Deuteron Reactions and the $A=14$ Polyad*

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Possible deuteron stripping and pickup reactions involving various states for $A=14$ and the two one-hole states for $A=15$ are discussed in detail. They would give considerable information about the $A=14$ wave functions (particularly the C^{14} and N^{14} ground states) and also some valuable measurements of the basic single-particle Butler cross section.

I. INTRODUCTION

RECENT experimental work by Sherr[1] and his collaborators and theoretical work by Jancovici and Talmi[2] and by Visscher and Ferrell[3] have intensified interest in the long-standing problem of the polyad $A=14$. It now seems that, as a result of this work, the ground state wave functions for N^{14} and C^{14} may be regarded as rather well known. However, because of the particular interest in these nuclei we discuss here the information which may be derived by deuteron stripping and pickup reactions involving the nuclei with $A=14$, 15. We consider mainly reactions involving the first two states for $A=14$ ($J=1$, $T=0$ and $J=0$, $T=1$) and the two one-hole states ($J=\frac{1}{2}^-,\frac{3}{2}^-$) for $A=15$. This case is unusual because, in the p^n configuration,[4] the $A=15$ states (and in fact some of the higher $A=14$ states too) are unique and involve no unknown parameters; also the anomalous lifetime of C^{14} makes it available as a target. Purely apart from the information to be gained about the $A=14$ wave functions, it will appear too that the experiments will give information about the stripping process which will be of value for other nearby nuclei.

II. RELATIVE DEUTERON CROSS SECTIONS AND THE $A=14$ WAVE FUNCTIONS

We first give expressions for the relative stripping or pickup cross sections involving the first two levels for $A=14$ and the two one-hole levels for $A=15$. There are four basic cross sections and these define four relative reduced widths, S_0, S_1, S_0^*, S_1^*; the subscript here defines the J value for $A=14$; S_0, S_1 involve the $A=15$ ground state and S_0^*, S_1^* the excited state. Specifically, we shall understand by any S the ratio of the reduced width divided by the appropriate isotopic spin coupling factor to the corresponding single-particle width. In the notation of Auerbach and French,[5] we have then $S=n\sum\beta_z^2$.

For the particular case of transitions from a two-hole state T_0, J_0 to one-hole states J, we find by direct summation in Eq. (6) of reference 5 that

$$\sum_J (2J+1)S(J)=(2T_0+1)(2J_0+1). \quad (1)$$

Thus we eliminate S_0^*, S_1^* by $S_i+2S_i^*=\frac{3}{2}$ and have then two parameters which we can hope to determine from stripping and pickup reactions. In terms of the wave function amplitudes for the $A=14$ states, we have, using the notation and phase convention[6] of Sherr *et al.*,[1]

$$\begin{aligned}2S_0&=1+y(y+2\sqrt{2}x),\\ 2S_1&=1+\tfrac{3}{2}\gamma^2+2\beta(\gamma\sqrt{\tfrac{5}{6}}-\alpha\sqrt{\tfrac{2}{3}}).\end{aligned} \quad (2)$$

In particular, for the experimental wave function set and the two theoretical sets as listed in reference 1, we have values for S_0 and S_1 as given in Table I.

We now write in Table II the relative cross sections for various reactions. Specifically, we give the product of the statistical and isotopic spin factors and the relative reduced widths S. We do not include the purely kinematic factors in Butler's theory; experimental cross sections should therefore be corrected for these (e.g., as in the Appendix of reference 5).

TABLE I. Relative reduced widths as calculated with the experimentally determined $A=14$ wave functions of Sherr *et al.*[a] and the theoretical ones of Jancovici and Talmi[b] and Visscher and Ferrell.[c]

	Sherr *et al.*	*J–T*	*V–F*
S_0	1.40	1.49	1.40
S_1	1.26	1.31	1.39

[a] See reference 1.
[b] See reference 2.
[c] See reference 3.

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[1] Sherr, Gerhart, Horie, and Hornyak, Phys. Rev. **100**, 945 (1955).

