Effects of Nondegeneracy of Nuclear Ground State on Low-Energy Neutron Reactions

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The imaginary part of the potential acting on a neutron moving in the nucleus is shown to be very sensitive to the departure of the nuclear ground-state wave function from that of an independent-particle model. The nucleon-nucleon interaction which gives rise to the neutron absorption also leads to correlation structure in the nuclear wave function. This manifests itself both in the partial emptying of the independent-particle states near the Fermi momentum and in the velocity dependence of the real part of the average potential seen by the neutron. These effects are determined by using interaction operators derived in studies of the nuclear saturation problem. The result is that the velocity dependence of the potential reduces V_{Im} by a factor of about eight; the departure of the nuclear state from complete degeneracy gives an increase of roughly 5 in the opposite direction. Thus, the two effects nearly cancel so that the final prediction for V_{Im} at a density corresponding to $R=1.40\times10^{-13}A^{\frac{1}{3}}$ cm is close to the empirical value. It is also found that V_{Im} decreases rapidly for higher nuclear densities, suggesting that the neutron absorption may be primarily a surface phenomenon.

I. INTRODUCTION

In a previous paper[1] (to be referred to as I) a theory of the interactions of low-energy neutrons with nuclei was developed, following closely methods used in the study of the properties of the ground state and the low excited states of nuclei.[2–10] It is the purpose of this paper to remove some of the approximations made in I and to give a more accurate determination of the imaginary part of the average potential acting on the neutron. In the course of the calculation, the physical significance of the general approximation method used will be developed in greater detail than in I.

The imaginary part of the average potential which acts on a nucleon moving through nuclear matter in its ground state is a measure of the rate at which the neutron through its interactions loses energy to nuclear excitation, i.e., to compound nucleus formation. An elementary way of estimating this rate was developed by Goldberger[11] for high energies and more recently applied by Lane and Wandel[12] to the low-energy neutron region. In these calculations the nucleus is idealized as a degenerate Fermi gas and the rate of excitation determined by a use of the experimentally determined nucleon-nucleon cross section to give the mean-free path for a nucleon in nuclear matter. The features of the process which are not treated correctly in this approximation are the alteration of the nucleon-nucleon scattering by the presence of the dense nuclear matter, and the departure of the nuclear wave function from that of an idealized independent-particle model. A further error arises from the assumption that only the simple excitations resulting from a single interaction of the incoming neutron with a bound nucleon need be considered. Consequently the effects of more complicated compound and collective excitations are not taken into account. A surprising feature of the calculations of Lane and Wandel based on this simple approximation scheme, however, is that their results are in semiquantitative agreement with experiment, both in energy dependence and in magnitude.

The calculation of the imaginary potential given in I is a determination of the rate at which the simplest excited states of the nucleus decay, transferring their energy into multiparticle excitations. These states are not simple independent-nucleon states in the sense of the shell model, but rather the simplest states of excitation of the actual nucleus, i.e., eigenstates of the true nuclear Hamiltonian with the particle-particle interactions included. To clarify the relationship between the actual nuclear states and the states of the independent-particle model, we shall restate here some of the results obtained in other applications of the methods of this paper, particularly in the theory of nuclear models[7] and in the theory of configuration mixing.[10] It is convenient to consider first a system of particles moving without mutual interaction in an external potential which for example may be taken to be the typical well of the independent-particle model. The independent-particle states may be filled uniformly to give a degenerate Fermi gas; this is the lowest state of the system. Other possible states may have, for example, one nucleon moving in an excited state, two particles in excited states, one hole in the filled states and one excited particle, etc. These typical independent-particle states are indicated schematically in Figs. 1(A), 1(B), and 1(C).

We next consider the alteration of the independent-particle states as the particle-particle forces are adia-

[1] Brueckner, Eden, and Francis, Phys. Rev. **100**, 891 (1955).
[2] Brueckner, Levinson, and Mahmoud, Phys. Rev. **95**, 217 (1954).
[3] K. A. Brueckner, Phys. Rev. **96**, 508 (1954).
[4] K. A. Brueckner, Phys. Rev. **97**, 1353 (1955).
[5] K. A. Brueckner and C. A. Levinson, Phys. Rev. **97**, 1344 (1955).
[6] K. A. Brueckner, Phys. Rev. **100**, 36 (1955).
[7] R. J. Eden and N. C. Francis, Phys. Rev. **97**, 1366 (1955).
[8] R. J. Eden, Phys. Rev. **99**, 1418 (1955), and R. J. Eden (to be published).
[9] Brueckner, Eden, and Francis, Phys. Rev. **98**, 1445 (1955).
[10] Brueckner, Eden, and Francis, Phys. Rev. **99**, 76 (1955).
[11] M. L. Goldberger, Phys. Rev. **74**, 1269 (1948).
[12] A. M. Lane and C. F. Wandel, Phys. Rev. **98**, 1524 (1955).

batically "turned on" from zero to their actual value. One consequence is that the energy levels all shift downward as a result of the perturbation. This interaction energy is of course the origin of the actual nuclear binding. Another effect of the perturbation is that the independent-particle levels are mixed with nearby levels so that the amplitude for occupying a given level is distributed among many nearby levels.[13] This alteration of the population of the independent-particle levels is indicated schematically in Figs. 1(D), 1(E), and 1(F).

The connection between the nuclear wave function and the independent-particle wave function can be expressed formally using the methods of the previous nuclear studies.[14] We consider an independent-particle wave function Ψ_0 describing $A+1$ nucleons occupying various independent-particle states. When the interactions are turned on, the wave function goes over into

$$\Psi=F\Psi_0, \tag{1}$$

where F is the correlation function or "model operator" (in the sense of Eden) which introduces the correlation structure into the wave function to account properly for the effects of the strong two-body interactions. At the same time the energy of the system shifts downward from the unperturbed energy

$$\epsilon=\sum_i k_i^2/2M,$$

to

$$E=\sum_i(k_i^2/2M)+\tfrac{1}{2}\sum_i\sum_j(K_{ij,\,ij}-\text{exchange}), \tag{2}$$

where the sums run over all occupied states. The $K_{ij,\,ij}$ are the diagonal elements of the two-body reaction matrix which is determined by the self-consistency procedure discussed in detail in reference 4.

