PHYSICAL REVIEW VOLUME 103, NUMBER 5 SEPTEMBER 1, 1956

p-p Interactions at 3 Bev*

R. Cester, T. F. Hoang,† and A. Kernan‡
Department of Physics, University of Rochester, Rochester, New York

(Received March 20, 1956)

Interactions initiated by 3-Bev protons of the Brookhaven Cosmotron were studied by photoemulsion technique. With appropriate criteria, 115 events are attributed to interactions of the incident beam protons with hydrogen nuclei (~55%) and with bound protons of other nuclei (~45%). A detailed analysis allowed the subdivision of the 115 events in categories, according to the number of π mesons ($N \geqslant 0$) produced in the collision. The ratio of elastic scattering to the total number of events was estimated to be $\sigma_{\mathrm{el}}/\sigma_{\mathrm{total}} = 0.20_{-0.07}^{+0.04}$. The observed cross section for pure elastic scattering is $\sigma_{\mathrm{el}} = 8.9 \pm 1.0$ mb.

The percentages of single, double, triple, and quadruple π-meson production are respectively: 34_{-20}^{+22}; 35.6_{-23}^{+20}; 9.6_{-4}^{+6}; $\sim 1.0^{+3.5}$.

Among the 20 most probable cases of single π-meson production—the estimated ratio of π^+ to π^0 is $\sigma_{\pi^+}/\sigma_{\pi^0} = 5.3_{-1.4}^{+0.3}$. The experimental results are not in agreement with the Fermi statistical-model theory (in particular the lower limit for the experimental ratio of triple to single production is given by $\sigma_3/\sigma_1 > \sim 1/10$ in contrast with the predicted ratio $\sigma_3/\sigma_1 = 1/67$) but are not inconsistent with the Peaslee excited-state-model theory.

I. INTRODUCTION

One of the problems of great interest in recent years has been that of the study of the interaction of high-energy nucleons. In particular, one would like to study the nucleon-nucleon interaction at high energies in order to obtain information on the nature and strength of the nucleon-nucleon interaction and the coupling of the nucleon field to the various types of meson fields. One of the primary questions relevant to the nature of the fundamental interactions is that related to the existence or nonexistence of multiple π-meson production in nucleon-nucleon collisions. A great wealth of experimental information bearing on this problem has been obtained from studies of the interactions of cosmic rays. These studies have, however, suffered from the facts that the intensity is extremely low, and the beam is not monoenergetic and has a complex composition. Because of these limitations it is not possible to study the nucleon-nucleon interaction directly. However, from the wealth of data obtained by many different laboratories[1] it seems rather conclusive that the nucleon-nucleon interaction is indeed a strong one and that at sufficiently high energies multiple π-meson production occurs. It was not possible in the experiments, however, to determine in a meaningful way the effective threshold for multiple production nor the relative probabilities for production of different numbers of π mesons. In fact, the cosmic-ray data at lower energies could be made consistent with the Heitler-Janossy[2] plural production mechanism. However, with the advent of the high-energy accelerators at Brookhaven National Laboratory and the Radiation Laboratory at Berkeley, monoenergetic sources of 3- and 6-Bev protons are available and it is then possible to study in a more definitive way the nucleon-nucleon interaction at a known energy. It was first shown by Shutt and his collaborators[3] that multiple π-meson production does indeed occur at incident nucleon energies as low as 2.2 Bev. It is the purpose of this paper to report on a study of the *p-p* interaction for 3-Bev incident protons as observed in nuclear emulsions and to compare the results with available theories.

II. EXPERIMENTAL

A small stack of Ilford G-5 stripped emulsions was exposed to one pulse of the 3-Bev internal beam at the Brookhaven Cosmotron. The plates were then developed by standard methods and scanned under high magnification with the on-track-scanning technique. This technique is preferred to the "area-scanning" technique for the following reasons: (1) The main purpose of the experiment was the study of *p-p* interactions originated by 3-Bev protons of the internal beam. In the majority of events of this type most of the visible prongs are usually at minimum grain density and the events could be easily missed by "area scanning." (2) The "on-track scanning" allows an unbiased detection of events and thus enables one to determine also characteristic mean free paths. With this procedure the plates were scanned and all interactions recorded.

We accepted for analysis as *p-p* interactions those events that obeyed the following criteria necessary to insure their compatibility with the interaction of an

---

* This work was assisted by the U. S. Atomic Energy Commission.

† In leave of absence from the Laboratoire Leprince-Ringuet, Ecole Polytechnique, Paris.

‡ American Association of University Women fellow.

