The results obtained by this latter method seem to favor the existence of spins greater than $\frac{1}{2}$. We hope that improved statistics may soon settle this important question.

Note added in proof.—W. D. Walker and W. D. Shepard have recently reported some new evidence for angular correlations in the Λ^0 decay [Phys. Rev. **101**, 1810 (1956)].

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Z Dependence of Bremsstrahlung for the Case of Complete Screening*

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The relative yield of 235-Mev photons from the elements Cu, Ta, and U for incident electrons of energy 500 Mev has been measured by using the photoproduction of positive pions from liquid hydrogen as an essentially monoenergetic photon detector. The observed photon yield per radiation length of uranium relative to copper is 0.878 ± 0.02, and of tantalum relative to copper is 0.937 ± 0.015. These results are consistent with the predictions of a theorem of Olsen which evaluates the integral bremsstrahlung cross section in terms of the integral pair-production cross section of Bethe, Maximon, and Davies.

I. INTRODUCTION

THE Bethe-Heitler theory[1] of bremsstrahlung and pair production as developed in 1934 has been superseded recently by the calculations of Bethe, Maximon, and Davies,[2–5] with some important contributions by Bethe, Maximon, and Low,[6] and by Olsen.[7] The essential difference between the present theories of Bethe, Maximon, and Davies and the Bethe-Heitler theory is that the current calculations are based on the use of Furry-type wave functions; whereas the original Bethe-Heitler calculations are based on the Born approximation which represents the electron wave functions in the initial and final states as plane waves and hence ignores the Coulomb distortion effect.

In the first calculations for bremsstrahlung made by Bethe, Maximon, and Davies,[2,3] both the initial and final electron wave functions were represented by a plane wave plus an outgoing spherical wave. In a later article Bethe, Maximon, and Low[6] pointed out that the final electron wave function should be represented instead by a plane wave plus an ingoing spherical wave. This was the basis for the later papers of Bethe, Maximon, and Davies[4,5] in which they concluded (in contradiction to their previous result[2,3]) that, in the limit of complete screening, their formulation for bremsstrahlung went over into the old Bethe-Heitler result. Since then, Olsen[7] has shown that the particular choice of the final state wave function of the electron is important if the differential bremsstrahlung cross section for a given direction of the scattered particle is to be calculated. However, if only the cross section integrated over all possible directions of the scattered particle is desired, then one can equally well use the plane wave plus outgoing spherical wave-type function for the final state. This permits the integral bremsstrahlung cross section to be deduced from the corresponding pair cross section by a detailed balancing transformation. Conversely, by the inverse transformation, the integral pair cross section may be derived from a knowledge of the corresponding bremsstrahlung cross section. This conclusion holds whether the screening is absent, partial, or complete. Using the Davies, Bethe, and Maximon result for pair production (which has had numerous experimental checks at high energies[5]), Olsen gives the following expression for the bremsstrahlung spectrum at high energies:

$$(d\sigma/dk)=(d\sigma/dk)_{\mathrm{BH}}-\frac{4\bar{\phi}}{k}\frac{1}{E_0^2}(E_0^2+E^2-\tfrac{2}{3}E_0E)f(Z),\quad(1)$$

where $(d\sigma/dk)_{\mathrm{BH}}$ is the Bethe-Heitler bremsstrahlung spectrum including the effect of atomic screening[1]; E_0 and E are the initial and final total electron energies; k is the photon energy; $\bar{\phi}=(Z^2/137)(e^2/mc^2)^2$; and $f(Z)$

* The research reported here was supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
[1] H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).
[2] L. C. Maximon and H. A. Bethe, Phys. Rev. **87**, 156 (1952).
[3] Handel, Davies, and Bethe, Phys. Rev. **87**, 156 (1952).
[4] H. A. Bethe and L. C. Maximon, Phys. Rev. **93**, 768 (1954).
[5] Davies, Bethe, and Maximon, Phys. Rev. **93**, 788 (1954).
[6] Bethe, Maximon, and Low, Phys. Rev. **91**, 417 (1954).
[7] H. Olsen, Phys. Rev. **99**, 1335 (1955).

is the same as is given by Davies, Bethe, and Maximon[5] for the pair cross section case. The Coulomb correction, which is given by the last term in Eq. (1), is, therefore, the same whether the screening is absent, partial, or complete.