The exact relationship between Ψ and Ψ_0 is determined by F; the distribution of the original independent-particle state among the nearby states is given by the matrix elements of F with respect to the independent-particle states. If the alteration of the independent-particle states is not too great; i.e., if F does not differ too much from unity, it is convenient to retain some of the language of the independent-particle model in characterizing the nuclear states. For example, the state schematically indicated by Fig. 1(D) is closely connected with the independent-particle state of Fig. 1(A). To keep this relationship in mind but to emphasize the lack of an exact one-to-one relationship between the actual nuclear states and the independent-particle states, we shall refer to the former as quasi-independent-particle states. With this distinction clearly in mind, we now return to our problem.

In I, to simplify the determination of the real part of the potential acting on a neutron, the nuclear state formed by the entrant neutron and the nucleus was

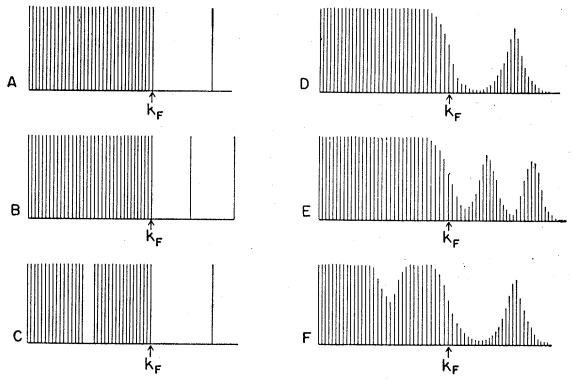

FIG. 1. Typical independent-particle-states [(A), (B), and (C)] and the related quasi-independent-particle nuclear states [(D), (E), and (F), respectively]. The relative population of the independent-particle states is schematically indicated by the vertical amplitudes. The shift of the levels in energy is not shown.

identified with a quasi-independent-particle excited state of the type indicated in Fig. 1(D). Taking k_0 as the momentum of the excited state to which the quasi-independent excited state is connected [as in the relationship between the states of Fig. 1(A) and 1(D)], the energy of the system is

$$E=\sum_{i=1}^{A}\frac{k_i^2}{2M}+\frac{k_0^2}{2M}+\tfrac{1}{2}\sum_{ij=1}^{A}(K_{ij,\,ij}-\text{exchange})+\sum_{i=1}^{A}(K_{0i,\,0i}-\text{exchange}), \tag{3}$$

where the third term in E is the internal interaction energy of the core and the fourth term is the interaction energy of particle "0" with the core particles. The solution of the problem is completed by adjusting k_0 so that the total energy of the system of A nucleons plus the one excited particle at momentum k_0 minus the energy of the nucleus with A nucleons is equal to the energy of the incoming neutron. This determination of the real part of the interaction energy,

$$V_R(0)=\text{Re}\sum_{i=1}^{A}(K_{0i,\,0i}-\text{exchange}), \tag{4}$$

is discussed in I. The result obtained for neutrons was

$$V_R=-\frac{M^*}{M}68\text{ Mev}+\left(1-\frac{M^*}{M}\right)E_n, \tag{5}$$

where $M^*=M$ is the effective nucleon mass for a nucleon moving at momenta near the Fermi momentum. For protons, the result is similar except that E_n should be replaced by E_p-E_c, where E_c is a mean value of the Coulomb energy of the proton in the nucleus. Equation (5) is valid at a density corresponding to a radius of $R=1.40\times10^{-13}A^{\frac{1}{3}}$. To correct this to the density de-

[13] A similar description is used by Lane, Thomas, and Wigner in their "intermediate coupling" model of the nucleus [Phys. Rev. 98, 693 (1955)].

[14] For a detailed discussion of the following, see references 5, 7, 8, and 10.

termined by Saxon's[15] analyses of scattering, which give $R=1.33\times10^{-13}A^{\frac{1}{3}}$, we somewhat arbitrarily multiply 68 Mev by $(1.40/1.33)^2$. The resulting potential acting on a proton (taking $E_c=14$ Mev) is shown in Fig. 2 for $\sigma=M^*/M=0.52$ and $\sigma=0.60$; the correct value of σ lies between these values. The agreement with the indicated experimental points is reasonably good.

In the approximation made in I which identified the neutron-nucleus state with a true nuclear eigenstate (a quasi-independent-particle state) absorption cannot take place if the energy of excitation is less than the energy at which particle emission can occur. This is clearly the case in an actual nucleus where the nuclear levels become metastable for such energies. Thus the imaginary part of the potential determined in the approximation of I vanishes at zero energy of the incoming neutron. At higher energies, the single-particle level is broadened since particle emission can occur. When the width of the single-particle levels is sufficiently great to overlap many compound levels, the methods of I gave an imaginary potential of

$$V_{\mathrm{Im}}=1.24\left(\frac{E-E_F}{8\ \mathrm{Mev}}\right)^2\left(\frac{M^*}{M}\right)^3\ \mathrm{Mev}. \tag{6}$$

Inserting values of M^*/M between 0.5 and 0.6 as determined in the saturation studies, the result for neutrons of 2 Mev is $V_{\mathrm{Im}}=0.3$ to 0.4 Mev, which is a factor of roughly five smaller than that determined empirically.[16] This considerable discrepancy between theory and experiment is somewhat puzzling at first sight. A possible source of the too-small value for V_{Im} lies in the neglect of more complex excitations: we shall show, however, that the principal error lies in the neglect of the difference between the quasi-independent particle and the true independent particle states.

It is apparent that while the methods of I predicted (correctly) the vanishing at zero energy of theimaginary potential acting on a particle in a quasi-independent-particle state, it cannot be correct that a neutron sees similarly a vanishing absorption as the energy tends to zero. The anomaly is resolved if we recall that the neutron state bears only an approximate one-to-one relation with the quasi-independent-particle state or conversely that the quasi-independent-particle wave function can be expanded into independent-particle wave functions in which expansion the single-nucleon state has an amplitude less than unity. Thus in I we have neglected the incomplete overlap of the incoming neutron wave function with the quasi-independent-particle function; it is the incompleteness of this overlap which leads to an increase in V_{Im} relative to that determined in I.

An alternative way of expressing this feature is to observe that the true nuclear state (itself degenerate in the sense that all of the low states are filled) expanded in independent-nucleon states is not a degenerate Fermi gas, but, because of the effects of the strong particle-particle forces, is only partially degenerate. Consequently, the incoming nucleon sees only the partially degenerate nucleus and is less affected in its absorption by the exclusion principle. We shall see that the consequence of this weakening of the exclusion effects leads to an increase in V_{Im} to bring it into approximate agreement with experiment.

We shall also in this paper improve our estimates of the effects of nuclear binding by using for the reaction matrices which determine the particle-particle interaction, a better approximation based on the results of the saturation studies of references 2 to 4.