[1] M. L. Vidale and M. Schein, Phys. Rev. **87**, 71 (1952); W. Bosley and H. Muirhead, Phil. Mag. **43**, 783 (1952); Weaver, Long, and Schein Phys. Rev. **87**, 531 (1952); A. B. Weaver, Phys. Rev. **90**, 86 (1953); Kusumoto, Miyake, Suga, and Watase, Phys. Rev. **90**, 998 (1953); McCusker, Porte, and Wilson, Phys. Rev. **91**, 384 (1953).

[2] W. Heitler and L. Janossy, Proc. Phys. Soc. (London) **62**, 669 (1949).

[3] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **95**, 1026 (1954).

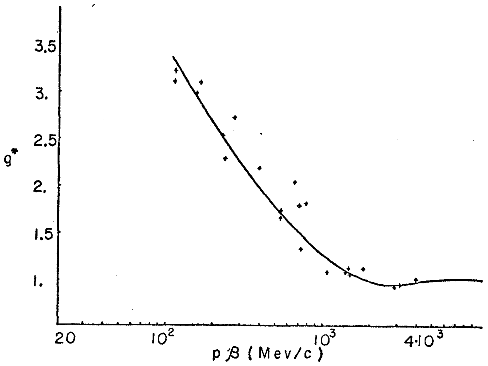


FIG. 1. Calibration curve for protons.

incident proton with either a free proton of the emulsion or a bound proton of an emulsion nucleus: (1) An even number of visible prongs emerging from the interaction, of which at most two are protons. (2) No evidence of nuclear recoil or of β-ray emission. (3) A proton emerging at an angle θ cannot have an energy greater than an elastically scattered proton at the same angle.

Criterion (1) is just that of conservation of charge and of number of interacting nucleons. Criterion (2) is peculiar to emulsion. In nuclear emulsion the greatest portion of the mass is contained in AgBr crystals. The A-1 isotones of Ag and Br are not β active while the A-1 isotopes are. A similar statement holds for the rest of the emulsion constituents. Therefore the lack of observation of a β ray is at least consistent with an interaction with a bound proton but does not of course prove it, since the efficiency for detecting β rays is not 100%. The condition on lack of nuclear recoil is a requirement that no nuclear cascade has occurred. Criterion (3) is less definite since the momentum of a bound proton in a nucleus is not well defined. We have taken as the limiting angle and energy that which corresponds to the interaction of the incident proton with a bound proton of maximum Fermi energy and directed opposite to the incident one.

Of 315.5 meters of track scanned by the more efficient scanner, 81 events could be interpreted as *p-p* collisions, giving a mean free path of 390 cm. By linearly extrapolating to 3 Bev the results of Shapiro, Chen, and Leavitt[4] for the total *p-p* cross section obtained by counter measurements and using the known composition of the emulsion, a mean free path of 704 cm is expected for collision with the free hydrogen of the emulsion. This indicates that approximately one-half of the accepted *p-p* interactions are with bound protons of emulsion nuclei. Any attempt to give a meaningful estimate of the *p-p* interaction cross section thus seems rather unfruitful. The unavoidable inclusion of edge collisions among the analyzed events does not, on the other hand, prevent the study of the relative probabilities of different modes of interaction (elastic scattering, single, double, triple . . . N π-meson production) which is the main object of our experiment. However, due to their inclusion, additional ambiguities other than those inherent from the limited obtainable data arise in the interpretation of some events; this is due to the fact that the momentum of the target proton, which is zero for pure *p-p* collisions, is indeterminate, both in magnitude and direction, for an edge collision, and allows in some events alternate classification depending on the assumed target proton momentum.

III. ENERGY AND MASS MEASUREMENTS

The kinetic energy of the beam was first checked by the relative scattering technique.[5] The result obtained was $E_p=2.95\pm0.15$ Bev, in good agreement with the *a priori* expected value of 3 Bev.

For each of the outgoing tracks in the accepted interactions, limiting mass and energy values were obtained by a combination of grain counting (g) and range and scattering measurements ($P\beta$), when possible.

As Voyvodic[6] has recently shown, the shape of the ionization $-P\beta$ curve for particles more massive than electrons is particularly sensitive in the region of minimum ionization to the processing of the emulsion. The first requirement was therefore to obtain the $g-P\beta$ calibration curve, which is shown in Fig. 1. This was done by measuring the $P\beta$ of 22 identified protons originating from interactions taking place in the emulsion, and having a length >2 cm and a grain density ranging from $28/75\,\mu$ (minimum) to $100/75\,\mu$. The "plateau" grain density was obtained by grain counting electron tracks from $\pi-\mu-e$ decays and shows a rise of less than 6% over the minimum grain density as obtained by grain counting of beam protons in the same region of the plate. Because of the small difference between "minimum" and "plateau" grain density it was impossible to differentiate by a combination of grain density and $P\beta$ measurements, between protons and pions for $P\beta>1.5$ Bev, even with good statistics.