As a direct consequence of the Olsen theorem and the pair production results of Bethe, Maximon, and Davies, it may be concluded, also, that the error in $(d\sigma/dk)$ as given by Eq. (1) should be less than $1/E$ and that the form of the bremsstrahlung spectrum is essentially unchanged by the Coulomb correction. In particular, in the limit of complete screening the percentage deviation from the Bethe-Heitler theory is given, to a good approximation, for both bremsstrahlung and pair production, by the expression $[100f(Z)/\log(183Z^{-\frac{1}{3}})]$ percent. Therefore, in the limit of complete screening, this result for the integral bremsstrahlung cross section is in contradiction to the 1954 Bethe and Maximon paper,[4] but is in agreement with their earlier results.[2,3]

The present experiment was begun before the results of Olsen were known, and had as its objective the experimental confirmation or denial of a deviation from Bethe-Heitler theory for the case of complete screening. A previous measurement in this laboratory, using Cu, Ta, Pb, and U radiators, had already confirmed a deviation in the region of incomplete screening[8] which, incidentally, is in good agreement with the Bethe, Maximon, and Davies, and Olsen predictions, as may be seen from Fig. 2. Previous results of Lanzl and Hanson,[9] Curtis,[10] and Fisher[11] have all shown negative deviations from Bethe-Heitler theory which are of the right order of magnitude to agree with Olsen's result; Fisher's data, especially, are in excellent agreement with Olsen's theorem. All of these data are summarized in Fig. 2.

II. EXPERIMENTAL METHOD

As it was pertinent to perform an experiment in the region of complete screening, or at least in a region approaching complete screening, the following experimental scheme was adopted which provided an essentially monoenergetic photon detector: The relative yield of photons produced by the elements Cu, Ta, and U for incident electrons of energy 500 Mev was detected by observing the photoproduction of positive pions from a liquid-hydrogen target. The pions were selected in both energy and angle so that only those pions produced by photons within the energy range 234 ± 6 Mev were detected. For these conditions, the screening parameter

$$\gamma = 100mc^2k/E_0EZ^{\frac{1}{3}}$$

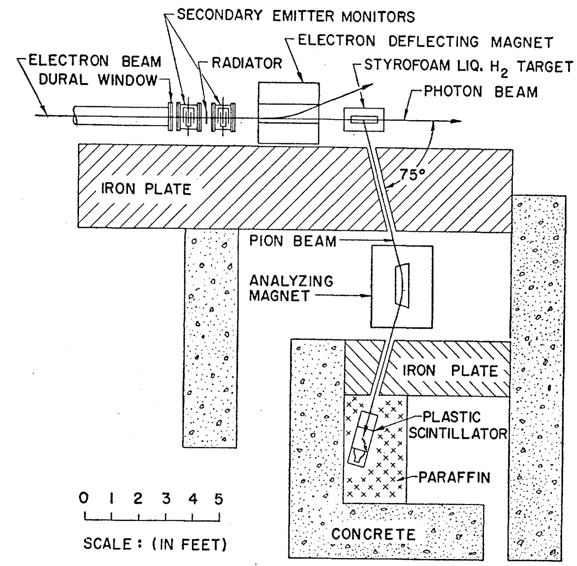

FIG. 1. Photon detection apparatus showing the pion geometry and target configuration.

is equal to 0.029, which is very close to the complete screening requirement that $\gamma=0$.

The 500-Mev electrons were extracted from the Stanford Mark III linear accelerator[12] by a double-deflecting magnetic-analyzing system.[13] This system provided an intense beam of purified electrons (i.e., free of neutron and gamma-ray contamination) of known energy which then passed through the experimental equipment shown in Fig. 1.

