Interpretation of the $H^3(p,n)$ Reaction*

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The angular and energy dependence of the $H^3(p,n)$ reaction at energies up to a few Mev have been much analyzed for their possible indication of resonances in the "compound nucleus" He^4. It is shown that the experimental data can be explained on the basis of "direct" interactions, without assuming that any compound-nucleus state is involved. A combination of "knock-on" and "pickup" effects is involved.

In addition to removing the $H^3(p,n)$ data as evidence for an excited state of He^4, the result of this work adds to previous evidence that the Born approximation can give a good account of the angular and energy variations in nuclear rearrangement collisions, even at energies as low as a few Mev.

INTRODUCTION

RECENTLY the $H^3(p,n)$ reaction has received considerable attention, especially because of the information one might hope it could give concerning (unbound) excited states of He^4.[1,2] It is the purpose of this note to point out that the broad maximum in the total cross section for this reaction, at about 3 Mev, can be understood on the basis of a simple picture of the reaction as a rearrangement collision,[3] so that the existence of a maximum does not give positive evidence for any excited states in He^4. That is, the energy dependence, and the nature of the angular distribution as well, can be understood on the basis of a "direct" interaction, without assuming that any "compound-nucleus" state is involved.

REMARKS ON THE APPROACH USED

The discussion given here is based on a rearrangement collision picture. This "direct interaction" viewpoint, as opposed to one involving a compound nucleus, has been applied to the (n,p) reaction by Austern, Butler, and McManus,[4] following the success of the Butler type theory in accounting for the results of stripping and pickup reactions. The Butler type theory itself represents a similar approach, corresponding essentially to a Born-approximation type calculation.[5]

In the usual form of application of the Butler type theory it is assumed that the nucleus is opaque, and that the reaction takes place outside the nucleus. However, it has been pointed out that this may not be an appropriate picture[6] and that in fact at higher energies, where the nucleus is more transparent, the interior region of the nucleus plays a dominant role.[7]

In the present note there are two differences from the treatment of reference 4. First, the nucleus is treated as transparent rather than opaque. With regard to the direct p-n interaction, this has the general effect of broadening and smoothing the angular distribution. Secondly, the interaction between the incoming proton and the "remainder" of the target nucleus (i.e., the part remaining when a neutron is removed) plays an important role, in this special case in which H^3 is the target nucleus. Namely, this interaction is responsible for the production of neutrons giving the backward peak observed, through a "stripping" process, in which a neutron is stripped from the "incoming" triton.

The nuclei involved, H^3 and He^3, are treated as transparent in the calculation of both processes (p-n, stripping). For these light nuclei, this transparency viewpoint is probably reasonable even at the low energies at which we propose to apply it (7 Mev and less),[1,2] especially in view of the relative success which the transparent model has shown for the (p,d) reaction in Be^9 down to comparable energies.[8,9] The errors associated with this transparent-nucleus approach in the present case may be expected in any event to be no larger than those associated with our use of Born and "impulse"[10] approximations or with our omission of possible compound-nucleus contribution to the cross section.

QUALITATIVE DISCUSSION OF THE REACTION

The calculations described below are made in a perturbation-theory way. We then have to do essentially with a matrix element between an initial and a final state. The wave function of the 3-nucleon target nucleus or final nucleus is taken as separable, in the manner indicated in Fig. 1. The (p,n) reaction in this case may be thought of as a rearrangement, which can be produced by either of two interactions. One of these is the p-n potential $V_{pn}(\mathbf{r}_1-\mathbf{r}_2)$, the other is equivalent in Born approximation to the interaction between the proton and the "residual" nucleus, $V_{pN}(\mathbf{r}_1-\mathbf{r}_N)$. (The coordinate labeling is explained in Fig. 1. There are

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[1] N. A. Vlasov *et al.*, Soviet Physics JETP **1**, 500 (1955).
[2] Willard, Bair, and Kington, Phys. Rev. **90**, 865 (1953).
[3] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949).
[4] Austern, Butler, and McManus, Phys. Rev. **92**, 350 (1953).
[5] See, e.g., R. Huby, Progr. Nuclear Phys. **3**, 177 (1953); E. Gerjuoy, Phys. Rev. **91**, 645 (1953), N. C. Francis and K. M. Watson, Phys. Rev. **93**, 313 (1954).
[6] P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952).
[7] W. Selove, Phys. Rev. **98**, 208 (1955).
[8] S. Glashow and W. Selove, Phys. Rev. **102**, 200 (1956).
[9] J. Dabrowski and J. Sawicki, Acta Phys. Polon. **14**, 143, 407 (1955).
[10] G. F. Chew, Phys. Rev. **80**, 196 (1950).