In an experiment of this type it is necessary to include all events without bias with respect to their geometry, since any criteria on favorable geometry may preclude a certain class of final states. This of course leads to further difficulties in interpretation, for there will then exist secondaries about which little information is obtainable. For those secondaries which were relatively short and at or near minimum ionization, no precise information is obtainable other than that from kinematic considerations. If the ionization of a secondary was appreciably above minimum, it was possible in some cases to differentiate between π mesons and protons by scattering measurements. In this differentiation no allowance was made for inclusion of

---

[4] C. P. Leavitt, *Proceedings of the Fifth Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1955), p. 41.

[5] M. Koshiba and M. F. Kaplon, Phys. Rev. **97**, 193 (1955).

[6] L. Voyvodic (private communication).

K mesons since their relative production cross section is low at these energies ($N_{K^+}/N_{\pi^+} \lesssim 1/500$) and the accuracy of the measurements is in general never precise enough to distinguish between K mesons and protons. In some cases, identification of tracks was possible from dynamical considerations.

(a) In the case of a pure p-p collision, a maximum angle of emergence of the proton in the lab system can be calculated from the well-known transformation formula[7]

$$\tan\theta_{L\,\max} = [(1-\beta_c^2)/(m^2-1)]^{\frac{1}{2}},$$

where $m=\beta_c/\beta_i$, β_c is $(1/c)$ times the velocity of the center of mass, and β_i is the velocity of the protons in the c.m. system when 1,2,3, . . . N pions are emitted at rest in the c.m. system (maximum value of β_i).

(b) In the case of edge collisions, the proton can be emitted at any angle but it is still possible to calculate a maximum energy of the proton in the lab system as a function of the angle θ of emission. Again the maximum condition corresponds to emission of the pions at rest in the c.m. system. From (a) and (b) we can, for example, immediately identify as a π meson a track at or near minimum ionization emerging at 90° with respect to the beam direction (this is an example of identification by dynamical considerations).

IV. RESULTS

(a) Elastic Scattering

The cases of elastic scattering could be identified by energy and angle measurements and by the coplanarity test. If elastic scattering takes place in a pure p-p collision, the energy of the emitted protons and the included angle between them is a single-valued function of the angle of emission, and in addition the plane containing the two emerging protons must also contain the incident proton. This interpretation can be carried out in most of the cases unambiguously. Fifteen of the 115 p-p collisions (of which approximately $\frac{1}{2}$ are pure) analyzed belong to this category; of these, only three could be alternatively interpreted as π-meson production. If elastic scattering occurs in an edge collision, the Fermi momentum of the target proton has the effect of destroying the uniqueness in the angle-energy correlation of the emitted protons. In many of these cases it was impossible to differentiate between an elastic edge collision and π^0 production. For this reason we are bound to assign lower and upper limits to the number of events included in this category. This number is estimated to be 8_{-5}^{+6}.

The results on elastic scattering support the conclusion, deduced from the total cross-section measurement, that ~55% of the analyzed events are pure p-p collisions.

The elastic scattering cross section σ_0, deduced from the 15_{-3}^{+0} cases of pure p-p collisions and the composition of the emulsion, is $7.9<\sigma_0<9.9$ mb. The corresponding value of $\sigma_0=9.8$ mb obtained by linear extrapolation to 3 Bev of the results of Smith, McReynolds, and Snow[4] agrees satisfactorily with this result and lends support to our procedure.

In Fig. 2 we have plotted the angular distribution in the center-of-mass system for protons emitted in pure elastic collisions. The strong peak in the forward direction indicates that most of the elastic scattering is diffraction scattering. In order to test this hypothesis, we have calculated the diffraction spectrum produced if the incoming proton beam felt the target proton as an opaque sphere of radius R. The value of the parameter R to be used in the calculation was obtained from the experimental total cross section by using the relation $\sigma_T=\pi(\lambda\!\!\!^{-}+R)^2$ and was found to be $R=1\times10^{-13}$ cm. The diffraction-scattering angular distribution[8] is also plotted in Fig. 2. A comparison with the experimental angular distribution shows that diffraction scattering accounts for most of the elastic scattering, though a better fit with experiment could be obtained by considering the target nucleon as a sphere with an opaque core and semitransparent edges. This would in fact have the effect of broadening the first maximum of the