The basic scheme was to produce bremsstrahlung in the Cu, Ta, and U radiators which followed the first secondary emitter monitor. The electrons were then deflected to the left by the sweeping magnet and the photon flux passed through the liquid-hydrogen target. The hydrogen target was viewed by a channel 2 in. wide by 4 in. high at 75° with respect to the incident photon flux. The analyzing magnet was adjusted to accept 68-Mev positive pions which were then channeled into the plastic scintillator and detected by observing the positrons from the $\pi-\mu-e$ decay. This meson detection scheme has been described in a previous article.[14,15] The over-all resolution of the equipment was such that positive pions of 68 ± 5 Mev corresponding to photons of 234 ± 6 Mev were detected. Hence, one had an essentially monoenergetic photon detector whose characteristics were independent of the radiator.

This experiment was sensitive only to the consistency of the meson and electron energies used and was quite insensitive to a knowledge of their absolute values.

[8] Barber, Berman, Brown, and George, Phys. Rev. **99**, 59 (1955).
[9] L. H. Lanzl and A. O. Hansen, Phys. Rev. **83**, 959 (1951).
[10] C. D. Curtis, Phys. Rev. **89**, 123 (1953).
[11] P. C. Fisher, Phys. Rev. **92**, 420 (1953).
[12] Chodorow, Ginzton, Hansen, Kyhl, Neal, and Panofsky, Rev. Sci. Instr. **26**, 134 (1955).
[13] W. K. H. Panofsky and J. A. McIntyre, Rev. Sci. Instr. **25**, 287 (1954).
[14] Crowe, Friedman, and Motz, Phys. Rev. **98**, 268(A) (1955).
[15] Panofsky, Newton, and Yodh, Phys. Rev. **98**, 751 (1955).

The primary electron beam energy was held constant to better than ± 0.1 of one percent throughout the experimental runs and the current through the meson-analyzing magnet was stabilized to approximately the same precision.

The size of the electron beam impinging upon the radiators was approximately $\frac{3}{8}$ in. by $\frac{3}{4}$ in. The hydrogen target (also described elsewhere[16]) consisted of an aluminum cup of 0.002-in. wall, $5\frac{1}{2}$ in. long, 3 in. wide, and 7 in. high. This cup was surrounded by a layer of Styrofoam, a liquid nitrogen jacket, and a second layer of Styrofoam. In order to avoid undesirable background, the nitrogen jacket was designed with large windows at the points of entry and exit of the photon beam. The liquid hydrogen consumption was found to be 2 to $2\frac{1}{2}$ liters per hour.

The relative electron beam intensity (integrated charge) passing through the Cu, Ta, and U radiators was monitored by secondary-electron monitors before and after the radiators. These monitors[17] are essentially energy-independent and have proved to be accurately reproducible. The ratio of the integrated charge of the two monitors was found to remain constant independently of the radiator between the monitors to within better than $\frac{1}{2}\%$ throughout the duration of each run.

A total of five independent runs were carried out using three different sets of Cu, Ta, and U radiators. The first three deviated from the above description in that the sweeping magnet was not available. In these, the electron beam also passed through the hydrogen target and the second beam monitor was positioned after the hydrogen target rather than before. Throughout the experiment the data were taken by cycling the Cu, Ta, and U radiators (plus a run without a radiator to evaluate background) so as to minimize the effects of drifts in the electronic equipment. The Cu, Ta, and U radiators, of approximately 0.03 radiation-length thickness, were always chosen to be as nearly equal in thickness (radiation lengths) as possible. Under these conditions, the corrections due to electron energy loss in the radiators and to multiple scattering were negligible.

The pions were counted through the decay positrons from the muons. The basic idea of the electronics has been recorded,[14,15] but was somewhat modified from the previous arrangement. The output pulses from the photomultiplier were recorded simultaneously in four delayed-time gates after the beam pulse and in three integral discriminator channels such that one had a 3×4 coincidence matrix. The first gate was delayed approximately 2.5 μsec after the electron beam pulse of the machine and had a gate width of 0.5 μsec. The second and third gates followed in succession after the first gate and had widths of 0.7 and 1.0 μsec, respectively. The fourth channel of 10-μsec duration was delayed 10 μsec after the beam pulse and served to evaluate background arising from thermal neutrons. This choice of gate lengths yielded an approximately equal number of counts in the first three time channels inasmuch as one is observing the 2.2-μsec decay of the muons.