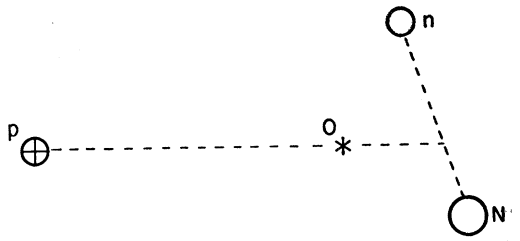

FIG. 1. Coordinates are measured from the center of mass, O. The vectors specifying the positions of the proton p, the neutron n, and the residual nucleus N (mass=2) are denoted $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_N$, respectively.

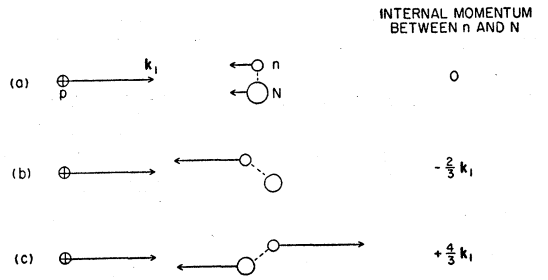

FIG. 2. Kinematics in the center-of-mass system. The lengths of the arrows represent velocities. In cases (b) and (c) the neutron momentum has the same magnitude as the proton momentum.

only two independent coordinates; the three coordinates used are connected by the relation $\mathbf{r}_1+\mathbf{r}_2+2\mathbf{r}_N=0$.)

The effect of V_{pn} may be pictured physically perhaps most easily in the high-energy limit, in terms of a "knock-on" collision in which the proton directly ejects the neutron and takes its place. The angular distribution of the emitted neutrons will reflect the angular distribution of the free p-n cross section. This V_{pn} process has been essentially treated by Chew and by Austern *et al.* It gives a very simple picture of the energy-dependence of the $H^3(p,n)$ reaction. Namely the reaction cross section σ_{H3} is found to be roughly proportional to the free p-n cross section σ_{pn} at an energy corresponding to (but not identical to) the energy of the incident proton. The $H^3(p,n)$ cross section consequently tends to decrease with increasing proton energy, because σ_{pn} decreases. The gentle decrease due to this effect, combined with the characteristic post-threshold energy dependence of a reaction, namely a steep rise followed by a leveling off, serves to explain the broad maximum followed by a gentle decrease, seen in the total $H^3(p,n)$ cross section. This argument when pursued predicts approximately the observed rate of decrease following the maximum.

However, this V_{pn} process does not predict the observed angular dependence. At low energies σ_{pn} itself is isotropic. The corresponding theoretical result for the $H^3(p,n)$ reaction at low energies is an angular distribution dropping slightly for backward angles.

The experimentally observed backward peak thus is not explained by this interaction. However, as remarked above, the V_{pN} interaction *does* give an explanation. The effect here is due to the appreciable velocity of the H^3 nucleus in the over-all center-of-mass system. The V_{pN} interaction serves to attach the proton to the "residual" nucleus, while the neutron is detached, and proceeds with the (total) momentum which it had before the collision. Now the magnitude of the final neutron momentum is fixed by the energetics of the reaction, since He^3 is left in a definite single state. For a neutron to be released in a given direction the H^3 nucleus must initially be in a suitable state of internal motion. Now comes the essential point: for a neutron which will be emitted in the "backward" direction—i.e., opposite to the initial proton direction—the center-of-mass motion of the H^3 serves to "catapult" the neutron out. Thus, as shown in Fig. 2, the internal momentum in the H^3 nucleus required to furnish a neutron in the backward direction is much less than for a forward-emitted neutron. (Momentum will be expressed here in units in which $\hbar=1$.) If we neglect the binding energy difference between H^3 and He^3 and take the initial proton momentum and the final neutron momentum to have the same magnitude, then from Fig. 2 we see that for a backward-emitted neutron the internal momentum required is only half that for a forward-emitted neutron—the kinetic energy only one-fourth as great. Moreover, it is easy to see that the internal momentum of the final He^3 nucleus has essentially the same magnitude as the internal momentum of the initial H^3 nucleus. Consequently, the matrix element for this "stripping" process produced by V_{pN} involves the *square* of the momentum wave function for the appropriate value of internal momentum. The dominant backward emission is thus connected with the fact that the internal momentum distribution for H^3 falls fairly rapidly for increasing momentum. Madansky and Owen[11] have also pointed out that this type of "heavy-particle stripping" can contribute a backward peak of neutrons.

