Angular Distribution of Fission Fragments at Threshold According to the Bohr Model*

LAWRENCE WILETS AND DAVID M. CHASE†
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
(Received May 31, 1956)

The sideways-peaked angular distribution of fragments from neutron-induced fission of Th^{232} near threshold, measured by Brolley and Henkel, is found to be consistent with the model of A. Bohr. It is assumed that the saddle-point states through which the fissioning nucleus passes include primarily only a few members of a particular rotational band. Indications are adduced that this band has $K=3/2$ and odd parity.

BROLLEY and Henkel[1] have found that the angular distribution of fission fragments from Th^{232} near threshold is concentrated in a direction normal to that of the incident neutrons. They are the first to report such sideways distributions in neutron-induced fission. While the considerations of Hill and Wheeler[2] and of Bohr[3] lead to the more generally observed forward-peaked fission distribution, we should like to point out in this note that *near threshold* the Bohr model predicts a sideways distribution for even-even target nuclei. Bohr has emphasized that near threshold a unique behavior is to be expected.

Regarded classically, the angular momentum vector of the incident neutron points approximately normal to the incident direction ($m_L=0$, $m_s=\pm 1/2$). The compound nucleus formed of an even-even target (spin 0) and an incident neutron is also characterized by an angular momentum vector pointing approximately normal to the incident direction.

Although the compound nucleus may live for many nuclear rotational periods, it is assumed that the fission process proceeds rapidly once the saddle-point shape is reached. Thus the direction of the nuclear symmetry axis at the saddle point determines the angular distribution of fission fragments.

With threshold energy at the saddle point, the nucleus is essentially "cold"; only the lowest nuclear state is occupied.[4] The lowest state is that for which the *rotational* angular momentum (and energy) is lowest. Thus the nuclear symmetry axis is nearly parallel to the angular momentum vector, which in turn lies nearly normal to the incident direction. The fission fragments then come out predominantly normal to the incident direction. For higher energies the nuclear symmetry axis rotates nearly normal to the angular momentum vector; in this case Bohr's theory leads to forward peaking.

This description can be made quantitative in the following way. In the unified model the wave functions describing the orientation of the nuclear symmetry axis are the functions[5]

$$\mathfrak{D}^I{}_{MK}(\theta,\phi,\psi), \tag{1}$$

properly symmetrized with respect to the sign of K, the component of angular momentum along the nuclear symmetry axis. For an odd nucleus, $I=K$, $K+1$, $K+2\cdots$. The rotational energy levels are given by

$$E=B[I(I+1)-K(K+1)], \tag{2}$$

where the constant B is a decreasing function of deformation and therefore is smaller for the saddle-point than for the ground-state shape. We presume that only a few rotational states are excited at the saddle point. For a spin-zero target nucleus, the spin I and parity π of the compound nucleus are related to the orbital angular momentum L of the contributing partial neutron wave by $I=L\pm 1/2$ and $\pi=(-1)^L$.

The orientation of the nuclear symmetry axis, and hence the angular distribution of fission fragments, is given by

$$W^I{}_K(\theta)\propto\int|\Psi|^2 d\phi d\psi\propto|\mathfrak{D}^I{}_{\pm\frac{1}{2}K}|^2+|\mathfrak{D}^I{}_{\pm\frac{1}{2}-K}|^2, \tag{3}$$

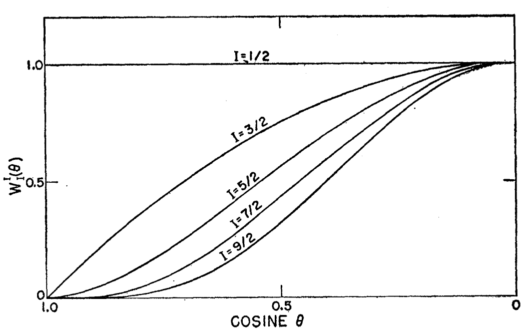

FIG. 1. Theoretical angular distributions of fission fragments from saddle-point states of the fissioning nucleus with spin I equal to K, the projection of I along the nuclear symmetry axis, for various values of I. The normalization is arbitrary.

* Work supported by the U. S. Atomic Energy Commission.
† Recently a visiting member of the Department of Physics, Iowa State College, Ames, Iowa.
[1] R. L. Henkel and J. E. Brolley, Phys. Rev. **103**, 1280 (1956).
[2] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).
[3] A. Bohr, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy* (United Nations, New York, 1956), Vol. 2, p. 151.
[4] We shall refer to the (decaying) states through which the fissioning nucleus may pass at the saddle point as "saddle-point states"; the term "transition states" has also been used [for example, see N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939)].
[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

since the $|\mathfrak{D}^I{}_{MK}|$ do not depend on ϕ or ψ. The distributions are independent of the sign of M. Explicit formulas, derived from expressions[6] for the $\mathfrak{D}^I{}_{MK}$, are given below for the first three rotational states

$$W^K{}_K(\theta)=\frac{(2K)!}{[(K-\frac{1}{2})!]^2 2^{2K}}\sin^{2K-1}\theta, \tag{4a}$$

$$W^{K+1}{}_K(\theta)=\frac{(2K+1)!}{[(K+\frac{1}{2})!]^2 2^{2K+2}}\sin^{2K-1}\theta \times[1+4K(K+1)\cos^2\theta], \tag{4b}$$

$$W^{K+2}{}_K(\theta)=\frac{(2K+3)!}{(K+\frac{1}{2})!(K+\frac{3}{2})!2^{2K+4}}\sin^{2K-1}\theta \times[1-4K\cos^2\theta+4K(K+2)\cos^4\theta]. \tag{4c}$$

The normalization is such that $\int_{-1}^{1}W^I{}_K d(\cos\theta)=1$. The functional dependence of $W^I{}_K(\theta)$ on θ near $\theta=0$ is determined by K only.