[2] B. Jancovici and I. Talmi, Phys. Rev. **95**, 289 (1954).

[3] W. M. Visscher and R. A. Ferrell, Phys. Rev. **99**, 649 (A) (1955).

[4] We shall consider only the p^n configuration; it seems likely that, except perhaps for the higher $A=14$ states, this is adequate. In fact, K. G. Standing [Phys. Rev. **101**, 152 (1956)] in commenting on the smallness of his $N^{14}(p,d)$ cross section to the first excited state of N^{13} concludes that the p^8s^2 and p^8sd configurations contribute negligibly to the N^{14} ground state. His conclusion is quite tempting but unfortunately the argument is not rigorous; for each of these configurations contains several $T=0$, $J=1$ states and then for either l value the (d,p) amplitude is a *coherent* sum of several contributions.

[5] T. Auerbach and J. B. French, Phys. Rev. **98**, 1276 (1955). See also A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953). The treatment of the stripping process itself is due to S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[6] The phase convention is the usual one for $A=15$, 14, described as 1- and 2-hole nuclei. Specifically, we have for the coefficient of fractional parentage $(66)^{\frac{1}{2}}\langle 1\frac{1}{2}\frac{1}{2}|L_0S_0T_0\rangle=[(2L_0+1)(2S_0+1)\times(2T_0+1)]^{\frac{1}{2}}$.

TABLE II. Relative cross sections for the various reactions connecting the first two states for $A=14$ and the one-hole states for $A=15$, in terms of the reduced width factors. Note that certain pairs of levels may also be connected by (d,t) and (d,He^3) reactions.

Reaction	Relative cross section
$C^{14}(d,n)N^{15}$	$2S_0$
$C^{14}(d,n)N^{*15}$	$3-2S_0$
$N^{14}(d,p)N^{15}$, $N^{14}(d,n)O^{15}$	S_1
$N^{14}(d,p)N^{*15}$, $N^{14}(d,n)O^{*15}$	$\frac{3}{2}-S_1$
$N^{15}(p,d)N^{14}$	$(9/4)S_1$
$N^{15}(p,d)N^{*14}$, $N^{15}(n,d)C^{14}$	$\frac{3}{4}S_0$

In order to determine the S values [and thus from Eq. (2), something about the wave functions], we first consider pairs of experiments in which the same reaction leads to two states of the same final nucleus. The simplest to interpret is $C^{14}(d,n)N^{15}$, and here the cross section specifies almost uniquely the C^{14} wave function, as indicated in Fig. 1. If it should turn out experimentally that the excited-state cross section is not much smaller than the ground-state cross section, the correctness of the recent conclusions concerning the $A=14$ polyad would be seriously in doubt. This experiment has not been done, but from the observations at very low deuteron energy and at only two angles made by Hudspeth, Swann, and Heydenberg,[7] one is tempted to conclude that $d\sigma^*/d\sigma$ is indeed small.

For $N^{14}(d,p)N^{15}$, absolute cross sections are given by Gibson and Thomas[8] with an accuracy which is, however, rather less than we would like. A very recent measurement by Green and Middleton[9] gives the excited-state cross section with excellent accuracy but a comparable ground-state cross section is missing. On comparing the two experiments, we find that we can safely write $1.15 \leqslant S_1 \leqslant 1.35$, and this value could be refined with a more accurate value of the ground-state cross section. On the other hand, the $N^{14}(d,n)O^{15}$ experiment has been done with satisfactory accuracy by Evans, Green, and Middleton.[10] From these results, we find $1.26 \leqslant S_1 \leqslant 1.37$. It is clear that these experiments support the conclusions of Sherr *et al.*[1] No

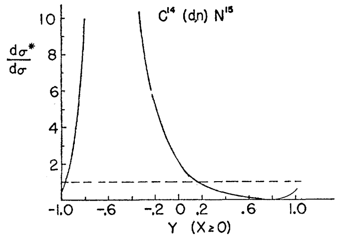

FIG. 1. Plotted *vs* the P state amplitude in the C^{14} wave function is the ratio of the (kinematically corrected) differential cross sections to the two one-hole states of N^{15}. $d\sigma^*=0$ for $y=(\frac{2}{3})^{\frac{1}{2}}$ corresponding to a $(p_{\frac{3}{2}})^8(p_{\frac{1}{2}})^2$ representation for C^{14}; $d\sigma=0$ for $y=-(\frac{1}{3})^{\frac{1}{2}}$ corresponding to a $(p_{\frac{3}{2}})^7(p_{\frac{1}{2}})^3$ representation.