The angular dependence of the stripping contribution can be used in principle to obtain information on the triton wave function. However, such an analysis is not pursued here, since the approximations involved in the present treatment are almost certainly too crude to provide very accurate information of this kind. To see whether this treatment is capable of accounting for the $H^3(p,n)$ data, however, requires only a rather crude description of the properties of the H^3 wave function—principally just its approximate range.

CALCULATION

In this report, effects due to spin and to the Pauli principle are not discussed in detail. The writer has investigated these effects in detail for $H^3(p,n)$ for the case of purely central forces and has found that in this case the effects do not modify the predictions as to energy and angular dependence of the cross section.

[11] L. Madansky and G. E. Owen, Phys. Rev. **99**, 1608 (1955).

They enter into the magnitude of the cross section, but only in simple ways such as factors of two or one-half. The theory used here is expected to give fairly good results for the energy and angular dependence of the cross section, but not for the absolute magnitude, judging from previous experience with the transparent-nucleus (p,d) theory.[12] Consequently, the effects of spin and Pauli principle are not important here, at least on the assumption of central forces. To keep the calculation of reasonable magnitude, the effect of tensor forces is not examined; at the low energies under consideration, this procedure is not likely to affect the results seriously.

The approximations made here in order to carry out the calculation involve principally (i) the form of the mass-3 nuclear wave function, (ii) the use of the impulse approximation for calculating the V_{pn} matrix element, and (iii) the use of the Born approximation for calculating the V_{pN} matrix element.

For the wave function of the mass-3 nucleus we take the form

$$\begin{aligned} v(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)&=u(\mathbf{r}_2-\tfrac{1}{2}(\mathbf{r}_3+\mathbf{r}_4))w(\mathbf{r}_3-\mathbf{r}_4). \\ &=u(\mathbf{r}_2-\mathbf{r}_N)w(\mathbf{r}_3-\mathbf{r}_4), \end{aligned} \tag{1}$$

where v is assumed to be symmetric in $\mathbf{r}_2$, $\mathbf{r}_3$, $\mathbf{r}_4$. In calculating matrix elements involving, say, the interchange of neutron 2 and proton 1, we shall neglect any effects due to structure of the residual nucleus, i.e., due to the form of w. Now we know the correct wave function can be expanded in a sum of products of the form $\sum_i u_i w_i$. To take only the single term (1) may seem roughly equivalent to assuming that the "residual" neutron-proton pair in H^3, say, are in a deuteron ground state. From recent work of Werner[13] we know this to be not a very good description. However, there is at least one approximate form for $v(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)$ which satisfies the requirement that it be symmetric with respect to the three nucleons and at the same time is separable in the form of (1), namely a Gaussian

$$\begin{aligned} v_g&=\exp[-(r_2^2+r_3^2+r_4^2)], \quad (\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4=0) \\ &=\exp\left[-\tfrac{2}{3}\left(\mathbf{r}_2-\frac{\mathbf{r}_3+\mathbf{r}_4}{2}\right)^2\right]\exp[-\tfrac{1}{2}(\mathbf{r}_3-\mathbf{r}_4)^2], \text{etc.} \end{aligned} \tag{2}$$

Hence the assumption of factorability in the form (1) is felt to be a reasonable one. As to the further assumption that the form of w does not play an important role, this is essentially in the spirit of the Born and impulse approximations. Henceforth we shall omit entirely the function w, and use, for example, $u(\mathbf{r}_2-\mathbf{r}_N)$ alone as the wave function for the initial nucleus.

