pared with the theoretical single-particle value of -13.45. Hence, the *M*1 radiation is considerably slowed down. The E2 radiation has a value of $\log\{\tau_{\gamma}A^{4/3}E_{\gamma}{}^{5}\}$ of -6.48; the single-particle value is -8.20. The comparative lifetime is normal for an unenhanced E2.

(c) 178, 199, and 308 kev

The dipole component of these mixtures, regardless of whether it is of E1 or M1 character, is 10^6 to 10^8 times slower than the theoretical single-particle value, whereas the quadrupole component is 10^2 to 10^3 times slower which is not unreasonable for transitions which are not enhanced. The comparative lifetimes for both components of the above mixtures are shown in Table V.

VII. AUGER LINE SCREENING CORRECTION

For Z=69, the screening correction ΔZ was found to be 0.51 ± 0.15 for the $L_{\rm I}$ shell, 0.58 ± 0.15 for the $L_{\rm II}$ shell and 0.76 ± 0.20 for the $L_{\rm III}$ shell. These values are to be compared with those obtained by Bergstrom and Hill¹⁰ for mercury (Z=80). They found $\Delta Z = 0.55$ for $L_{\rm I}$ and $L_{\rm II}$ shells and 0.76 for the $L_{\rm III}$ shell.

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Angular Distribution of Fragments from Neutron-Induced Fission of U^{238} and Th^{232}

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The angular anisotropy of neutron-induced fission in Th²³² and U²³⁸ has been investigated with a double fission chamber whose common center high-voltage electrode was also a collimator. Ratios of $\sigma_F(0^\circ, E_n)/$ $\sigma_F(90^\circ, E_n)$ were measured at a number of neutron energies from the thresholds near 1 MeV to 20 MeV. Larger variations were found in the anisotropy of fission fragments for U²³⁸ and Th²³² than in earlier measurements for other nuclei, particularly at neutron energies immediately above the fission thresholds. For Th²³², there seems to be direct correlation between the variations in the anisotropy and those in the fission cross section. At 1.6 Mev, corresponding to a peak in the Th²³² fission cross section, the angular distribution of fission fragments showed a maximum at 90° to the neutron direction. All previous measurements have shown maxima along the neutron direction.

INTRODUCTION

IN earlier studies¹⁻³ it was shown that the fragments of fission induced by high-energy neutrons were anisotropically distributed in the center-of-mass system. In the first experiments^{1,2} the anisotropy was observed in varying degree for different nuclei bombarded with 14-Mev neutrons. These data were represented by angular distributions of the form $1+A \cos^2\theta$. More recent measurements³ showed that the anisotropy changed considerably when the bombarding neutron energy was changed. In addition, more careful angular distribution measurements were fitted by $1+A \cos^2\theta + B \cos^4\theta$ with the $B \cos^4\theta$ term being dominant. The only study of angular anisotropy as a function of neutron energy³ was for U²³⁵ which has a negative neutron energy threshold. It seemed useful to extend the work to include fissionable nuclei having positive neutron energy thresholds for fission. It has been suggested⁴ that near the threshold the number of energy levels involved might be small enough to make possible quantitative theoretical

analysis. The present work gives the measurements with U²³⁸ and Th²³² in an effort to furnish pertinent data to assist in the theoretical interpretation of the fission process.5-7

EXPERIMENTAL METHOD

The collimated double fission chamber used in the present measurements was the same as that used in the previous work.3 The details of the chamber, the associated electronic apparatus, the testing and calibration of the chamber, are all described in detail in reference 3, as well as experimental details concerning the neutron sources and background measurements. Since this information is identical for the present experiment, it will not be repeated here.

The Th²³² foil having a thickness of 0.9 mg/cm² was evaporated on a 0.9-mg/cm² gold foil backing. Similarly, a 1.06-mg/cm² foil of depleted U²³⁸ was evaporated on a 1-mg/cm² gold foil backing. The angular distribution data were obtained by counting the relative number of fissions produced for different rotational positions of the chamber while the foils were

[†] Work performed under the auspices of the U.S. Atomic Energy Commission. ¹W. C. Dickinson and J. E. Brolley, Jr., Phys. Rev. 90, 388

^{(1953).} ² J. E. Brolley, Jr., and W. C. Dickinson, Phys. Rev. 94, 640

^{(1954).} ⁸ Brolley, Dickinson, and Henkel, Phys. Rev. 99, 159 (1955).