II. FORMAL EXPRESSION FOR V_{Im}

The expression[17] given in I [Eq. (1)] for the average potential acting on a neutron is (we do not indicate exchange effects explicitly)

$$W=\sum_{i=1}^{A}(\Psi(A),K_{0i}\Psi(A)), \tag{7}$$

where $\Psi(A)$ is the true nuclear ground-state wave function. K_{0i}, the reaction matrix for particles 0 and i, is related to the potential acting between the neutron and the ith bound nucleon by the integral equation

$$K_{0i}=v_{0i}+v_{0i}(E-H)^{-1}K_{0i}, \tag{8}$$

where H is the sum of the exact Hamiltonian for the nucleus and the Hamiltonian for the neutron moving in the "optical potential" of the nucleus. In order to solve this integral equation, it is necessary to introduce the exact nuclear wave functions so that $(E-H)^{-1}$ can be expressed as a simple diagonal operator.

To avoid this complication, in the computations of I Eq. (8) was replaced by a simpler equation analogous to that used in the saturation studies. A simple extension of the methods used there led to the equation for the potential acting on a quasi-particle state (connected with the single-nucleon state by the F transformation)

$$V(0)=\sum_{i=1}^{A}(\Psi_0(A),K_{0i}\Psi_0(A)), \tag{9}$$

where

$$\Psi(A)=F\Psi_0(A), \tag{10}$$

and the reaction matrix K is computed by solving an equation similar to Eq. (8) except that H is replaced by an operator diagonal with respect to the independent-particle or model states. It is this approximation which we wish now to remove.

We shall compute the potential directly from Eq. (7) for W. The real part of the average potential is only slightly affected by the difference between Ψ and Ψ_0; the

[15] Melkanoff, Moszkowski, Nodvik, and Saxon, Phys. Rev. **101**, 507 (1956).

[16] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[17] For a detailed discussion of some of the following points, see N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953).

imaginary part of the potential, however, is much more sensitive to the difference. Thus we shall attempt to improve the determination of V_{Im} only. To do this, we need first a good approximation to the nuclear wave function Ψ. It is convenient to base our study on the independent-particle wave function Ψ_0; thus we shall express the departure of the true nuclear wave function from Ψ_0 by determining the alteration of the population of the independent-particle levels. We shall neglect surface effects and treat the nuclear medium as infinite; this allows us to use plane waves for the independent particle states. We cannot expect in this approximation to determine the fluctuation in the potential associated with the discrete structure of the nucleus, but only the average value.

The transformation or correlation function F which connects Ψ with Ψ_0 is given by the set of coupled equations[5,7]

$$F=1+\frac{1}{e}\sum_{ij} I_{ij}F_{ij},$$
$$F_{ij}=1+\frac{1}{e}\sum_{ij\neq kl} I_{kl}F_{kl}, \tag{11}$$

where the operators I are the nondiagonal part of the reaction matrix. We shall in this calculation assume that the departure of F from unity is primarily the result of the two-body correlation in the nucleus, i.e., we take

$$F\cong 1+\frac{1}{e}\sum_{ij} I_{ij}. \tag{12}$$

The two-body operator I operating on the state Ψ_0 of the system (degenerate Fermi gas) then has the effect of emptying two states and filling two previously empty states. We return to the explicit determination of F in this approximation in the next section.

The remaining problem is the determination of the imaginary part of the potential. In determining this we make the following approximations: for the change of the energy of the nucleus upon the change of state of particles, we use the reduced mass approximation[4]

$$E(k)=k^2/2M^*+\text{constant}, \tag{13}$$

where M^* is the effective nucleon mass. We take for the matrix element of the energy denominator in a state where particles occupying levels k, l have made a transition to levels i, j

$$1/e_{kl,\,ij}=1/(E_k+E_l-E_i-E_j+i\eta), \tag{14}$$

where η is a positive real infinitesimal which specifies the nature of the singularity on the energy shell. The sign of η must be chosen positive since loss from the initial state (or incident beam, in scattering terminology) is occurring. This approximate specification of the imaginary part of the energy denominator is valid only when many levels are summed over and when the energy broadening of the initial state due to absorption is large compared with the compound level widths. These conditions are well satisfied in this application.

The imaginary part of $K_{1i,\,1i}$ is

$$\mathrm{Im}K_{1i,\,1i}=\mathrm{Im}[v_{1i,\,1i}+\sum_{mn} v_{1i,\,mn}(E_1+E_i-E_m-E_n+i\eta)^{-1}K_{mn,\,1i}]$$
$$=-\pi\sum_{mn}|K_{1i,\,mn}|^2\delta(E_1+E_i-E_m-E_n). \tag{15}$$

At this stage, we approximate $K_{1i,\,mn}$ by the real reaction matrix; this is a good approximation as long as ImK is smaller. The sum over m, n is over all empty states of the state Ψ; if we were to approximate Ψ by Ψ_0, the degenerate state, then the sum would run only over states above the Fermi momentum. Finally, the imaginary part of the potential acting on particle 1 is

$$V_{\mathrm{Im}}(1)=\sum_{i\ (\mathrm{filled})}\ \sum_{mn\ (\mathrm{empty})}|K_{1i,\,mn}|^2\times\delta(E_1+E_i-E_m-E_n), \tag{16}$$

where again the sum over i is over all filled states of the physical nucleus Ψ rather than the model Ψ_0. The remaining problem is to compute $F\Psi_0$ which gives the departure of Ψ from the degenerate gas Ψ_0 and then to carry out the sums of Eq. (16).

III. DETERMINATION OF WAVE FUNCTION

A. Reaction Matrix

For the reaction matrix, we shall use a result which follows from the studies of the saturation problem.[4] We shall use the same form for the reaction matrix both in determining the departure of the ground state from a degenerate Fermi gas and in estimating the transition rate in evaluating the imaginary part of the potential. In the latter case, since transitions are occurring, it is not quite correct to use the reaction matrix which is applicable only to the stationary state. Instead the appropriate scattering matrix should be determined. When the transition rate is low, however, as it is in the cases we study, the difference between the reaction matrix and scattering matrix is very small and will be neglected here.