[7] Hudspeth, Swann, and Heydenberg, Phys. Rev. **80**, 643 (1950).
[8] W. M. Gibson and E. E. Thomas, Proc. Roy. Soc. (London) **A210** (543).
[9] T. S. Green and R. Middleton, Proc. Phys. Soc. (London) **A69**, 28 (1956).
[10] Evans, Green, and Middleton, Proc. Phys. Soc. (London) **A66**, 108 (1953).

$N^{15}(p,d)$ experiments [or the analogous (d,t) and (d,He^3)] have been done.

III. REACTIONS WHICH MEASURE THE SINGLE-PARTICLE CROSS SECTIONS

It will be observed that in Sec. II we have refrained from comparing cross sections except those for reactions of the same type with the same initial nucleus, although Table II gives us a warrant for making more general comparisons. We must recall, however, two sources of difficulty in extracting the relative reduced widths, S, from the experimental cross sections. The first, which is a difficulty in extracting the actual reduced width, comes about because the Butler theory may not give a good account of the absolute cross sections.[11] Thus it seems safe to compare reactions for close-lying states (particularly when the kinetic energies are high); but the error in comparing, for example, the two levels of N^{15} (which are 6.3 Mev apart) may not be small, and in the same way comparison of widths deduced from reactions of different type[12] may not be safe. The second essential difficulty is in deducing from the observed reduced widths the values (or relative values)

TABLE III. Relative reduced widths S for reactions connecting the $A=15$ ground state with the higher p^n states (T,J) for $A=14$.

T	J	S
1	1	9/4
0	2	5/4
0	3	0
1	2	$(5/4)(P-\sqrt{2}D)^2$

of the quantity S; the point here is that the single-particle width may well vary with the state of excitation of the nucleus.[13]

It seems altogether likely that both of these difficulties, which must be resolved if the deuteron-reaction analysis is to become a trustworthy quantitative technique,[14] will have to be clarified empirically, and for this purpose the reactions involving $A=14$, 15 supply many useful cases. Firstly we note from Eq. (1) (or Table II) that, as long as we ignore the variation of the single-particle width with excitation,[14] the pairs of experiments which end in the two states for $A=15$ are "self-normalizing" in that a measurement of the *relative* cross sections will produce the *absolute* S values

[11] J. Horowitz and A. M. L. Messiah, Phys. Rev. **92**, 1326 (1953); W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955); J. E. Bowcock, Proc. Phys. Soc. (London) **A68**, 512 (1955); W. Tobocman, Phys. Rev. **102**, 588 (1956).
[12] For some pertinent comparisons of (d,p) and (d,n) cross sections, see Calvert, Jaffe, and Maslin, Phys. Rev. **101**, 501 (1956).
[13] A. M. Lane, Atomic Energy Research Establishment, Harwell Report T/R 1289 (unpublished).
[14] Note, however, that since experimentally the (d,p) cross section [and probably also the (d,n) cross section] to the N^{15} excited state is small, these difficulties would not seriously affect the conclusions about the $A=14$ wave functions.

and thus the absolute single-particle cross sections. Other experiments which give the single-particle cross sections in this region of A (but for quite different kinetic energies) are $O^{16}(n,d)$ and its inverse and $O^{16}(p,d)$.