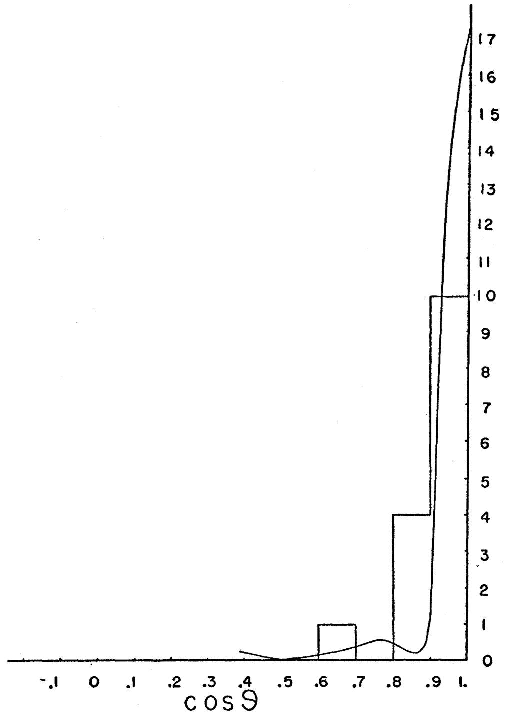


FIG. 2. Elastic scattering angular distribution. The curve is the diffraction scattering angular distribution calculated for $l_{\max}=6$ and $R=1\times10^{-13}$ cm and normalized to the total experimental cross section.

---

[7] Bradt, Kaplon, and Peters, Helv. Phys. Acta 23, 24 (1950).

[8] Normalized with respect to the total area covered by the histogram which represents the angular distribution of the 15 most probable cases of pure elastic collision.

diffraction spectrum, and shift to the right the secondary maxima. Because of the large number of partial waves involved, this result could, however, not be obtained in an unique way. The contribution of Coulomb scattering was also calculated and was found to be negligible for $\theta > 3°$.

(b) Meson Production—2-Prong Events

Most of the two-prong events could not be interpreted as elastic scattering because: (a) one or both of the emitted particles were positively identified as π mesons, or (b) the measured energy of the emitted proton was incompatible with its angle of emission. The compatibility of these events with one of the following interaction schemes (which conserve charge and nucleon number) was then studied.

(a) $p+p \rightarrow p+\pi^+ + n$
$\rightarrow p+\pi^+ + n + N\pi^0, \quad N \geqslant 1.$
(b) $p+p \rightarrow p+p+N\pi^0, \quad N \geqslant 1.$
(c) $p+p \rightarrow 2\pi^+ + 2n$
$\rightarrow 2\pi^+ + 2n + N\pi^0, \quad N \geqslant 1.$

Let us first consider the cases in which both of the emitted particles were positively identified by mass measurement or by dynamical consideration, and their energy and momentum fairly well determined. The direction cosines of the charged prongs were determined and the missing mass obtained from the relation

$$[P_0 + P - (P_1 + P_2)]^2 = M^2, \tag{1}$$

where P_0, P, P_1, P_2 are the 4-vectors of momentum and energy, respectively, of the beam proton ($\mathbf{P}_0$), the target proton ($\mathbf{P}$), and the two emitted particles ($\mathbf{P}_1, \mathbf{P}_2$). If the two particles were identified as one proton and one pion [case (a)] and the missing mass was found to be $M \geqslant$ a nucleon mass (for $\mathbf{P}=0$), then the interaction was accepted as a p-p event and was included (1) in the single π^+ production category if the mass was in good approximation equal to one nucleon mass, and (2) in the multiple production category if $(M^2 - M_n^2)$ was found to be $\geqslant \mu^2 + 2\mu M$ (where M is the calculated missing mass and M_n, μ are respectively the mass of the nucleon and of the π).[9] If the missing mass was less than a nucleon mass, the interaction could still be interpreted as an edge collision and the Fermi momentum of the target nucleon taken into consideration. If after consideration of the Fermi momentum of the target nucleon, the missing mass remained negative, the event was rejected as a p-p collision, some nuclear cascade effect being probably involved in the emission process. Some attempt was also made to differentiate between double and triple (or more) π production, simply by considering as double production the events in which the missing mass M satisfies $M_n + 2\mu M_n \leqslant M \leqslant M_n + 3\mu M_n \approx 1.5$ Bev. The events with $M > 1.5$ Bev are regarded as most probably double production events but are also included in the computation of the upper limit of triple production events. In a similar way the interactions of class (b) were studied (two identified protons emitted). In this case the mass of the nucleon has to be substituted for the pion mass in all the written conditions, as a consequence of the fact that the neutral particles emitted are always π^0 mesons. The classification into the categories of (b) is then made in similar way to that of (a).

TABLE I. Two-prong events—total number = 97.

Elastic scattering	Single π prod.	Single or double π prod.	$\geq 2\pi$ production
23_{-8}^{+6}	20_{-3}^{+5}	39_{-6}^{+6}	15

The single π-meson production in pure p-p collisions could be completely reconstructed both in the lab and in the c.m. systems, the momentum, energy and direction cosines of the neutral particle being determined by energy and momentum conservation arguments. If the two particles were both identified as π mesons, single production is obviously excluded and the calculated mass must be $\geqslant 2M_n$. Unless this condition was satisfied the event was rejected.