In the saturation studies it was shown that an accurate approximation to the two-body reaction matrix can be obtained by solving the two-body problem in the presence of the external velocity-dependent nuclear potential. The approximation used in the saturation studies was to replace the actual two-body central and noncentral potentials by square walls with repulsive cores which predict both the two-body scattering and the reaction matrices correctly for low momentum values. Since it will be evident as we proceed that in this problem it is the small momentum transfers which are most important, we shall adopt the same approximation

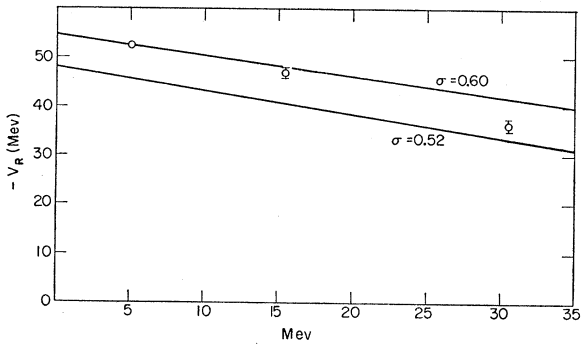

FIG. 2. The energy dependence of the real part of the potential acting on a neutron. The theoretical curve, corrected approximately for density, is given for two values of the effective nucleon mass. The experimental points are taken from reference 14.

to the reaction matrices here. The diagonal elements of K are given by

$$(\mathbf{k}_1\mathbf{k}_2|K|\mathbf{k}_1\mathbf{k}_2)_{s,t}=\frac{4\pi}{M^*}\lambda\tan\delta_{s,t}\left(\frac{\mathbf{k}_1-\mathbf{k}_2}{2}\right), \quad (17)$$

where M^* is a value for the effective nucleon mass valid for states of low momentum. By the subscript s, t we mean singlet or triplet spin state. The phase shift δ is parametrically a function of a mean value M^{**} of the effective mass which is valid for the high momenta which occur when the two particles are interacting. The average of the resulting reaction matrices for singlet and triplet spin states (the relevance of the sum will appear in the following) is given graphically for two values of the density in Fig. 2. (The density parameter η corresponds to a nuclear radius $R/\eta=1.40\times10^{-13}A^{\frac{1}{3}}$ cm.) For simplicity in the following we introduce a simple analytic form for the sum of the matrices, i.e.,

$$(\mathbf{k}_1\mathbf{k}_2|K|\mathbf{k}_1\mathbf{k}_2)=\frac{4\pi V_0}{\alpha}\frac{\alpha^2}{[\alpha^2+\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)^2]^2}, \quad (18)$$

which gives a quite accurate fit to the computed curves over the important low momentum region. The values of the constants are

$$\left.\begin{aligned}V_0/\alpha&=1.044\\ \alpha^2&=2.65\mu^2\end{aligned}\right\}\eta=1, \quad M^*/M=0.517,$$
$$\left.\begin{aligned}V_0/\alpha&=0.950\\ \alpha^2&=2.65\mu^2\end{aligned}\right\}\eta=0.9, \quad M^*/M=0.498, \quad (19)$$

where $\mu=$meson mass$=140$ Mev (we set $\hbar=c=1$). Finally, to extend these results to the nondiagonal element of K, we assume the reasonable form

$$(\mathbf{k}_1'\mathbf{k}_2'|K|\mathbf{k}_1\mathbf{k}_2)=4\pi V_0\alpha\{\alpha^2+[\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)]^2+[\tfrac{1}{2}(\mathbf{k}_1'-\mathbf{k}_2')]^2\}^{-2}. \quad (20)$$

This approximation does not introduce an appreciable uncertainty into our results since, as we shall see, the contributions to the potential come largely from the region of small momentum transfers where Eq. (20) is a good approximation.

It is interesting to compare the reaction matrix obtained from the saturation studies with other possible choices. If we wished to make a Born approximation calculation, a possible choice for K would be the scattering from an exponential well. For the well

$$V(r)=V_0e^{-2\beta r}, \quad (21)$$

the matrix element is

$$(\mathbf{k}_1'\mathbf{k}_2'|V|\mathbf{k}_1\mathbf{k}_2)=8\pi V_0\beta[\beta^2+(\mathbf{k}_1-\mathbf{k}_1')^2]^{-2}. \quad (22)$$

We choose the ranges and depth as average values[18] for the singlet and triplet states, i.e.,

$$V_0/\beta=0.506, \qquad \beta=1.98\mu. \quad (23)$$

The s-wave scattering from this potential then is, for low relative momentum values,

$$(\mathbf{k}_1\mathbf{k}_2|V|\mathbf{k}_1\mathbf{k}_2)=8\pi V_0\beta[\beta^2+\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)^2]^{-2}. \quad (24)$$

We compare this with the reaction matrices of Eq. (20) in Fig. 3. It is apparent that the Born approximation matrix elements are considerably smaller than the actual reaction matrix, the Born approximation being particularly incorrect for low momentum values. The discrepancy is also particularly marked for the lower density, $\eta=1$. For comparison we also give the average of the total p-p and n-p cross sections in Fig. 3. This is the average used by Lane and Wandel[12] in their determination of V_{Im}. From Fig. 3 it is apparent that the average scattering agrees roughly in average magnitude with the reaction matrices. Closer examination of these results shows that this agreement is the result of compensation of one effect by another. On one hand, the reaction matrix is larger than the scattering matrix by roughly the ratio $\tan^2\delta/\sin^2\delta$. (This ratio follows from the fact that the real reaction matrix is proportional to $\tan\delta$ while the complex scattering matrix is proportional to $\sin\delta e^{i\delta}$.) This increase is partially offset by the effects of the velocity dependence of the medium, i.e., by the reduced-mass effect. Both of these alterations in the interaction of neutron with bound nucleons are very probably present and reflect characteristic effects of the binding.