Besides this, the $A=14$ polyad has higher p^{10} states which are unique (i.e., only one multiplet) and the $N^{15}(p,d)$ experiment to these levels (involving *two* unique states) would supply other values of the single-particle cross section for different kinetic energies and excitations. Table III lists all the higher states with their S values (few if any of these states have yet been experimentally identified). Besides these, there are three states with $T=0$, $J=1$ and two with $T=1$, $J=0$; the S values are given by Eq. (2) but the vectors, of course, are different. For $T=1$, $J=2$ there are two states; P, D are the amplitudes for the ${}^{33}P$ and ${}^{31}D$ components respectively; in the jj limit $(p_{\frac{3}{2}}{}^{7}p_{\frac{1}{2}})$, S has the large value 15/4. Measurement of the cross sections to any of these states would be very interesting.[15] We emphasize too that experimental S determinations for the higher "nonunique" $A=14$ states would supply additional parameters which should be considered in any theoretical study of the $A=14$ polyad.

[15] The determination of the channel spin ratio by angular correlation measurements would also be useful. [See T. Auerbach, thesis, University of Rochester, 1954 (unpublished); and for the $(d,p\gamma)$ case, in particular, O. Hittmair, Z. Physik 143, 465 (1955).]

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p-He^3 Scattering at 9.75-Mev Proton Energy*

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The differential cross section for the elastic scattering of 9.75-Mev protons from He^3 has been measured in the range of center-of-mass angles from 30 to 150 degrees. The cross section varies from 345 millibarns/steradian at 30 degrees, through a minimum of 18.2 mb at 110 degrees, and rises to 84.5 mb at 150 degrees. Comparison of the data with the distributions calculated theoretically by Swan gives only rough agreement, the closest being for ordinary forces in the four-body system. This conflicts with the good agreement obtained by Swan with the 14-Mev n-T data of Coon *et al.*, in which a symmetric exchange force scheme was clearly preferable.

INTRODUCTION

THE elastic scattering of protons by He^3 has been studied at Van de Graaff energies,[1] and more recently, a beam of He^3 ions from a cyclotron has been used to extend these observations to the neighborhood of 4 Mev energy in the center-of-mass system.[2]

Swan[3] has compared these lower energy data, together with the 14-Mev n-T scattering measurements of Coon, Bockleman, and Barschall,[4] to angular distributions derived from a model of the four-particle system based on the resonating group structure of Wheeler.[5] In the case of the n-T data, this theory agrees well with experiment, provided that a symmetric mixture of exchange forces is assumed. However, Swan's agreement with the p-He^3 experiments is quite poor. Regardless of the exchange force scheme invoked, the theory predicts a continually decreasing differential scattering cross section with increasing scattering angle up to about 5-Mev proton energy, whereas all the experimental distributions exhibit pronounced minima at around 90 degrees in the center-of-mass system.

The availability of a homogeneous beam of protons of approximately 10-Mev energy from the first section of the Minnesota linear accelerator has made possible the extension of these measurements into an energy region where the agreement between experiment and theory might be expected to improve, as evidenced by the 14-Mev n-T data.

EXPERIMENTAL, GENERAL

The Minnesota proton linear accelerator is built in three sections, exclusive of the 500-kev injector, which raise the beam energy to 10, 38, and 68 Mev, respectively. Protons deflected from the machine axis into the area where this work was done have a precise energy of 9.89 Mev, and are homogeneous in energy to ± 70 kev. The total proton current available after deflection was about 1.4×10^{-7} ampere, but the collimation used in this experiment reduced the beam intensity to approximately 5×10^{-9} ampere.

The scattering occurred in a two-foot diameter chamber which was built as a general purpose facility for

* Work supported in part by the U. S. Atomic Energy Commission.
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[1] Famularo, Brown, Holmgren, and Stratton, Phys. Rev. 93, 928(A) (1954).
[2] D. R. Sweetman (unpublished).
[3] P. Swan, Proc. Phys. Soc. (London) A66, 740 (1953).
[4] Coon, Bockelman, and Barschall, Phys. Rev. 81, 33 (1951).
[5] J. A. Wheeler, Phys. Rev. 52, 1107 (1937).