# Letters to the Editor

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## Influence of Exchange and Correlation on Electron Transport in Metals

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IN a paper[1] with the above title, Blatt has calculated the change produced in the conductivity and thermoelectric power of metals when one uses the Bohm and Pines $\epsilon-k$ curve[2] instead of the usual standard parabolic curve. The value he found for the conductivity was not very different from that obtained in the usual theory. Unfortunately, there are errors in his paper and when these are corrected, it is found that the conductivity changes markedly. In this paper we discuss only these errors and do not consider the validity of the use of the Bohm and Pines density-of-states curve nor of the use of Fermi-Dirac statistics to describe an assembly of interacting particles.

Near the Fermi energy, the $\epsilon-k$ relation is, in the theory of Bohm and Pines,

$$\epsilon(k)=(3.68/r_s^2)(m/m^*)(k^2/k_F^2)-(0.611/r_s)\times\left[1-2\beta+\frac{\beta^2k_F^2-k_F^2+3k^2}{2k_Fk}+\frac{k_F^2-k^2}{k_Fk}\ln\left(\frac{k_F+k}{k_F\beta}\right)\right], \quad (1)$$

where $\epsilon$ is in rydbergs, $r_s$ is the average interelectronic distance in units of the Bohr radius, $\beta$ is a measure of the screening of the Coulomb interaction ($\beta=0.353\ r_s^{\frac{1}{2}}$), and $k$ is the wave number vector, $k_F$ being its value at the Fermi energy.

We wish to compare the formulas for the transport phenomena for such a band with the corresponding formulas for the standard band

$$\epsilon_0(k)=(3.68/r_s^2)(m/m^*)(k^2/k_F^2). \quad (2)$$

The subscript 0 is used to describe the standard band. It is assumed that both bands contain the same number of electrons, so that the value of $k_F$ is the same in the two cases and $\epsilon_F$, the Fermi energy, correspondingly different.

The band (1) is spherically symmetric in **k** space but is nonparabolic in profile. Such bands have been studied by Radcliffe[3] and Barrie.[4] In the formal solution of Boltzmann's equation, the formulas for the non-standard band are obtained from those of the standard band by replacing $m^*$ by $\hbar^2k(d\epsilon/dk)^{-1}$. For lattice scattering in the high-temperature limit, the same substitution is made in the formula for the time of relaxation.

The time of relaxation for the standard band is[5]

$$\tau=Ck^3(m^*)^{-2}, \quad (3)$$

and this therefore becomes

$$\tau=C\hbar^{-4}k(d\epsilon/dk)^2. \quad (4)$$

(In his formula for the time of relaxation, Blatt had the second derivative of $\epsilon$ appearing; this derivative should not have entered the problem.) In the above relation, $C$ is a constant which is unchanged after the transition to the Bohm and Pines formalism.[6]

Following Blatt, we write

$$(kd\epsilon/dk)_{\epsilon_F}\equiv X_1=2A+B[\tfrac{1}{2}\beta^2-2+2\ln(2/\beta)], \quad (5)$$

where $A=(3.68/r_s^2)(m/m^*)$ and $B=0.611/r_s$. Thus the change in the time of relaxation, evaluated at the Fermi level, is

$$\tau/\tau_0=(2A/X_1)^{-2}. \quad (6)$$

The corresponding change in the density of states at the Fermi level is

$$N(\epsilon_F)/N_0(\epsilon_F)=2A/X_1. \quad (7)$$

For a spherically symmetric band, the formulas for the conductivity ($\sigma$) and the thermoelectric power ($S$) are

$$\sigma=\frac{e^2}{3\pi^2\hbar^2}K_1, \quad (8)$$

$$S=-\frac{k_0}{e}\left[\frac{1}{k_0T}\frac{K_2}{K_1}-\frac{\epsilon_F}{k_0T}\right], \quad (9)$$

with

$$K_n=-\int\tau k^2(d\epsilon/dk)\epsilon^{n-1}(\partial f_0/\partial\epsilon)d\epsilon. \quad (10)$$

In these, $k_0$ is Boltzmann's constant, $f_0$ is the Fermi-Dirac distribution function, and the other symbols have their usual meaning. Using these formulas, in the approximation of large $\epsilon_F/k_0T$, we have

$$\sigma/\sigma_0=(2A/X_1)^{-3}, \quad (11)$$

and

$$S/S_0=A(X_1+X_2)/X_1^2, \quad (12)$$

where

$$(k^2d^2\epsilon/dk^2)_{\epsilon_F}\equiv X_2=2A+B[3-\beta^2-2\ln(2/\beta)]. \quad (13)$$

The second derivative of $\epsilon$ has appeared merely because we have evaluated the integrals (10) for large $\epsilon_F/k_0T$;

TABLE I. Conductivity and thermoelectric power of the alkali metals.

| | Li | Na | K | Rb | Cs |
|---|---|---|---|---|---|
| $r_s$ | 3.22 | 3.96 | 4.87 | 5.18 | 5.57 |
| $m^*/m$ | 1.45 | 0.98 | 0.93 | 0.89 | 0.83 |
| $\beta$ | 0.634 | 0.703 | 0.78 | 0.80 | 0.834 |
| $N(\epsilon_F)/N_0(\epsilon_F)$ | 0.837 | 0.90 | 0.935 | 0.945 | 0.965 |
| $\tau/\tau_0$ | 1.43 | 1.23 | 1.14 | 1.12 | 1.07 |
| $\sigma/\sigma_0$ | 1.70 | 1.37 | 1.22 | 1.19 | 1.11 |
| $S/S_0$ | 0.81 | 0.91 | 0.99 | 1.01 | 1.05 |

the thermoelectric power is a second-order phenomenon and hence the second derivative of part of the integrand appears. (Blatt took the conductivity to be proportional to $N\tau$, further invalidating his calculations; the conductivity is proportional to $N\tau v^2$ and $v$ is replaced by $\hbar^{-1}(d\epsilon/dk)$. A similar mistake was made in his formula for the thermoelectric power.)

