Structure of Magnetohydrodynamic Shock Wave in a Plasma of Infinite Conductivity

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RECENTLY a work with this title was published in this Journal by Sen.[1] The purpose of this letter is to point out certain effects occurring in a highly ionized plasma which he overlooked.

Sen takes the Prandtl number $\mu c_p/k$ to be $\frac{3}{4}$ and correctly observes that his conclusions depend only on the order of magnitude of this number. In a fully ionized gas, however, the Prandtl number is at least an order of magnitude less than this value. The physical reason for this can be seen by comparing simple kinetic theory expressions for viscosity and thermal conduction. For an electron gas alone

$$\mu_e = c(m_e KT)^{\frac{1}{2}}/\sigma, \quad k_e = 15K\mu_e/4m_e,$$

and for an ion gas alone

$$\mu_i = c(m_i KT)^{\frac{1}{2}}/\sigma, \quad k_i = 15K\mu_i/4m_i.$$

Here m_e and m_i are the electron and ion masses, σ is the mean cross section ($\sigma \propto T^{-2}$), c is a numerical constant of order-of-magnitude unity, and K is Boltzmann's constant. For a gas mixture, we expect $\mu \approx \mu_1$ because $\mu_i/\mu_e = (m_i/m_e)^{\frac{1}{2}}$ but we expect $k \sim k_e$ because k_e is greater than k_i by this same factor. Thus we expect $\mu c_p/k \approx (8/9)(m_e/m_i)^{\frac{1}{2}} \approx 0.031A^{-\frac{1}{2}}$, where A is the atomic weight of the ions. Detailed calculations by Chapman and Cowling[2] show that a further factor of about two comes in, and their expressions lead to a value $\mu c_p/k = 0.065A^{-\frac{1}{2}}$, which is small even for hydrogen.

This small value of the Prandtl number leads to interesting modifications in the shock structure which have been discussed by the author.[3,4] For weak shocks the velocity, density, and temperature change smoothly over a width $\approx 20A^{\frac{1}{2}}$ mean free paths—very much greater than that predicted by Sen. In this case the profile is determined by the thermal conduction and viscous effects are negligible. But for stronger shocks, when

$$u_1^2 < p_1/\rho_1 + H_1^2/4\pi\rho_1,$$

where u_1, p_1, H_1, and ρ_1 are the values behind the shock of the velocity relative to the shock, pressure, magnetic field, and density respectively, then this smooth variation of velocity, density, and temperature is followed by a narrow region, of width $\approx$ a mean free path, in which the velocity and density change rapidly while the temperature remains almost constant. If there is no magnetic field, the above condition reduces to $\rho_1/\rho_0 > 1.5$.

Thus all shocks in an ionized gas of infinite electrical conductivity have widths very much greater than those predicted by Sen and for the stronger shocks the front is made up of two distinct regions.

The author has also examined the structure of these shocks in a gas of finite electrical conductivity and it was found that the width of the front in this case is $\approx c^2/4\pi\sigma u_0$ (provided that this length exceeds the lengths mentioned in the foregoing) where c is the velocity of light, σ is the electrical conductivity, and u_0 is the velocity of the gas ahead of the shock. This can be very large (≈ 2 cm) at some temperatures. The front again divides into distinct regions depending on the magnitude of the field and upon the shock strength.

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[2] S. Chapman and T. Cowling, *The Mathematical Theory of Non-uniform Gases* (Cambridge University Press, Cambridge, 1939), p. 167.
[3] W. Marshall, Proc. Roy. Soc. (London) **A233**, 367 (1956).
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Relation between Energy and Half-Thicknesses for Absorption of Beta Radiation

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IT has been shown[1,2] that under conditions of cylindrical geometry in which the sample is placed cylindrically around the detecting Geiger counter, the absorption curve obtained by interposing varying thicknesses of absorbers between the sample and

Table I. Absorption data.

Isotope	Maximum energy of beta spectrum (Mev)	Absorbing material	Half-thickness (mg/cm²)
T	0.0189	He	0.050
Zr^{93}	0.060	Al	0.35
Sm^{151}	0.0755	Al	0.63
C^{14}	0.155	Al	1.9
		Mylar plastic[a]	2.2[a]
S^{35}[a]	0.167[a]	Al[a]	2.3[a]
		Mylar[a]	2.7[a]
Rb^{87}	0.270	Al	4.85
Ca^{45}[a]	0.255[a]	Al[a]	4.9[a]
Tc^{99}	0.296	Al	6.09
Tl^{204}	0.762	Al	22
Cl^{36}[a]	0.716	Al[a]	32[a]
		Cu[a]	26[a]
		Sn[a]	21[a]
		Sn[a]	18[a]
K^{40}	1.36	Al	67.0
P^{32}	1.708	Al	84
		Cu[a]	60[a]
		Sn[a]	50[a]
Y^{90}[a]	2.275[a]	Al[a]	130[a]

[a] New data; other data from reference 1.