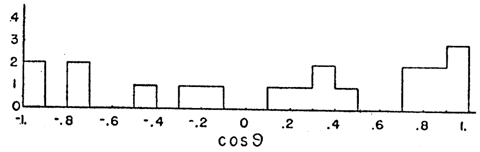


FIG. 3. Differential angular distribution in the c.m. system of the pions emitted in single production events.

The most probable interpretation of the analyzed two prong events with lower and upper limits of error is given in Table I. The angular distribution in the c.m. for the single emitted π mesons is plotted in Fig. 3. As should be expected, the distribution is approximately symmetric with respect to 90°

(c) Meson Production—4-Prong Events

Seventeen of the thirty-four 4-prong events analyzed can be interpreted as p-p collisions. (Those excluded had a negative missing mass, allowing for all limits of error and Fermi momentum.) The ratio of four-prong events to inelastic collisions is then $17/92 = 0.185$, in good agreement with the corresponding ratio 27/147 obtained in cloud chamber experiments with a beam energy of 2.7 Bev.[10]

---

[9] This condition was obtained in the following way: When two neutral particles are emitted in one of the (a)-type interactions, the explicit form of the right side of (1) becomes $M^2 = (P_n + P_{\pi^0})^2 = M_n^2 + \mu^2 + 2E_{\pi^0}E_n - 2\mathbf{P}_\pi \cdot \mathbf{P}_n$ (where the E_i are the total energies and the $\mathbf{P}_i$ the momenta of the neutral particles). This expression obviously has a minimum when $\mathbf{P}_n = \mathbf{P}_{\pi^0} = 0$, which is equivalent to the statement that double production is only possible for interactions of the (a) type if $(M^2 - M_n^2) \geqslant 2M\mu + \mu^2$. The generalized condition for the case of N π^0-meson production is obtained with an analogous procedure and is $M \geqslant M_n + N\mu M_n$.

[10] R. P. Shutt, *Proceedings of the Fifth Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1955), p. 44.

In the majority of the cases considered, it was not possible to identify positively all of the four charged particles emitted. Collisions of the type $p+p\rightarrow p+p+\pi^{+}+\pi^{-}$ could, however, be unambiguously classified and completely reconstructed both in the lab system and in the c.m. system when at least two of the emitted particles were known and their energy and momentum accurately determined. This was done by missing-mass determination and the transverse-momentum-balance test. Eleven of the seventeen 4-prong p-p interactions could not be classified as double production because (a) 3 or more of the emitted particles were positively identified as π mesons; (b) no transverse momentum balance consistent with energy conservation could be obtained. Of these eleven events, seven were positively identified as 3π production and could be completely reconstructed both in the lab system and in the c.m. system, the momentum, energy, and angle of emission of the neutral particle being determined by energy- and momentum-conservation arguments. The remaining cases could be interpreted either as triple or quadruple production. A definite case of 4π production was also identified among the four 6-prong events analyzed.

TABLE II. Summary of results. Total number of interactions = 115.

	Elastic scattering	Single prod.	Double prod.	Triple prod.	Quad-ruple prod.
Experimental results	23_{-8}^{+5}	39_{-23}^{+25}	41_{-27}^{+23}	11_{-5}^{+7}	1_{-0}^{+4}
Experimental percentage	20_{-7}^{+5}	34_{-20}^{+22}	35.6_{-23}^{+20}	9.6_{-4}^{+6}	$0.9^{+3.5}$
Predicted by Fermi theory (percent)	3	67	29	1	...