Let C_{Cu}, C_{Ta}, C_{U}, and C_o be the observed counts per integrated charge from the Cu, Ta, U radiators and the radiator-out cases. Then the quantities of interest are the ratios

$$Y\left(\frac{\mathrm{Ta}}{\mathrm{Cu}}\right)=\frac{C_{\mathrm{Ta}}-C_o}{C_{\mathrm{Cu}}-C_o} \quad \text{and} \quad Y\left(\frac{\mathrm{U}}{\mathrm{Cu}}\right)=\frac{C_{\mathrm{U}}-C_o}{C_{\mathrm{Cu}}-C_o}. \qquad (2)$$

It is important to understand the origin of the background C_o. These "radiator-out" counts arose primarily from the bremsstrahlung produced in the Dural output window, the secondary emitter monitors, and in the air spaces between these components (see Fig. 1). In the first three runs, when the sweeping magnet was not in use, a substantial contribution to C_o also arose from the electron-pion production in the hydrogen target and from the bremsstrahlung produced in the target. However, all of these sources of C_o remained constant throughout any given experiment and, hence, C_o could be determined accurately even though its magnitude was large. Furthermore, the importance of C_o was minimized by selecting the radiator-thicknesses so as to produce as nearly as possible the same number of photons from each radiator.

In the first three runs (without the sweeping magnet) C_o was found to be approximately one-half of the observed counts with the radiators in. However, when the sweeping magnet became available, this dropped to one-fourth. As may be seen from the data in Table I, the outcome of the experiment was the same with or without the sweeping magnet. Background runs were, of course, made with the liquid-hydrogen target empty (i.e., hydrogen gas only). The resulting count rate for hydrogen gas was, in all cases, only a few percent of the full-target rate and was furthermore found to be proportional to the "target full" count rates. An additional statistical error must be added to the results to

TABLE I. Summary of the experimental results showing the consistency of the five independent runs. $R(E_0,Z)$ is the ratio of the observed bremsstrahlung yield from an element Z to the observed yield from copper, all normalized to the yields predicted by the Bethe-Heitler theory.

Run	$R(E_0,Z)$ Tantalum/copper	Uranium/copper
A	0.940 ± 0.022	0.873 ± 0.025
B	0.921 ± 0.028	0.793 ± 0.032
C	0.938 ± 0.023	0.917 ± 0.025
D	0.978 ± 0.045	0.927 ± 0.038
E	0.932 ± 0.026	0.876 ± 0.025
Average	0.937 ± 0.015	0.878 ± 0.02

[16] Panofsky, Woodward, and Yodh (to be published).
[17] G. W. Tautfest and H. R. Fechter, Rev. Sci. Instr. **26**, 229 (1955).

account for nonuniformities in foil thicknesses. This amounted to ±1% for the Cu-Ta data and $1\frac{1}{2}$% for the U-Cu data. After each independent run, 1-in. diameter circles were punched from the foils with their centers corresponding to the location of the beam center (this could be located very accurately) and then weighed. This weight was used to determine the best density of the foils and the deviation of the density of larger areas from this value was used to evaluate the magnitude of the foil nonuniformities. These nonuniformities were the determining factors in the ultimate precision of the experiment. An additional 1% absolute correction was applied to the uranium data to account for oxygen content.[18]

RESULTS

The corrected results of the five independent runs are tabulated in Table I. The quantity $R(E_0,Z)$ is the ratio of the corrected experimental bremsstrahlung yields of an element Z to that of copper normalized to the theoretically expected yields using Bethe-Heitler theory. The Wheeler-Lamb[19] correction for the contribution of atomic electrons to the bremsstrahlung yield was also included in these data. Figure 2 gives a plot of the quantity $R(E_0,Z)$ as a function of $Z^2/100$. It is clear from Fig. 2 that the present data substantiate the Olsen theorem (which utilized the Bethe, Maximon, and Davies result for pair production). However, the data are not sufficiently precise to prove or disprove the slight deviation from a Z^2 law which the function $f(Z)$ of Bethe, Maximon, and Davies predicts.[5] Therefore, one is justified in representing the 500-Mev data as showing a $-(1.42\pm0.2)\times10^{-3}Z^2$% deviation from Bethe-Heitler theory. Also plotted in Fig. 2 are the previous results of Fisher, Curtis, Lanzl, and Hanson, and of the Mark II group in this laboratory, all of which tend to substantiate the Olsen theorem.