B. Determination of the Wave Function

We next determine Ψ. The amplitude for finding the (previously filled) states i, j empty and the (previously empty) states k, l filled is given by the matrix elements of the reaction matrix for this transition. With an

[18] See, for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

obvious notation, the amplitude is

$$A(k_i^{-1},k_j^{-1},k_k,k_l) = \frac{2M^*}{k_i^2+k_j^2-k_k^2-k_l^2}(\mathbf{k}_k\mathbf{k}_l|K|\mathbf{k}_i\mathbf{k}_j), \quad (25)$$

where because of momentum conservation we must have

$$\mathbf{k}_i+\mathbf{k}_j=\mathbf{k}_k+\mathbf{k}_l. \quad (26)$$

We first fix our attention on the probability for emptying the state k_i. This we determine by summing the square of the probability amplitude over all initial states which are compatible with a fixed state k_i and with the momentum conservation condition, i.e.,

$$P(k_i^{-1}) = \sum_{k_jk_kk_l} |A(k_i^{-1},k_j^{-1},k_k,k_l)|^2\delta_{\mathbf{k}_i+\mathbf{k}_j,\mathbf{k}_k+\mathbf{k}_l}. \quad (27)$$

The summation over momentum states alone can be replaced by an integral,

$$\int d\mathbf{k}/(2\pi)^3, \quad (28)$$

leaving only a sum over spins and isotopic spins. This is most simply evaluated, taking proper account of the exclusion principle, in the following way. First we break the probability $P(k_i^{-1})$ into terms arising from transitions out of triplet and out of singlet spin states, i.e., we write

$$P(k_i^{-1}) = P_{\text{singlet}}(k_i^{-1})+P_{\text{triplet}}(k_i^{-1}). \quad (29)$$

If we fix our attention on a proton, the sum over the neutrons is given by

$$2\{\tfrac{3}{4}P_{\text{triplet}}+\tfrac{1}{4}P_{\text{singlet}}\}, \quad (30)$$

the factor of 2 coming from 2 neutrons per momentum state, and $\frac{3}{4}$ and $\frac{1}{4}$ being the *a priori* weighting for the two spin states. For the protons, since we are considering only s-state interactions, only singlet spin states contribute. The result is

$$2\times(2/4)P_{\text{singlet}}, \quad (31)$$

the factors coming from two protons per state, $\frac{1}{4}$ *a priori* probability for a singlet spin state, and 2 from the identity of the two protons. Combining these, we have for the net probability

$$3(P_{\text{triplet}}+P_{\text{singlet}})/2. \quad (32)$$

Thus it is convenient to consider the average of singlet and triplet as was anticipated in the discussion of the foregoing reaction matrix.

The integral of Eq. (27) is carried out in Appendix I; the result is

$$P(k_i^{-1}) = \rho f[k_F(k_F-k_i)/\alpha^2], \quad (33)$$

where

$$\rho = \tfrac{1}{2}(V_0M^*/2\pi\alpha k_F)^2, \quad (34)$$

and

$$f(x) = 1+6x-2x(2+3x)\ln[1+(1/x)]. \quad (35)$$

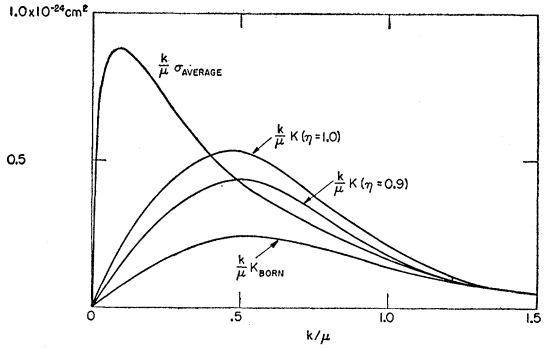

FIG. 3. The average of the singlet and triplet reaction matrices $(\mathbf{k}|K|\mathbf{k})$ as a function of k. For comparison, the Born approximation to the s-wave scattering from an exponential well and the average of experimental values for singlet and triplet scattering is given.

The probability $P(k_k)$ that a previously empty state of momentum k_k is now occupied is, in the approximation used in evaluating the integral of Eq. (27), identical with Eq. (33) except that k_F-k_i is replaced by k_k-k_F. The distribution is given in Fig. 3 for the two choices of the constants V_0 and M^* appropriate to the densities $\eta=1.0$ and $\eta=0.9$. The result is sensitive to the density since the reaction matrices decrease appreciably in going to higher densities and the velocity dependence of the nuclear potential also increases somewhat, resulting in a lower effective mass.

IV. DETERMINATION OF THE IMAGINARY PART OF THE POTENTIAL

We now combine the results of the last section with Eq. (16) for the imaginary part of the potential. The integrals cannot be carried out in closed form; they can be considerably simplified, however, if we make use of the fact that for a neutron of low energy and therefore with momentum close to k_F, the exclusion principle forces all final momenta to lie near to k_F. This is true even when the nondegeneracy of the ground state is taken into account since this affects appreciably only states near k_F. The integral to be evaluated is, again going from summation to integration and introducing the proper spin weighting,

$$V_{\text{Im}} = \frac{3}{4}\frac{4\pi}{(2\pi)^6}\int d\mathbf{k}_i d\mathbf{k}_i'(\mathbf{k}_1'\mathbf{k}_i'|K|\mathbf{k}_1\mathbf{k}_i)^2 \times\delta(E_1+E_i-E_1'-E_i')p(k_i) \times[1-p(k_1')][1-p(k_i')], \quad (36)$$

where K^2 is again the average over singlet and triplet spin states. For a degenerate state,

$$p(x)=1, \quad x<k_F, \qquad =0, \quad x>k_F. \quad (37)$$

For the actual nuclear case, we use $p(x)$ as given by Fig. 4.

The integral of Eq. (36) can be evaluated without great difficulty if the departure from a fully degenerate state is treated as a perturbation, and if the incident neutron has low energy so that its momentum is close to the Fermi momentum. The integration is carried out in Appendix II; the result is

$$V_{\mathrm{Im}}(k)=\pi\left(\frac{V_0}{2\pi\alpha}\right)^2\frac{M^*}{\alpha^2}[\epsilon^2+\epsilon\rho F_2+\rho F_1], \quad (38)$$

where ρ is defined by Eq. (34) and

$$\epsilon=k-k_F. \quad (39)$$

The functions F_1 and F_2 which express the effects of nondegeneracy are given by

$$F_1=36\int_0^\infty\frac{ydy}{(1+y^2)^2}g\left(\frac{\alpha}{\sqrt{2}}y\right),$$
$$F_2=36\int_0^\infty\frac{ydy}{(1+y^2)^2}h\left(\frac{\alpha}{\sqrt{2}}y\right), \quad (40)$$

where

$$g(s)=\int_0^{2s}xf(x)dx+\frac{4}{3}\int_s^{2s}(s-x)f(x)dx,$$
$$h(s)=\int_0^{2s}f(x)dx+\tfrac{2}{3}sf(2s)-\tfrac{2}{3}\int_s^{2s}f(x)dx. \quad (41)$$

The function $f(x)$ in these integrals is defined by Eq. (35); in the approximation of complete degeneracy $f=0$ and $F_1=F_2=0$.