V. COMPARISON WITH THEORIES

In Table II we give the complete results of the analysis with upper and lower limits of error. The percentages for the various processes are compared with those calculated on the basis of the Fermi statistical theory.[11] As is well known, the basic idea of this theory is that in nucleon-nucleon collisions at high energy, statistical equilibrium is attained among all possible final states in a volume Ω with radius $\sim\hbar/\mu c$. The matrix element for a transition from an initial state with two nucleons to a final state in which n pions are produced, becomes then simply proportional to $(\Omega/V)^{\frac{1}{2}(2+N)}(1/N!^{\frac{1}{2}})$, where V is a normalization volume and the factor N takes care of the degeneracy due to the indistinguishability of the pions. Conservation of isotopic spin is assumed, and the neutral, positive, and negative pions have therefore to be considered as different charge states of one single particle. The density of final states in momentum space was calculated from the approximate formula[12] of Lepore and Stuart, and the resulting relative probabilities were weighted in the isotopic spin space following the procedure used by Fermi. From Table II, we read that the Fermi theory predictions underestimate elastic scattering and triple production (the calculation was done neglecting 4π production). A comparison between the predicted and experimental ratio σ_2/σ_1 is not meaningful due to the large uncertainties in the differentiation of these two modes of interaction. However, the lower limit for the experimental ratio σ_3/σ_1 is 1/10 compared to 1/67 predicted for the Fermi theory. It appears that Fermi's theory does not give the correct results at 3-Bev energy. Of the 20 reasonably definite cases of single production, 16 are π^{+} and 3 are π^{0} production events (the remaining one could be interpreted either as π^{+} or π^{0} emission); this yields a ratio $\sigma_1(\pi^{+})/\sigma_1(\pi^{0})=5.35_{-1.4}^{+0.3}$, in apparent disagreement with the prediction of the Fermi theory, $\sigma_1(\pi^{+})/\sigma_1(\pi^{0})=3$. The ratio $\sigma_2(\pi^{+},\pi^{0})/\sigma_2(\pi^{+},\pi^{-})$ appears also to be higher (by a factor $\sim$2) than the value $\frac{3}{2}$ predicted by Fermi, although in this case an accurate estimate is not possible.[13] The c.m. momentum distribution of the π-mesons emitted in single production was also plotted and compared with Fermi's theoretical prediction (Fig. 4). The maximum of the histogram which gives the experimental distribution, is shifted toward the lower energy region with respect to the maximum of the spectrum (solid curve) calculated on the basis of the Fermi theory.[14]

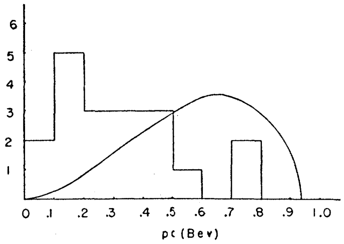


FIG. 4. Momentum distribution in the c.m. system of the pions emitted in single-production events. The curve represents a theoretical prediction from the Fermi statistical theory.

The experimental results can also be compared with the predictions of the excited-state model of Peaslee,[15] which assumes that pion production takes place in two steps: (1) one or both of the colliding protons are excited to a state of isotopic spin $\frac{3}{2}$, and (2) the excited state decays, emitting a π meson. In order to explain triple and quadruple π-meson production, the possibility of excited states of higher isotopic spin have to be taken into consideration. This theory predicts a ratio

---

[11] E. Fermi, Progr. Theoret. Phys. (Japan) **5**, 570 (1950); E. Fermi, Phys. Rev. **92**, 452 (1953); Phys. Rev. **93**, 1434 (1954).

[12] J. V. Lepore and R. N. Stuart, Phys. Rev. **94**, 1724 (1954).

[13] The process $p+p\rightarrow p+n+\pi^{+}+\pi^{0}$ seems to be most frequent one between the modes of double π-meson production, whereas double π^{+}-meson and double π^{0}-meson production seems to be rather rare.

[14] C. N. Yang and R. Christian, Internal Report of the Brookhaven National Laboratory (unpublished).

[15] D. Peaslee, Phys. Rev. **94**, 1085 (1954); **95**, 1580 (1954).

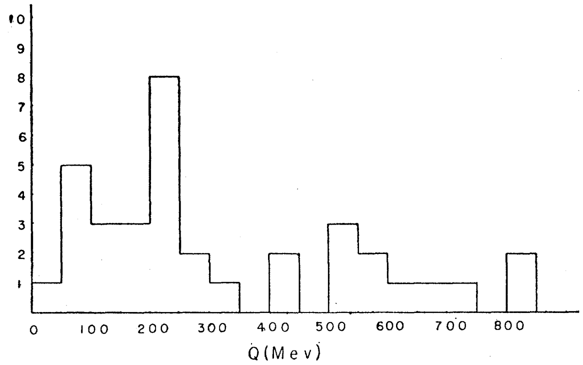


FIG. 5. Sum of the two possible sets (A and B) of Q values calculated for the decay of an excited nucleon ($T=\frac{3}{2}$) into (A) a pion and the outcoming nucleon with less energy in the c.m. system (B) a pion and the outcoming nucleon with largest energy in the c.m. system for the cases of single π-meson production events.