The matrix element involving an incoming proton of momentum $\mathbf{k}_1$ and an outgoing neutron of momentum $\mathbf{k}_2$ can then be written, by standard techniques,[10] as

$$\begin{aligned} |H'| = \int & d\mathbf{r}_2 d(\mathbf{r}_1-\mathbf{r}_N) \\ &\times\exp[-i\mathbf{k}_2\cdot(4/3)\mathbf{r}_2]u_f^*(\mathbf{r}_1-\mathbf{r}_N) \\ &\times[V_{pn}(\mathbf{r}_1-\mathbf{r}_2)+V_{nN}(\mathbf{r}_2-\mathbf{r}_N)]\Psi, \end{aligned} \tag{3}$$

where Ψ is the exact wave function of the system. (The small numerical factors appearing ubiquitously result from reduced-mass effects.) We discuss the two terms separately.

For the calculation of $|V_{pn}|$ we start with the impulse approximation. As discussed above, we may have some hope that even at the low energies involved here this approximation may give the essential features which would result from a more accurate calculation. The matrix element takes the form[4,10]

$$|V_{pn}| = \int d\mathbf{p}\varphi_f^*(\mathbf{q})\varphi_0(\mathbf{p})\langle\mathbf{k}'|r_{pn}|\mathbf{k}\rangle, \tag{4}$$

where φ_0 and φ_f are, respectively, the Fourier transforms of $u_0(\mathbf{r}_2-\mathbf{r}_N)$ and $u_f(\mathbf{r}_1-\mathbf{r}_N)$, $\mathbf{q}$ satisfies the momentum conservation condition

$$\mathbf{q}=\mathbf{p}+\tfrac{2}{3}(\mathbf{k}_1-\mathbf{k}_2), \tag{5}$$

and the matrix element $\langle\mathbf{k}'|r|\mathbf{k}\rangle$ corresponds to the scattering amplitude for an isolated pn system, from an initial relative momentum

$$\mathbf{k}\equiv\tfrac{1}{2}(\mathbf{k}_p-\mathbf{k}_n)_{\text{initial}}=\tfrac{1}{2}[(4/3)\mathbf{k}_1-\mathbf{p}] \tag{5a}$$

to a final relative momentum

$$\begin{aligned} \mathbf{k}'\equiv\tfrac{1}{2}(\mathbf{k}_p-\mathbf{k}_n)_{\text{final}}&=\tfrac{1}{2}[\mathbf{q}-(4/3)\mathbf{k}_2] \\ &=\tfrac{1}{2}(\tfrac{2}{3}\mathbf{k}_1+\mathbf{p}-2\mathbf{k}_2). \end{aligned} \tag{5b}$$

The r_{pn} matrix elements are in general off the energy shell. For $p\ll k_1$, k_2, one can nevertheless argue that $|r_{pn}|$ can be moved outside the integral (4). This condition is not really met in the present case, and one cannot be very certain of the validity of such a procedure. However, another approach to the problem, which we now discuss, suggests that such an approximation may indeed be reasonably good. We consider the use of the Born approximation to calculate $|V_{pn}|$. In this approximation direct and exchange contributions are treated separately, and take slightly different forms. (This is in contrast to the impulse-approximation treatment, where the over-all properties of the potential V_{pn} are handled at once.)

One can make an argument that the Born approximation may give the principal features of the results for a rearrangement collision even though it may be quite inadequate for treating a nucleon-nucleon collision itself. The argument is on the basis that the target nucleon involved is "spread out," so that the replacement of the exact wave function Ψ in (3) by the "initial-

[12] See reference 8, and also W. Selove, Phys. Rev. **101**, 231 (1956).

[13] A. Werner, Nuclear Phys. **1**, 9 (1956).

state" wave function $\exp[i\mathbf{k}_1\cdot(4/3)\mathbf{r}_1]u_0(\mathbf{r}_2-\mathbf{r}_N)$ may be a reasonable approximation. Moreover, we again can note the experience with the (p,d) rearrangement collision, where the Born approximation seems to give good results.[8] Finally, we may try to improve the Born approximation, in somewhat the spirit of the impulse approximation, by replacing any nucleon-nucleon matrix elements, that arise, by the equivalent experimental values.