After inserting the explicit forms for $f(x)$, numerical integration for the functions $g(s)$ and $h(s)$ and for F_1 and F_2 gives

$$-V_{\mathrm{Im}}=36.2\ \mathrm{Mev}\left[\frac{\epsilon^2}{k_F^2}+0.0829\frac{\epsilon}{k_F}+0.0175,\right]\quad \eta=1.0$$
$$=36.0\ \mathrm{Mev}\left[\frac{\epsilon^2}{k_F^2}+0.0414\frac{\epsilon}{k_F}+0.00732\right],\quad \eta=0.9. \quad (42)$$

Finally, making the replacement

$$\frac{\epsilon}{k_F}=\frac{k-k_F}{k_F}\cong\frac{k^2-k_F^2}{2M^*}\frac{M^*}{k_F^2}$$
$$=\frac{E-E_F}{8\ \mathrm{Mev}}\frac{8M^*\ \mathrm{Mev}}{k_F^2}, \quad (43)$$

we obtain

$$V_{\mathrm{Im}}=\left[0.265\left(\frac{E-E_F}{8\ \mathrm{Mev}}\right)^2+0.257\left(\frac{E-E_F}{8\ \mathrm{Mev}}\right)+0.633\right]\mathrm{Mev},\quad \eta=1.0$$
$$=\left[0.161\left(\frac{E-E_F}{8\ \mathrm{Mev}}\right)^2+0.100\left(\frac{E-E_F}{8\ \mathrm{Mev}}\right)+0.264\right]\mathrm{Mev},\quad \eta=0.9. \quad (44)$$

These results are also given in Fig. 5 as a function of the neutron energy, taking 8 Mev as the binding energy.

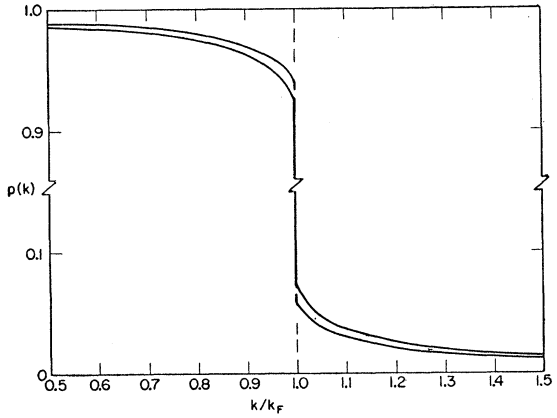

FIG. 4. Departure of the nuclear state from a Fermi gas at densities corresponding to $\eta=1.0$ and $\eta=0.8$.

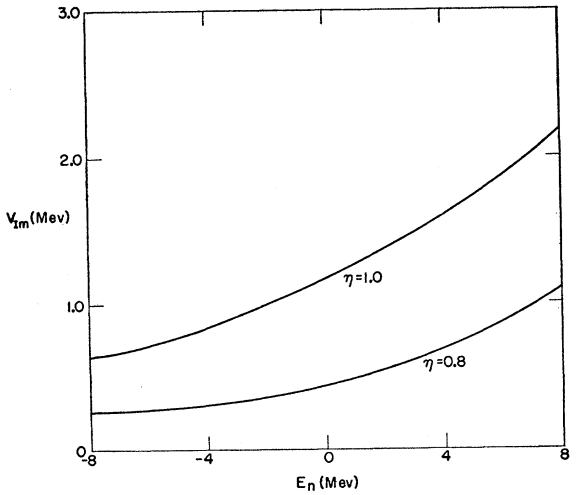

FIG. 5. The imaginary potential seen by a nucleon moving in the nucleus. The two curves are computed for densities corresponding to $R=1.40\times10^{-13}A^{\frac{1}{3}}$ cm ($\eta=1.0$) and $R=1.12\times10^{-13}$ cm $A^{\frac{1}{3}}$ ($\eta=0.80$). The zero of the energy scale has been assumed to correspond to $E-E_F=8$ Mev.

The term quadratic in $E-E_F$ is the only contribution which does not vanish for the fully degenerate state; the other terms arise from the partial emptying of the state near the Fermi momentum. These terms give the major contribution to the imaginary potential for the low excitations for which the approximations used in deriving Eq. (44) are valid. It is interesting to note the reasons for the approximate equality of these results with those of Lane and Wandel.[12] If at $\eta=1$ we take only the contribution for the fully degenerate state, i.e.,

$$V_{\mathrm{Im}}(\text{degenerate})=0.265\left(\frac{E-E_F}{8\ \mathrm{Mev}}\right)^2, \tag{45}$$

then our result[19] is less than theirs by a factor of about 6. This difference originates primarily in the velocity dependence of the potential, which not only lowers the density of states into which transitions can occur by a factor of almost exactly two, but also reduces the difference between the neutron momentum inside the nucleus and the momentum of the last filled levels. These effects depend on the cube of the effective mass which therefore gives a factor of $(M^*/M)^3\sim\frac{1}{8}$ in the result. This very marked reduction is compensated by the correction for the absence of complete degeneracy which, as Eq. (44) shows, increases the imaginary potential at $\eta=1.0$ by about a factor of five. Thus there are two large but partially compensating effects which must be included to give a reasonable result.

It is obvious that the extreme sensitivity of the imaginary potential to slight density changes makes it impossible to draw other than qualitative conclusions from our results. It is not clear that the rapid change of V_{Im} with η is completely meaningful; this feature does suggest, however, that the absorbing part of the potential may be weaker in the central regions of higher density and smaller η than in the peripheral regions.

V. CONCLUSIONS

In concluding, we would like to point out some general features of our results. We have shown that the departure of the nuclear ground state from a pure independent-particle state has a pronounced effect on the imaginary potential. The departure is the consequence of position correlations in the wave function due to the strong particle-particle forces; these are also responsible for the interaction of the incoming neutron with the bound nucleons which causes the excitation of the bound particles and the absorption of the neutron. Thus a realistic model must simultaneously introduce both of these effects. The velocity dependence of the effective potential also may be a quite general feature of nuclear structure. It is closely related to the saturating character of the nuclear forces; it is also an alternative way of describing the symmetry energy of the nucleus. Therefore its presence has strong empirical as well as theoretical basis.

To summarize: our results emphasize the importance of a detailed knowledge of the nuclear ground state properties since they reveal a perhaps unexpected sensitivity of the imaginary potential to the precise correlation structure of the nucleus.

[19] This result is very close to that obtained in I; the agreement, however, is partially coincidental since a different range, strength, and exchange character were assumed for the particle-particle interaction.