The modified results are shown in Table I, the last three rows being different from Blatt's results. It is to be noted that $\sigma/\sigma_0$ for lithium is 1.70, whereas Blatt found a value 1.07. This represents a considerable change in the conductivity.

The author would like to express his thanks to the Admiralty for permission to publish this work.

[1] F. J. Blatt, Phys. Rev. **99**, 1735 (1955). [Quoted by D. Pines, in *Solid State Physics*, edited by F. Seitz and D. Turnbell (Academic Press, Inc., New York, 1955), Vol. 1, p. 413].
[2] D. Bohm and D. Pines, Phys. Rev. **82**, 625 (1951); **85**, 836 (1952); **92**, 609 (1953); D. Pines, Phys. Rev. **92**, 626 (1953).
[3] J. M. Radcliffe, Proc. Phys. Soc. **A68**, 675 (1955).
[4] R. Barrie, Proc. Phys. Soc. (to be published).
[5] A. H. Wilson, *Theory of Metals* (Cambridge University Press, New York, 1953), second edition, p. 263.
[6] J. Bardeen and D. Pines (to be published).

## Cyclotron Resonance in Tin and Copper

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THE recent measurements of cyclotron resonance absorption in semiconductors[1] have stimulated interest in the possibility of observing this phenomenon in metals.[2,3] In normal metals at microwave frequencies the theoretical treatment is complicated by the intervention of anomalous skin effect conditions when the condition $\omega\tau\gg1$ is satisfied ($\omega$ being the angular frequency and $\tau$ the relaxation time of the electrons). When the dc magnetic field is applied perpendicular to the metal surface, the equations describing the behavior of a free-electron model are formally analogous to those describing the anomalous skin effect in the absence of a field, and Azbel' and Kaganov[4] and Chambers[3] have shown that in this case the surface impedance is independent of the field in the extreme anomalous limit. The more interesting case when the magnetic field is parallel to the surface and the metal is in the extreme anomalous region has been analyzed recently by Azbel' and Kaner.[5] They find that when $\omega\tau\gg1$, the resistance plotted as a function of the field should exhibit a series of resonance peaks, which occur when the cyclotron resonance frequency is near the subharmonics of the microwave frequency. For all $\omega\tau$, the resistance should decrease uniformly with increasing field for sufficiently large fields.[6] The preliminary measurements on tin and copper presented below appear to be consistent with this description. A brief report on this work has already appeared.[7]

The resistance is measured calorimetrically by a method similar to one described previously.[8] The specimen, in the form of a disk, is suspended in a vacuum chamber immersed in liquid helium, and faces the open end of a square wave guide. Microwaves at a frequency of 24 kMc/sec may be propagated in the wave guide in either of the two principal modes so that the direction of current flow is either parallel or perpendicular to the magnetic field, which is applied as nearly as possible parallel to the surface of the specimen. A link of high thermal resistance is provided between the specimen and the helium bath so that the specimen temperature rises, with a fairly short time constant, by an amount proportional to the heat generated by the incident microwaves, which under constant current conditions is a measure of the resistance. The rise in temperature is measured by means of a carbon composition thermometer attached to the back of the specimen. Spurious heating of the thermometer by microwaves leaking from the wave guide past the edge of the specimen is prevented by suitably disposed circular choking grooves. The values of $\omega\tau$ were calculated from the measured dc resistances of the specimens at 4.2°K

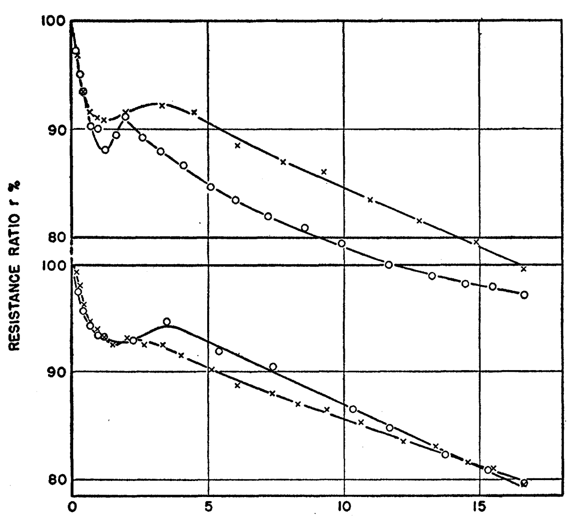


FIG. 1. Cyclotron resonance absorption in tin at 4.2°K for plane polarized radiation near 24 kMc/sec and magnetic field parallel to the metal surface. Specimen orientation—$\theta=59°$, $\phi=43°$; upper curves—$J\|X$, lower curves—$J\|Y$; ○—$H\|J$, ×—$H\perp J$; $\omega\tau=27$.