Dealing here with a nonexchange potential, the Born-approximation form of $|V_{pn}|$ can be put into the form (dropping a Jacobian)

$$|V_{pn}|=\int d\mathbf{r}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\frac{2}{3}\mathbf{r}}V(\mathbf{r})\times\int d\mathbf{R}e^{-i(\mathbf{k}_2-\mathbf{k}_1)\cdot\frac{3}{4}\mathbf{R}}u_f^*(\mathbf{R}+\tfrac{1}{2}\mathbf{r})u_0(\mathbf{R}-\tfrac{1}{2}\mathbf{r}),\quad(6)$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, and $\mathbf{R}=\mathbf{r}_1+\mathbf{r}_2$. We expect the range of V to be considerably smaller than that of u_f or u_0, and it is consequently a good approximation to take

$$u_f^*(\mathbf{R}+\tfrac{1}{2}\mathbf{r})u_0(\mathbf{R}-\tfrac{1}{2}\mathbf{r})\approx u_f^*(\mathbf{R})u_0(\mathbf{R}).\quad(7)$$

(6) now is factorable. The second integral can be converted back to momentum space, whereupon (6) can be written

$$|V_{pn}|\approx\int d\mathbf{r}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\frac{2}{3}\mathbf{r}}V(\mathbf{r})\times\int d\mathbf{p}\varphi_f^*(\mathbf{p}+\tfrac{2}{3}[\mathbf{k}_1-\mathbf{k}_2])\varphi_0(\mathbf{p}).\quad(8)$$

It is of interest also to write down the form which (6) takes if we convert u_f and u_0 to momentum space, *without* making the approximation (7). In this case, we obtain

$$|V_{pn}|=\int d\mathbf{p}\varphi_f^*(\mathbf{p}+\tfrac{2}{3}[\mathbf{k}_1-\mathbf{k}_2])\varphi_0(\mathbf{p})\times\int d\mathbf{r}e^{-i(\mathbf{p}-\mathbf{k}_2-\frac{1}{3}\mathbf{k}_1)\cdot\mathbf{r}}V(\mathbf{r}).\quad(9)$$

Comparison of (8) and (9) shows that the approximation (7) has the effect in the Born-approximation matrix element (9) of replacing the internal momentum $\mathbf{p}$ of the struck neutron by

$$\mathbf{p}_{\text{effective}}=\tfrac{1}{3}(\mathbf{k}_2-\mathbf{k}_1)\quad(10)$$

in the second integral of (9), i.e., in the nucleon-nucleon scattering factor.

On comparing (4) and (9) we see that the r_{pn} matrix element, which corresponds to the momentum change

$$\Delta\mathbf{k}=\mathbf{k}'-\mathbf{k}=\mathbf{p}-\mathbf{k}_2-\tfrac{1}{3}\mathbf{k}_1,\quad(11)$$

corresponds to the Born-approximation matrix element for this same momentum change—that is, the second factor of (9). In the spirit of modifying the Born approximation toward an impulse-approximation form, we may retain the identification of $|r_{pn}|$ and the Born factor even after the approximation (7). We infer then that under that approximation we may make the replacement (10) in $|r_{pn}|$ as well as in the equivalent Born factor. We can now remove $|r_{pn}|$ from the integral in (4), which is the procedure we set out to justify. (4) then reads

$$|V_{pn}|\approx\langle|r_{pn}|\rangle_{\text{effective}}F(\mathbf{k}_2-\mathbf{k}_1),\quad(12a)$$

where

$$\langle|r_{pn}|\rangle_{\text{effective}}=\left\langle\frac{\mathbf{k}_1-5\mathbf{k}_2}{6}|r_{pn}|\frac{5\mathbf{k}_1-\mathbf{k}_2}{6}\right\rangle^{14}\quad(12b)$$

and

$$F(\mathbf{k}_2-\mathbf{k}_1)=\int d\mathbf{p}\varphi_f^*(\mathbf{p}+\tfrac{2}{3}[\mathbf{k}_1-\mathbf{k}_2])\varphi_0(\mathbf{p})=\int d\mathbf{R}e^{-i\frac{3}{4}(\mathbf{k}_2-\mathbf{k}_1)\cdot\mathbf{R}}u_f^*(\mathbf{R})u_0(\mathbf{R}).\quad(12c)$$