APPENDIX A. INTEGRATION FOR $F\Psi_0$

The integral of Eq. (27) is

$$P(k_i^{-1})=\frac{3}{(2\pi)^6}\int d\mathbf{k}_k d\mathbf{k}_l\left\{\frac{8M^*\pi V_0\alpha}{k_i^2+k_j^2-k_k^2-k_l^2}\times\frac{1}{\{\alpha^2+[\frac{1}{2}(\mathbf{k}_i-\mathbf{k}_j)]^2+[\frac{1}{2}(\mathbf{k}_k-\mathbf{k}_l)]^2\}^2}\right\}^2, \tag{A1}$$

with total momentum conservation giving $\mathbf{k}_i+\mathbf{k}_j=\mathbf{k}_k+\mathbf{k}_l$. We then introduce the new variables

$$\begin{aligned}\mathbf{k}&=\tfrac{1}{2}(\mathbf{k}_i-\mathbf{k}_j),\\ \mathbf{k}'&=\tfrac{1}{2}(\mathbf{k}_l-\mathbf{k}_k),\end{aligned} \tag{A2}$$

which bring the integral to a particularly simple form:

$$P(k_i^{-1})=\frac{24}{(2\pi)^6}(4\pi V_0\alpha)^2(M^*)^2\times\int d\mathbf{k}d\mathbf{k}'\frac{1}{(k^2-k'^2)^2}\frac{1}{(\alpha^2+k^2+k'^2)^4}. \tag{A3}$$

The requirement of the exclusion principle is that

$$\begin{aligned}|\mathbf{k}_i-\mathbf{k}+\mathbf{k}'|&\geqq k_F,\\ |\mathbf{k}_i-\mathbf{k}-\mathbf{k}'|&\geqq k_F,\end{aligned} \tag{A4}$$

and the integration over k_j, extending only over the filled states, gives a restriction

$$|\mathbf{k}_i+2\mathbf{k}|\leqq k_F. \tag{A5}$$

The integration over the angles of $\mathbf{k}$, taking $\mathbf{k}_i-\mathbf{k}$ as the polar axis, gives

$$\begin{aligned}\int d\Omega'&=4\pi[k'^2+(\mathbf{k}_i-\mathbf{k})^2-k_F^2]/2k'|\mathbf{k}_i-\mathbf{k}| &&\text{if}\quad |\mathbf{k}_i-\mathbf{k}|^2>k_F^2-k'^2\\ &=0 &&\text{if}\quad |\mathbf{k}_i-\mathbf{k}|^2<k_F^2-k'^2\\ &=4\pi &&\text{if}\quad (k'-|\mathbf{k}_i-\mathbf{k}|)^2>k_F^2.\end{aligned} \tag{A6}$$

The integration over the angles of $\mathbf{k}$ is facilitated by the introduction of the variable $\mathbf{p}=\mathbf{k}_i-\mathbf{k}$, so that

$$d\Omega_k=2\pi d\mu_\alpha=2\pi p dp/k_i k. \tag{A7}$$

The angular integrals then give

$$(2\pi)^2\int\frac{pdp}{k_ikk'}\frac{p^2+k'^2-k_F^2}{k'p}, \tag{A8}$$

where the lower limit on the p integration is the larger of $|k_i-k|$ and $(k_F^2-k'^2)^{\frac{1}{2}}$. The upper limit on the p integration must be chosen so that

$$p_{\max}\leqq k_i+k, \tag{A9}$$

but also so that Eq. (A5) is satisfied. This requires that

$$k_i^2+4kk_i\mu+4k^2\leqq k_F^2, \tag{A10}$$

or in terms of p,

$$p^2\leqq\tfrac{1}{2}(k_F^2+k_i^2)-k^2. \tag{A11}$$

The remaining integration can be readily carried out if we now make use of the fact that the integrals over k and k' are strongly convergent so that most of the contributions to the integral come from small values of k and k'. Thus in the limits of integration on p we choose

$$p_{\max}=\tfrac{1}{2}(k_F^2+k_i^2)-k^2 \tag{A12}$$

and

$$p_{\min}=k_F^2-k'^2, \tag{A13}$$

since for small k and k', these will be the correctly chosen limits. We then change variables, letting $p^2+k'^2-k_F^2=x^2$. The integral over p becomes

$$\int_0^{\frac{1}{2}(k_i^2-k_F^2)+k'^2-k^2}\frac{x^3dx}{p}. \tag{A14}$$

Finally, since we are interested primarily in k_i^2 near to k_F^2, we replace p by k_F. The integral over x then gives

$$\frac{1}{4k_F}\left(\frac{k_F^2-k_i^2}{2}+k^2-k'^2\right)^2. \tag{A15}$$

The remaining integral over k and k' is

$$P(k_i^{-1})=\frac{3}{(2\pi)^6}(4\pi V_0\alpha M^*)^2\frac{(2\pi)^2}{4k_F^2}\int_0^\infty kdk\times\int_{[\frac{1}{2}(k_F^2-k_i^2)+k^2]^{\frac{1}{2}}}^\infty k'dk'\left(\frac{k_F^2-k_i^2}{2}+k^2-k'^2\right)^2\times(\alpha^2+k^2+k'^2)^{-4}(k^2-k'^2)^{-2}. \tag{A16}$$

In this result, to be consistent with our other approximations, we replace the multiplying factor $1/k_i$ by $1/k_F$. This integral is now of standard form; it is most easily carried out if we write

$$(\alpha^2+k^2+k'^2)^{-4}=-\frac{1}{6}\frac{d^3}{d(\alpha^2)^3}(\alpha^2+k^2+k'^2)^{-1}. \tag{A17}$$

The result of the final integrations is given in Eq. (33).