Plots of the distributions $W^I{}_I(\theta)$ for $I\leq 9/2$, arbitrarily renormalized to unity at $\cos\theta=1$, are shown as functions of $\cos\theta$ in Fig. 1. All display sideways peaking except the curve for $I=1/2$. The distributions for $I>K$ tend toward forward peaking. In the inset of Fig. 2 are plotted $W^{3/2}{}_{3/2}$, $W^{5/2}{}_{3/2}$, and $W^{7/2}{}_{3/2}$.

The experimental data of Brolley and Henkel at 1.6 Mev are shown in Fig. 2. The steep rise in the distribution near $\theta=0$ strongly indicates a saddle-point band with $K=3/2$, since the $W^I{}_K$ for all other K rise more slowly, having vanishing slopes at $\theta=0$. A mixture of angular distributions corresponding to the first three rotational states of the band $K=3/2$ and an isotropic contribution has been fitted to the data (by least squares). The result is

$$W(\theta)=0.63W^{3/2}{}_{3/2}(\theta)+0.18W^{5/2}{}_{3/2}(\theta)+0.33W^{7/2}{}_{3/2}(\theta)+0.68\times\tfrac{1}{2}. \tag{5}$$

This (unnormalized) function is plotted in Fig. 2. The constant (isotropic) term is interpreted as due to further "wobbling" of the nuclear axis past the saddle point, to nonaxially symmetric shape, or possibly to a nearby $K=\frac{1}{2}$ band. The coefficients of the $W^I{}_K$ give the relative probabilities of fission through the several saddle-point states.

The following observations can be made concerning (5). All members of a particular saddle-point rotational band have the same parity; hence only partial neutron waves of the same parity contribute to fission through states of this band. In particular, the saddle-point state $K=3/2$, $I=7/2$ can be formed via either the partial wave $L=3$ or the wave $L=4$, but not both. At the incident neutron energy 1.6 Mev, the product kR_0 of neutron wave number and average radius of the Th^{232} nucleus is $\simeq 2.4$ (for $R_0\simeq 1.42\times 10^{-13}A^{1/3}$ cm). Since one expects the contribution to compound nucleus formation (or any other process) from the partial waves to fall off steeply with L when $L\gtrsim kR_0$, the large admixture of $W^{7/2}{}_{3/2}$ in the angular distribution (5) suggests that the $L=3$ wave, rather than the $L=4$ wave, is the contributing one.[7] According to this supposition, only the $L=3$ wave contributes to the $W^{5/2}{}_{3/2}$ term in (5) (and only $L=1$ to the $W^{3/2}{}_{3/2}$ term). A statistical factor of 4/3 then tends to increase the $W^{7/2}{}_{3/2}$ term relative to the $W^{5/2}{}_{3/2}$ term. On the other hand, larger I implies less energy available for deforming the nucleus and therefore smaller probability for fission; this effect tends to decrease the former term relative to the latter. In view of the uncertainty of the experimental data and the incomplete state of the theory, the coefficients of (5) are at least qualitatively reasonable.

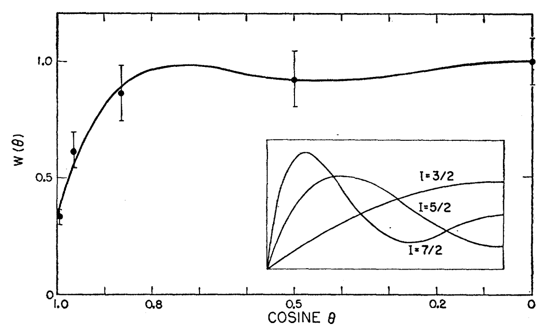

FIG. 2. Angular distribution of fission fragments from $n+Th^{232}$ at 1.6-Mev incident energy. The points represent experimental data of Brolley and Henkel. The curve constitutes a least-squares fit based on assumption of a $K=3/2$ saddle-point rotational band. Inset: plots of the normalized angular distribution functions $[W^I{}_K(\theta)]$ for the saddle-point states $K=3/2$, $I=3/2$, 5/2, 7/2.

ACKNOWLEDGMENTS

The authors wish to thank Dr. Brolley and Dr. Henkel for extensive discussion of their experiment and for permission to publish this note concurrently with their report of the experiment. They also wish to thank Dr. David L. Hill for stimulating discussions.

[6] See, for example, E. P. Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren* (Friedr. Vieweg and Sohn, Braunschweig, 1931), p. 180, Eqs. (27).

[7] This point is strengthened by the fact that single-particle resonances for the $L=3$ and $L=1$ waves are expected for energies and mass numbers in the neighborhood of those of concern here. [See Feshbach, Porter, and Weisskopf, Phys. Rev. 96, 448 (1954).]