We have thus finally arrived at the expression (12), from which we see that $|V_{pn}|$ takes the form, familiar for such problems, of the product of a nucleon-nucleon scattering amplitude with a nuclear "form factor." The latter, F, corresponds to the form factor which arises in a treatment of the scattering of waves of wave number $\frac{2}{3}\mathbf{k}_1$ by a density distribution $u_f^*(\mathbf{R})u_0(\mathbf{R})$, $\approx|u_0(\mathbf{R})|^2$ in the present case. The nucleon-nucleon amplitude $\langle|r_{pn}|\rangle_{\text{effective}}$ is essentially *on* the energy shell, since $k_2\approx k_1$, and so can be directly related to a free pn scattering, with a relative momentum and a momentum change obtainable from (12b). One sees that for forward emission of neutrons ($\mathbf{k}_2\approx\mathbf{k}_1$) the effective collision momentum is $\frac{2}{3}\mathbf{k}_1$ and the equivalent pn scattering is backwards; and for backward emission the effective collision momentum is $\mathbf{k}_1$ and the equivalent pn scattering is forwards. Thus the behavior of $\langle|r_{pn}|\rangle_{\text{effective}}$ with angle is such as to make forward emission of neutrons slightly preferred. For a $H^3(p,n)$ reaction with protons of laboratory energy E_0, the equivalent free pn

[14] The fact that the approximation (7) has the effect of making the replacement (10) in $\langle|r|\rangle$ can be given a rough physical interpretation. It can be noted that the value (10) gives the minimum product of the internal kinetic energies in the initial and final nuclei. This product is proportional to $|\mathbf{p}|^2|\mathbf{q}|^2$; with the condition (5), minimizing this product just gives (10). The fact that the approximation (7), which rests on the assumption that V is of considerably shorter range than u, results in the minimization of this product can be understood in the following way: The dominant contribution in the matrix element (4) comes from values of $\mathbf{p}$ in the neighborhood of the value which maximizes $\varphi_f^*(\mathbf{q})\varphi_0(\mathbf{p})\langle|r|\rangle$. If V is of considerably shorter range than u_0 or u_f, then the dependence of the third factor on $\mathbf{p}$ (in Born approximation this factor corresponds just to the Fourier transform of V) will be weaker than the dependence of the first two factors. The dominant contribution in (4) then comes from the region of $\mathbf{p}$ values which maximizes $\varphi_f^*(\mathbf{q})\varphi_0(\mathbf{p})$. If u_0 and u_f are of long range then φ_0 and φ_f will fall rapidly with increasing momentum, and the product $\varphi_f^*(\mathbf{q})\varphi_0(\mathbf{p})$ will become a maximum near a minimum of $|\mathbf{p}||\mathbf{q}|$.

collision for forward-emission is at E_0, that for backward-emission is at $(9/4)E_0$. (In the present case, for energies of a few Mev, the angle of the equivalent free collision is unimportant, since the experimental scattering is isotropic.)

The form factor F will also have a behavior favoring forward emission. F is equal to unity for forward emission. Its value for backward emission can be estimated by using a reasonable approximation for the wave function u. Such an estimate gives a value of the order of 0.85 for $E_0=5$ Mev.

If one now calculates the value of the total cross section, including the density-of-states (phase space) factor which plays a prominent role near threshold, according to

$$\sigma_t=\int\sigma d\Omega\sim\int d\Omega|V_{pn}|^2[1-(4E_1/3E_0)]^{\frac{1}{2}}, \quad (13)$$

where E_1 is the center-of-mass threshold energy, one can fit the energy dependence of the total cross section reasonably well. The absolute magnitude of σ_t is not given well by this procedure, but this is not surprising in view of similar experience in the (p,d) case,[8] in which the Born approximation has been found to give a good account of energy and angular variations while giving absolute values incorrectly by a factor of the order of 5.

As for the angular dependence of σ, the results so far would give a forward peaking, with a ratio $\sigma_{0°}:\sigma_{90°}:\sigma_{180°}$, at 5 Mev, of about 1.3:1.0:0.7. When this is compared with the experimental ratios of about 2:1:6 it is seen that the theoretical result gives the qualitative form of the forward-hemisphere data, but does not give the backward peak. As discussed above, the latter can be explained by a stripping process. Or, in alternative words, a "pickup" process, in which the incoming proton "picks up" the "residual nucleus" unit and continues forward, as He^3, leaving then the neutron going backwards. Most of the contribution to the total cross section comes from angles near 90°, where the stripping process does not contribute strongly; hence the previous discussion of the energy dependence of the total cross section is not changed by inclusion of the stripping term.