APPENDIX B. INTEGRATION FOR $V_{\text{Imaginary}}$

We wish to evaluate the integral of Eq. (36):

$$I(k_1)=\int d\mathbf{k}_id\mathbf{k}_i'(\mathbf{k}_1'\mathbf{k}_i'|K|\mathbf{k}_1\mathbf{k}_i)^2\delta(E_1+E_i-E_1'-E_i')\times p(k_i)[1-p(k_1')][1-p(k_i')]. \tag{B1}$$

We change variables to

$$\begin{aligned}\tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_i)&=\mathbf{k},\\ \tfrac{1}{2}(\mathbf{k}_1'-\mathbf{k}_i')&=\mathbf{k}',\end{aligned} \tag{B2}$$

bringing Eq. (B1) to the form

$$I(k_1)=8\int d\mathbf{k}d\mathbf{k}'\frac{(4\pi V_0\alpha)^2}{(\alpha^2+k^2+k'^2)^4}\delta\left(\frac{k^2-k'^2}{M^*}\right)p(|\mathbf{k}_1-2\mathbf{k}|)\times\{1-p[(p^2-2kp\mu'+k^2)^{\frac{1}{2}}]\}\times\{1-p[(p^2+2kp\mu'+k^2)^{\frac{1}{2}}]\}, \tag{B3}$$

where we have written $k_i-k=p$ and $k'p=kp\mu'$. The delta function on energy can be removed by using

$$\int dk'\delta\left(\frac{k^2-k'^2}{M^*}\right)=\frac{M^*}{2k} \tag{B4}$$

and everywhere setting $k'=k$. The angular integral over $\mathbf{k}$ is again simplified by the use of the variable p. The angular integrations are

$$\int d\Omega_kd\Omega_{k'}=\frac{(2\pi)^2}{kk_1}\int_{k_1-k}^{k_1+k}pdp\int_{-1}^1d\mu'p[(2p^2+2k^2-k_1^2)^{\frac{1}{2}}]\times\{1-p[(p^2+2kp\mu'+k^2)^{\frac{1}{2}}]\}\times\{1-p[(p^2-2kp\mu'+k^2)^{\frac{1}{2}}]\}. \tag{B5}$$

These integrals cannot be carried out in closed form, but they can be brought to manageable simplicity if we work only to second order in the small quantities

$$\begin{aligned}\epsilon&=k_1-k_F,\\ \epsilon'&=p-k_F,\end{aligned} \tag{B6}$$

and ρ as defined in Eq. (34). To make the dependence on these small quantities more explicit, we write

$$\begin{aligned}(p^2+2kp\mu'+k^2)^{\frac{1}{2}}&\cong k_F+\epsilon'+k\mu',\\ (p^2-2kp\mu'+k^2)^{\frac{1}{2}}&\cong k_F+\epsilon'-k\mu',\\ (2p^2+2k^2-k_1^2)^{\frac{1}{2}}&\cong k_F+2\epsilon'-\epsilon.\end{aligned} \tag{B7}$$

We then note that

$$\begin{aligned}p(x)&=1-\rho f(k_F-x), && x<k_F\\ &=\rho f(x-k_F), && x>k_F.\end{aligned} \tag{B8}$$

Working only to leading order in the small quantities, we have, for $\epsilon'>0$,

$$\int_{-1}^{1} d\mu'[1-p(k_F+\epsilon'+k\mu')][1-p(k_F+\epsilon'-k\mu')] = 2\left[\frac{\epsilon'}{k}+\frac{\rho}{k}\int_{\epsilon'}^{|\epsilon'-k|} f(x)dx\right]. \quad \text{(B9)}$$

If $\epsilon'<0$, the integral gives

$$\begin{cases} \frac{2}{k}\rho\int_{\epsilon'-k}^{2\epsilon'} f(x)dx & \text{for} \quad -\epsilon'<k, \\ 0 & \text{for} \quad -\epsilon'>k. \end{cases} \quad \text{(B10)}$$

When we introduce the variable ϵ' of Eq. (B6), the next integral over p becomes, if $\epsilon<k$,

$$\frac{2}{k}\rho\int_{\epsilon-k}^{0} d\epsilon' p(k_F+2\epsilon'-\epsilon)\int_{\epsilon'-k}^{2\epsilon'} f(x)dx + \frac{2}{k}\rho\int_{0}^{\epsilon+k} d\epsilon' p(k_F+2\epsilon'-\epsilon)\int_{\epsilon'}^{|\epsilon'-k|} f(x)dx. \quad \text{(B11)}$$

The last contribution can be dropped since in the integration over k, the contribution for k less than ϵ is of order $\epsilon^2\rho$ and hence can be neglected to our desired accuracy. The integration over ϵ' then gives, to first order in ϵ^2, $\epsilon\rho$, and ρ,

$$\frac{2}{k}\left\{\frac{\epsilon^2}{8}+\rho\int_{\epsilon-k}^{0} d\epsilon'\int_{\epsilon'-k}^{2\epsilon'} f(x)dx + \rho\int_{0}^{\frac{1}{2}\epsilon} d\epsilon'\int_{\epsilon'}^{|\epsilon'-k|} f(x)dx + \int_{\frac{1}{2}\epsilon}^{\epsilon+k} \epsilon' d\epsilon' f(2\epsilon'-\epsilon)\right\}. \quad \text{(B12)}$$

From this result we find the following for the term quadratic in ϵ:

$$\epsilon^2/4k. \quad \text{(B13)}$$

The term independent of ϵ is

$$\frac{2\rho}{k}\left[\int_{-k}^{0} d\epsilon'\int_{\epsilon'-k}^{2\epsilon'} f(x)dx+\int_{0}^{k}\epsilon' d\epsilon' f(2\epsilon')\right], \quad \text{(B14)}$$

which by a change in the order of integration becomes

$$\frac{2\rho}{k}\left\{\tfrac{1}{2}\int_{0}^{k} dx x f(x)+\int_{k}^{2k}(k-\tfrac{1}{2}x)f(x)dx +\tfrac{1}{4}\int_{0}^{2k} x dx f(x)\right\}. \quad \text{(B15)}$$

The term linear in ϵ is

$$\frac{\epsilon\rho}{k}\left\{\int_{0}^{k} f(x)dx+kf(2k)+\tfrac{1}{2}\int_{0}^{2k} f(x)dx\right\}. \quad \text{(B16)}$$

Finally the last integral over k is

$$I(k_1)=\frac{\delta(4\pi V_0\alpha)^2(M^*)^2(2\pi)^2}{2k_1}\int_{0}^{\infty}\frac{kdk}{(\alpha^2+2k^2)^4} \times\left\{\frac{\epsilon^2}{4}+\epsilon\rho\left[\int_{0}^{k} f(x)dx+kf(2k)+\tfrac{1}{2}\int_{0}^{2k} f(x)dx\right] +\rho\left[\int_{0}^{k} x f(x)dx+\tfrac{1}{2}\int_{0}^{2k} f(x)dx +\int_{k}^{2k}(k-\tfrac{1}{2}x)f(x)dx\right]\right\}. \quad \text{(B17)}$$

After slight rearrangement, this integral becomes

$$\frac{(4\pi\alpha V_0 M^*)^2(2\pi)^2}{12\alpha^6 k_1}\{\epsilon^2+\epsilon\rho F_2+\rho F_1\}, \quad \text{(B18)}$$

where F_1 and F_2 are defined in Eq. (40).