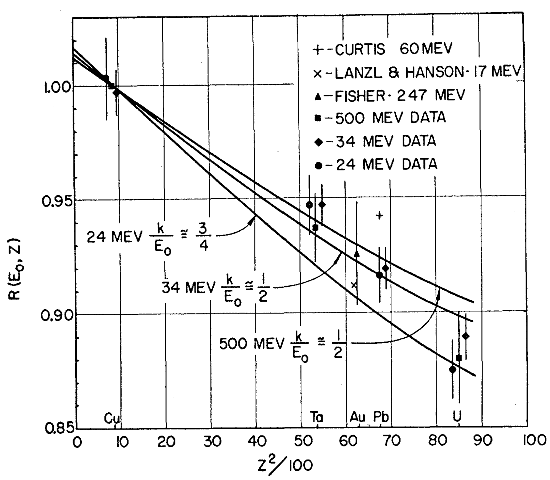

FIG. 2. The data labeled with the squares represent the results of this experiment. The experimental points are the ratios of the observed bremsstrahlung yield of an element Z to that of copper, normalized to the Bethe-Heitler theory. Hence, $R(E_0,29)$ is by definition unity. The solid curves show the ratio of the expected yield, using the Bethe, Maximon, Davies, and Olsen calculations, to the yield predicted by the Bethe-Heitler theory. These theoretical curves are normalized to unity for copper in order to make comparisons with experiments. The deviation from Bethe-Heitler theory for a given element Z is given by the difference between the intercept at $Z=0$ and the ordinate for the element Z. The 34- and 24-Mev data are those of reference 8. The symbol E_0 is the initial electron energy and k is the photon energy.

Recently, a very precise experiment (statistical errors less than $\frac{1}{2}$%) has been performed at Harwell by Moffatt, Thresher, and Wilson[20] on the total absorption cross section of 96-Mev photons in C, Cu, Ag, Pb, and U. The relative cross sections are believed to be known to better than 0.2%. From these results, Moffatt, Thresher, and Wilson are able to show that the correction to the Bethe-Heitler theory of pair production does indeed have a slight deviation from a Z^2 law as is predicted by Davies, Bethe, and Maximon.[5] The absolute deviation from Bethe-Heitler theory which they find is within $\sim\frac{1}{2}$% of that predicted. From the Olsen theorem, it may therefore be concluded that the bremsstrahlung cross section as given by Eq. (1) should be as valid at high energies as is the pair cross section. The error in Eq. (1) for the bremsstrahlung cross section resulting from theoretical approximations, is expected to be less than $1/E$ where E is the total energy of the final electron. It should be mentioned that at electron energies below about 10 Mev and for k/E_0 approaching unity, there are experiments which show a positive correction to the Born approximation theory.[21,22] This is not surprising inasmuch as Sommerfeld's exact calculations in the nonrelativistic limit[23] show that the Born approximation calculations yield an answer which is too low.

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[18] We are indebted to K. Street and D. R. Bomberger of the University of California Radiation Laboratory at Livermore for the analytical analysis of our uranium foils.

[19] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939); **101**, 1836 (1955).

[20] Moffatt, Thresher, and Wilson (private communication, to be published).

[21] N. Starfelt and H. W. Koch, Bull. Am. Phys. Soc. Ser. II, **1**, 10 (1956).

[22] J. W. Motz, Phys. Rev. **100**, 1560 (1955).

[23] A. Sommerfeld, Ann. Physik **11**, 257 (1931).