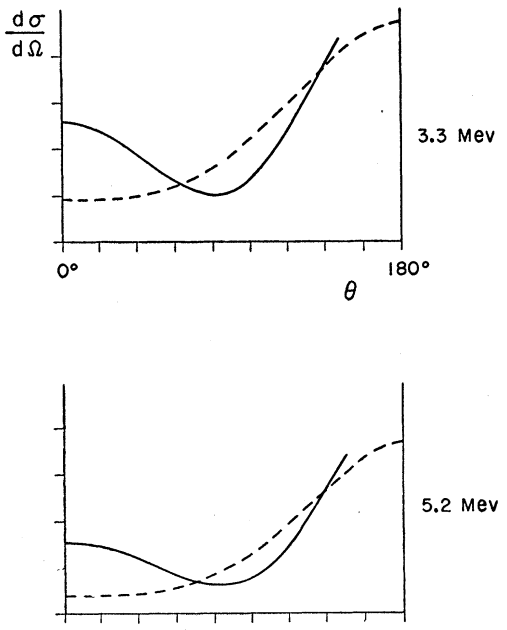

FIG. 3. Angular distributions. The experimental data are shown by solid lines. Calculations for the angular dependence of the stripping contributions are shown dashed, normalized at 140°.

This stripping process occurs as a result of the interaction $V_{nN}(\mathbf{r}_2-\mathbf{r}_N)$. The stripping term was discussed above qualitatively as due to the interaction V_{pN} rather than V_{nN}. Now to be precise one cannot speak of either of these potentials as "causing" the reaction—the transition probability amplitude given by (3) involves an integral equation for the exact wave function Ψ. However, in Born approximation the matrix element does have the perturbation-theoretic characteristic form $\int\psi_{\text{final}}^*V\psi_{\text{initial}}$. In this approximation the matrix elements of the two potentials V_{pN} and V_{nN} are equal, according to the well-known "post-prior" equivalence.[3] Thus in the Born approximation the matrix element may be written in the form $\int\psi_{\text{final}}^*V_{pN}\psi_{\text{initial}}$, which happens to be also more physically suggestive than if written with V_{nN} instead.

We then proceed to evaluate the matrix element V_{pN} in Born approximation. Since the residual nucleus is being treated as structureless, the result is identical in form to that for the (p,d) pickup process as first written down by Chew and Goldberger.[15]

$$|V_{pN}|=\int d(\mathbf{r}_1-\mathbf{r}_N)d(\mathbf{r}_2-\mathbf{r}_N)\times\exp[-i\mathbf{k}_2\cdot(4/3)\mathbf{r}_2]u_f^*(\mathbf{r}_1-\mathbf{r}_N)V_{pn}(\mathbf{r}_1-\mathbf{r}_N)\times\exp[i\mathbf{k}_1\cdot(4/3)\mathbf{r}_1]u_0(\mathbf{r}_2-\mathbf{r}_N). \quad (14)$$

On expressing $\mathbf{r}_1$ and $\mathbf{r}_2$ in terms of $\mathbf{r}_1-\mathbf{r}_N\equiv\mathbf{r}$ and $\mathbf{r}_2-\mathbf{r}_N\equiv\mathbf{s}$, this becomes

$$|V_{pN}|=\int d\mathbf{s}e^{-i(\mathbf{k}_2+\frac{1}{3}\mathbf{k}_1)\cdot\mathbf{s}}u_0(\mathbf{s})\times\int d\mathbf{r}u_f^*(\mathbf{r})V_{pN}(\mathbf{r})e^{i(\mathbf{k}_1+\frac{1}{3}\mathbf{k}_2)\cdot\mathbf{r}}. \quad (15)$$

This is now factorable. The second factor can be transformed in the usual way (see reference 15), giving finally

$$|V_{pN}|=-\varphi_0(\mathbf{k}_2+\tfrac{1}{3}\mathbf{k}_1)\varphi_f^*(\mathbf{k}_1+\tfrac{1}{3}\mathbf{k}_2)\times[(4mB/3\hbar^2)+(\mathbf{k}_1+\tfrac{1}{3}\mathbf{k}_2)^2], \quad (16)$$

where B is the binding energy of a proton in He^3, m is the mass of a nucleon, and φ_0 and φ_f are the Fourier transforms of u_0 and u_f, as before. Thus the differential

[15] G. Chew and M. Goldberger, Phys. Rev. **77**, 470 (1950).

cross section for this process has the behavior

$$\sigma \sim |\varphi(n)|^4 \left(\frac{4mB}{3\hbar^2}+n^2\right)^2, \tag{17a}$$

$$n=|\mathbf{k}_1+\tfrac{1}{3}\mathbf{k}_2|. \tag{17b}$$

For a proton energy (laboratory) E_0, one readily finds $n^2=E_0(5+3\cos\theta)/8$, where θ is the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$, and n^2 is here expressed in energy units.