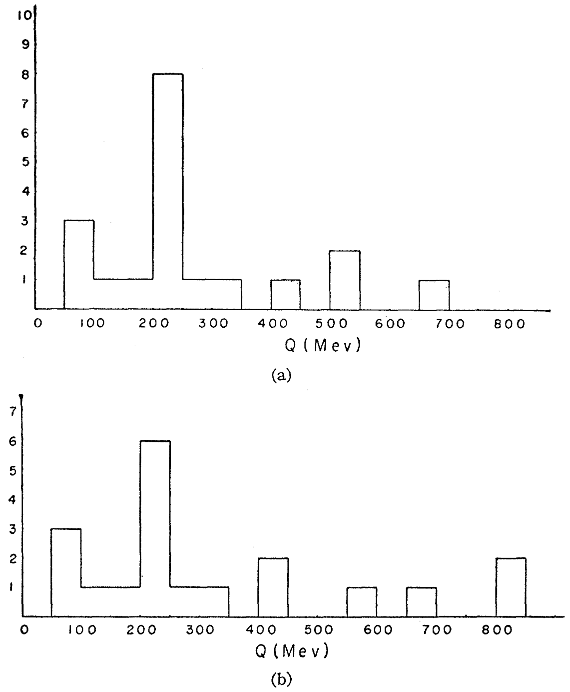
$\sigma_1(\pi^+)/\sigma_1(\pi^0)=5$, in good agreement with the value we have found experimentally. This type of theory also explains qualitatively the shift of the maximum of the c.m. pion momentum distribution toward low energies with respect to a statistical theory prediction. An excited-state model would account for a shift to a lower momentum for the pion since the phase space available is reduced. If the concepts of the Peaslee model are correct, a histogram of Q values calculated for the $\mathfrak{N}^* \rightarrow \mathfrak{N}+\pi$ reaction should show some evidence of a peak. A difficulty arises in the calculation of the distribution of Q values since we do not know *a priori* which one of the two outcoming nucleons is, together with the pion, the decay product of the excited state. An attempt to overcome this difficulty was made in the following way: we first plot in Fig. 5 the two series of Q values obtained if (a) the decay products are the pion and the nucleon of lower energy in the c.m. system, or (b) the excited state decay products are the pion and the nucleon with larger energy in the c.m. system. The sum of these two distributions shows a definite peak for $200 \leqslant Q \leqslant 250$ Mev. If this feature is due to the existence of an intermediate excited state, we should expect one and only one of the possible two Q values calculated for every single π-meson production event to fall in the region of the peak. (A certain spread is expected in the Q-value distribution, mainly due to the errors in the measured values of energy and momenta of the particles involved.)

In Fig. 6(a) we have plotted a distribution obtained by choosing in every case the Q value closer to the peak region $200 \leqslant Q \leqslant 250$ Mev. This allows a criterion for establishing which of the two outcoming nucleons is, together with the pion, the decay product of the excited state, and consequently permits an estimate of the ratio $\sigma_1(T_z'=\frac{3}{2},\ T_2=-\frac{1}{2})/\sigma_1(T_z'=\frac{1}{2},\ T_z=\frac{1}{2})$, where T_z is the z component of the isotopic spin and the prime refers to the excited state. This ratio is found to be equal to 1 and does not agree with the prediction of the Peaslee theory, which gives $\sigma_1(T_z'=\frac{3}{2},\ T_z=-\frac{1}{2})/\sigma_1(T_z'=\frac{1}{2},\ T_z=\frac{1}{2})=3/1$. If, on the other hand, we choose the Q value set so that only those events contribute to $\sigma_1(T_z'=\frac{1}{2},\ T_z=\frac{1}{2})$ which cannot have an intermediate state $(T_z'=\frac{3}{2},\ T_z=-\frac{1}{2})$ because (1) a π^0 is emitted or (2) the Q value corresponding to a $(T_z'=\frac{3}{2},\ T_z=\frac{1}{2})$ intermediate state is negative, we obtain for the ratio $\sigma_1(T_z'=\frac{3}{2},\ T_z=-\frac{1}{2})/\sigma_1(T_z'=\frac{1}{2},\ T_z=\frac{1}{2})$ the value 14/5 which agrees with the one predicted by Peaslee. Moreover, the distribution of Q values so obtained [Fig. 6(b)] still shows a fairly sharp peak for $200 \leqslant Q \leqslant 250$ Mev.

VI. CONCLUSIONS

The following conclusions have been drawn from an analysis of 115 p-p collisions with incident laboratory beam kinetic energy $E_p=3$ Bev: The measured elastic cross section for pure p-p interactions is in good agreement with the linearly extrapolated value of the results obtained from counter measurements. The ratio σ_1/σ_2 (of single to double π-meson production) is not very well determined but agrees, within the limits of error, with the preliminary results reported by the Brookhaven cloud chamber group. They obtained a value $\sigma_1/\sigma_2 \simeq 1/1.5$. There is also evidence that 4-pion production starts at this energy.