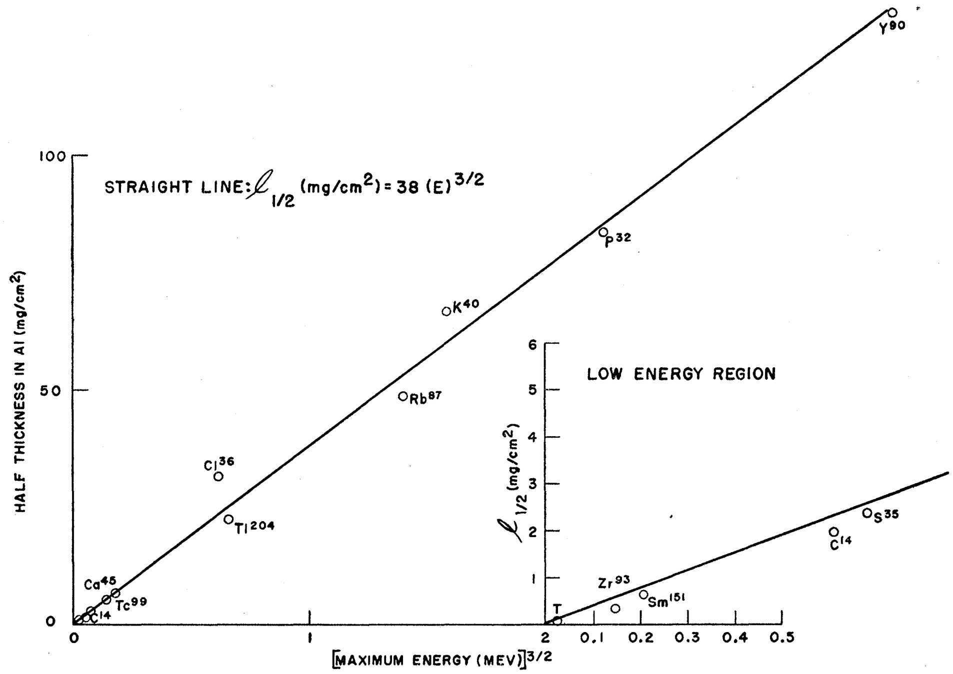

FIG. 1. Relation between half-thickness in aluminum and the $\frac{3}{2}$ power of the maximum beta-ray energy.

detection instruments is exponential for all beta-radioactive isotopes in which single spectra are involved, i.e., a single final state and a single original state—this is so even though the transition may be highly forbidden as in the case of K^{40}. Table I gives data for typical beta-radioactive isotopes of varying characteristics for various absorbers.

It has been discovered that there is a particularly simple relation between the half-thickness in aluminum and the upper energy limit, E, of the beta spectrum involved. This relation is shown graphically in Fig. 1 and is given by the following equation:

$$l_{\frac{1}{2}}(\text{mg/cm}^2)=38\times(E^{\frac{3}{2}}), \qquad (1)$$

where E is the upper energy limit in Mev.

For other absorber materials the relation of Lerch[3] should be used. It is that the absorption half-thickness is inversely proportional to the expression

$$1+M/100,$$

where M is the mean atomic weight of the absorbing material. The generality of the relation shown in Fig. 1 is considerable. It is to be noted, however, that the most highly forbidden spectra, those of Cl^{36} and K^{40}, deviate somewhat.

It is important to this relation that the sample be placed close to the counter wall and in cylindrical shape with axis in common with that of the counter. It is under these conditions that the absorption curves are exponential and thus yield values of the half-thickness or absorption coefficient. The earlier methods of Feather and Bleuler and Zünti[4] depend on the use of nonexponential absorption curves and the determination of the total range of the radiation. The range-energy relation is more complicated than Eq. (1) which uses the absorption coefficient or half-thickness under conditions of exponential absorption. The dependence of the shape of the absorption curves on the placement of the sample relative to the counter is caused by scattering of the beta radiation.

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Observation of Long-Lived Neutral *V* Particles*

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THE application of rigorous charge conjugation invariance to strange particle interactions has led to the prediction of rather startling properties for the θ^0-meson state.[1] Some of these are: (I) the existence of a second neutral particle, θ_2^0, for which two-pion decay is prohibited; (II) the consequent existence of a second