Calculations have been made from (17a) for the angular distribution resulting from the stripping process. For convenience in the calculation $u(r)$ was taken to be a Hulthén function $(e^{-\alpha r}-e^{-\beta r})/r$, with α determined from the binding energy and β taken as 1.7α, a value which makes $u''=0$ for $r=3.5\times10^{-13}$ cm. This approximation to u probably has too strong high-momentum components.

The results of using this approximation are compared with the experimental data, in Fig. 3. The theoretical peaks are broader than the experimental ones. At least part of this difference could be made up by taking a u "smoother" than the one used, while still having proper asymptotic behavior. It does not seem worthwhile to pursue this, since the peak could also be narrowed by interference effects between $|V_{pn}|$ and $|V_{pN}|$, and since the entire calculation is only approximate. (The rather sharply-featured dip in the angular distribution is suggestive of interference effects, but could also be produced by a model in which H^3 and He^3 are taken to be partially opaque rather than completely transparent.) The essential conclusion to be drawn from the present work is that the major features of the data can be accounted for on the rearrangement-collision viewpoint.

The results of this work are of interest from two standpoints. Firstly, the $H^3(p,n)$ reaction data have been considered to give the principal evidence for the existence of an excited (although unbound) state of He^4. From the results of the present work, the $H^3(p,n)$ data do not provide such evidence for a state of He^4, and thus they give no evidence for the existence of a state of H^4 at corresponding energy. The existence of these states would be important for the interpretation of certain scattering and hyperfragment data.

Secondly, this work provides additional evidence that the Born approximation can give a good account of nuclear re-arrangement collisions, even at energies as low as a few Mev. This result is of interest because it is difficult to establish a criterion for the validity of the Born approximation in a rearrangement collision—no such simple criterion can be stated as can be, for example, for the use of the Born approximation in simple scattering. The results of the present work add to previous evidence that the Born approximation can give a good account of the angular and energy variations in rearrangement collisions, although the absolute values may not be given with much accuracy.

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Spin of N^{16}†

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Observations have been made on the intensity ratio of the 6.13- to 7.11-Mev gamma rays of O^{16} following decay of N^{16} in order to test the possibility that the ground and first excited states of N^{16} have spin 0− and 3− as preliminary results of recent calculations by Elliott have predicted. The ratio was obtained with N^{16} made by the $O^{16}(n,p)N^{16}$ reaction and by the $F^{19}(n,\alpha)N^{16}$ reaction. For the first reaction, the ratio was obtained for two ages of the N^{16}. The constancy of the resulting intensity ratios implies that the theoretical prediction is not correct.

THE spin of N^{16} is commonly taken to be 2− on the basis of the character of the beta decay to the ground state and excited states of O^{16}.[1] Preliminary results of recent calculations by Elliott at Harwell predicted[2] four low-lying states (including the ground state) in agreement with experiment. These states were found all to have negative parity with spins 0, 3, 2, and 1 but the order was unreliable because of their closeness. Although the same calculations gave good agreement for the odd-parity levels and gamma-decay branching ratios in O^{16}, there appeared to be a serious disagreement in the beta decay of N^{16} to O^{16}. Assuming N^{16} to have 2− for its ground state gave reasonable agreement for the *ft* values to the O^{16} 2− state at 8.87 Mev[3] and the 3− state at 6.13 Mev,[4] but there was a factor of the order of 10^3 between theory and experiment for the *ft* value to the 1− state at 7.11 Mev.[4]

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[1] Millar, Bartholomew, and Kinsey, Phys. Rev. **81**, 150 (1951).
[2] D. H. Wilkinson, private communication.
[3] Wilkinson, Toppel, and Alburger, Phys. Rev. **101**, 673 (1956).
[4] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).