A comparison of the results with the Fermi statistical-



(a)

(b)

FIG. 6. (a) Distribution of Q values calculated for the decay of an excited nucleon ($T=\frac{3}{2}$) into a nucleon and a pion for the cases of single π-meson production. For every event, of the two possible Q values, that value was plotted which falls the nearest to the peak region $200 \leqslant Q \leqslant 250$ Mev. (b) Alternative distribution of Q values obtained by imposing the condition (suggested by the Peaslee model predictions) for the ratio $\sigma_1(T_z'=\frac{3}{2},\ T_z=-\frac{1}{2})/\sigma_1(T_z'=\frac{1}{2},\ T_z=\frac{1}{2})$ to be as large as possible.

theory model and with the Peaslee excited-intermediate-state model shows that: (i) The predictions of the Fermi theory, in its present formulation, do not agree at this energy with the experimental results. In particular, it predicts a ratio σ_3/σ_1 which is at least 7 times smaller than the experimental one. (ii) The Peaslee model agrees, at least from a qualitative point of view, with experimental results. There is also an indication of the possible existence of an intermediate excited state which decays with $200 \leqslant Q \leqslant 250$ Mev, although the statistics do not allow a positive statement concerning the existence of such an intermediate state.

VII. ACKNOWLEDGMENTS

We wish to thank Dr. M. Kaplon for his advice and encouragement throughout the course of this experiment and Mrs. V. Miller and Mrs. J. Milks for the assistance in scanning the plates. We are also indebted to the Brookhaven Cosmotron staff for their assistance in obtaining the exposure.

PHYSICAL REVIEW VOLUME 103, NUMBER 5 SEPTEMBER 1, 1956

Pais-Piccioni Experiment*

K. M. Case
H. M. Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan

(Received April 30, 1956)

Two experiments to check the Gellmann-Pais "particle mixture" suggestion are analyzed, by using a detailed, if crude, phenomenological description. A cloud-chamber experiment of the type proposed by Pais and Piccioni is compared to an experiment using a liquid-xenon bubble chamber. It is found that for likely values of the parameters involved, both experiments are considerably more difficult than envisaged by Pais and Piccioni. For intensity reasons, the bubble-chamber experiment seems preferable. However, if accidentally the relevant cross sections have rather special values, experiments of this type would be particularly easy.

I. INTRODUCTION

RECENTLY Pais and Piccioni[1] have proposed a cloud-chamber experiment to verify the Gell-Mann-Pais[2] suggestion that the θ^0 should be considered as a "particle mixture." In view of the availability of a xenon-filled bubble chamber,[3] it is of some interest to compare the feasibility of a bubble-chamber experiment with the Pais-Piccioni experiment.

Below, a set of phenomenological equations describing a crude model of these experiments is given. Solutions for the two types of experiments are obtained. From these it follows that for likely values of the parameters involved: (a) both experiments are considerably more difficult than anticipated by Pais and Piccioni; (b) bubble-chamber experiments are somewhat more feasible; (c) if the $\bar{\theta}^0$ absorption cross section and the θ_1^0, θ_2^0 mass difference are miraculously just right, experiments of this sort will be especially straightforward.

II. PHENOMENOLOGICAL DESCRIPTION

Our model is the following: The wave functions describing θ^0 and $\bar{\theta}^0$ particles are to be linear combinations with prescribed phases of functions describing θ_1^0 and θ_2^0 particles. Specifically, we have

$$\Psi(\theta^0)=(\Psi_1+i\Psi_2)/\sqrt{2}, \tag{1a}$$

$$\Psi(\bar{\theta}^0)=(\Psi_1-i\Psi_2)/\sqrt{2}. \tag{1b}$$

The θ_1^0 is to undergo the familiar two-π decay with lifetime $\tau_1 \sim 1.5\times 10^{-10}$ second. The θ_2^0 has a completely different set of decay modes with a lifetime $\tau_2 \gg \tau_1$. (To obtain detailed numerical results, we will idealize this and put $\tau_2=\infty$.) In passing through matter, the $\bar{\theta}^0$ can be absorbed while the θ^0 cannot. (For simplicity we follow reference 1 and omit consideration of all other processes.) This absorption will be described by an effective lifetime τ. Clearly,

$$1/\tau = N\rho v\sigma_a, \tag{2}$$

where v is the velocity of the $\bar{\theta}^0$, ρ the density of absorber material, N the number of absorbing particles per gram, and σ_a the absorption cross section per particle.

Let the wave function describing the state at time t after a θ^0 is produced be

$$\Psi(t)=\alpha_1(t)\Psi_1+\alpha_2(t)\Psi_2. \tag{3}$$

We have

$$\frac{d\Psi(t)}{dt}=\left[\frac{d\Psi(t)}{dt}\right]_{\rm ph}+\left[\frac{d\Psi(t)}{dt}\right]_{d}+\left[\frac{d\Psi(t)}{dt}\right]_{a}. \tag{4}$$

Here $[d\Psi(t)/dt]_{\rm ph}$ describes the phase change due to

---

* Supported in part by the Office of Naval Research, U. S. Navy Department.

[1] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[2] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[3] Brown, Glaser, and Perl, Phys. Rev. **102**, 586 (1956).