrak{N}$		
b	$\bar{K}\mathfrak{N} \to \bar{K}\mathfrak{N}$	$a=b$	$\Delta(a,b,c) \geqslant 0$
c	$\to K\Xi$		
2a	$K\mathfrak{N} \to K\mathfrak{N}\pi$		
b	$\bar{K}\mathfrak{N} \to \bar{K}\mathfrak{N}\pi$		Same as 1
c	$\to K\Xi\pi$		
3a	$K\mathfrak{N} \to \Lambda KK$		
b	$\bar{K}\mathfrak{N} \to \Lambda\bar{K}K$	$\Delta(a,b,c) \geqslant 0$	$\Delta(a,b,c) \geqslant 0$
c	$\to \Lambda K\bar{K}$		
4a	$K\mathfrak{N} \to \Sigma KK$		
b	$\bar{K}\mathfrak{N} \to \Sigma\bar{K}K$		Same as 3
c	$\to \Sigma K\bar{K}$		
5a	$Kd \to Kd$	$a=b$	
b	$\bar{K}d \to \bar{K}d$		
6a	$Kd \to \mathfrak{N}\mathfrak{N}K$		
b	$\bar{K}d \to \mathfrak{N}\mathfrak{N}\bar{K}$	$a=b$	$(2c+2d)^{\frac{1}{2}} \geqslant \lvert a^{\frac{1}{2}} - b^{\frac{1}{2}} \rvert$
c	$\to \mathfrak{N}\Xi K$		
d	$\to \Xi\mathfrak{N}K$		
7a	$Kd \to dK\pi$	$a=b$	
b	$\bar{K}d \to d\bar{K}\pi$		

(4). Perhaps the simplest test of CI_2 in Pais's theory would consist of a comparison of the elastic scattering of K^+ and K^- mesons by self-conjugate nuclei, e.g.,

$$K^+ + d \to K^+ + d, \tag{5a}$$

$$K^- + d \to K^- + d; \tag{5b}$$

the equality of the two cross sections is actually based on CS_1 and CS_2 [Table VI (5)]. As has already been noted, a test of the various inequalities that are listed for the SP theory would have to be performed at energies high enough so as to render the $\mathfrak{N}$-Ξ mass difference unimportant, except for reactions (3) and (4) of Table VI for which, curiously enough, the implications are the same in both the Pais and SP theories.

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[12] More generally, the state of the target must be isotropic in isotopic-spin space; the target can therefore also be d, He^4, C^{12}, etc.

[13] These are the only two species of elementary particles that are assigned spin unity in I_1 space.

[14] The only reactions that are then energetically capable of producing π mesons are those listed in Table IV (14 and 15) and $K^- + d \to \Lambda^0 + n + \pi^+ + \pi^-$, etc.; for the latter case, there are two equalities similar to those given in Table IV (15).