⁴ Breit, Bohr, and Wheeler (private communications).

⁵ D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953). ⁶ Peter Fong, Phys. Rev. 89, 332 (1953). ⁷ A. Bohr, Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, June 1955 (United Nations, New York, 1956), Vol. II, p. 151.

irradiated in a monoenergetic flux of neutrons. Various neutron energies from threshold (1 Mev) to about 20 Mev were employed. The neutrons were produced by the $T(p,n)He^3$, $D(d,n)He^3$, and $T(d,n)He^4$ reactions using bombarding particles from the Los Alamos large electrostatic accelerator. The effects resulting from background neutrons of various kinds were ignored since errors due to backgrounds were much smaller than the statistical errors.

The counting rates were so low that it was not possible to obtain statistically significant information of the angular distribution of the low- and high-energy groups of fission fragments separately. Hence, all angu-

TABLE I. Ratio of the fission fragment yield at 0° to the neutron beam, to the yield at 90° . The values have been corrected for the finite angular resolution of the apparatus.

Ne	utron energy	0°/90° Ratio of fragments			
		U238			
1.	260 ± 0.071	-	0.989 ± 0.32		
1.	348 ± 0.081		1.09 ± 0.16		
1.	474 ± 0.103		1.70 ± 0.12		
1.	979±0.060		1.36 ± 0.06		
2.	498 ± 0.056		1.40 ± 0.06		
3.	246 ± 0.061		1.18 ± 0.05		
4.	870±0.215		1.25 ± 0.09		
6.	008 ± 0.140		1.47 ± 0.08		
7.	265 ± 0.105		1.69 ± 0.08		
1	4.5 ± 0.500		1.40 ± 0.14		
17	1.77 ± 0.300		1.26 ± 0.12		
20	0.28 ± 0.120		1.36 ± 0.10		
		Th^{232}			
1.	400 ± 0.072		0.68 ± 0.10		
1.	607 ± 0.067		0.10 ± 0.05		
1.	607 ± 0.067		0.15 ± 0.07		
1.	800 ± 0.064		0.98 ± 0.20		
2.	260 ± 0.057		1.74 ± 0.17		
2.	400 ± 0.054		1.65 ± 0.21		
3	3.00 ± 0.046		1.39 ± 0.14		
4	4.00 ± 0.038		1.15 ± 0.08		
(ó.00±0.028		1.12 ± 0.16		
6.	230 ± 0.140		2.21 ± 0.22		
7.	147 ± 0.113		2.41 ± 0.23		
7.	$.777 \pm 0.100$		2.20 ± 0.22		
9.	562 ± 0.054		1.79 ± 0.16		
1	14.5 ± 0.30		1.96 ± 0.38		
1	14.5 ± 0.30		1.85 ± 0.80		
10	5.36±0.252		1.34 ± 0.16		
17	7.86 ± 0.150		1.79 ± 0.37		
20	0.32 ± 0.085		2.10 ± 0.46		

lar data are given by the integral over both groups of the spectrum measured at various angles.

EXPERIMENTAL RESULTS

The ratio of the fission fragment yield at 0° to the neutron beam to that at 90°, $\sigma_F(0^\circ, E_n)/\sigma_F(90^\circ, E_n)$, was measured at a number of energies and the data are shown in Table I. Angular variations of the differential fission cross section were measured for three cases and the results are compiled in Table II. The energy limits indicated in Table I and Table II represent the halfspread of neutron energy intercepted by the foil resulting from degradation of charged particle energy in the gas target and by geometrical factors. Errors shown



FIG. 1. Energy dependence of the anisotropy of fission fragments and the total fission cross section for U238.

are simply those due to statistical fluctuations since estimated systematic errors were small in comparison. Estimated corrections were made to the results shown in Table I to account for the effect of the finite angular resolution of the apparatus. This effect shows up in Fig. 2 by the "zero" degree data being plotted at an effective angle of about seven degrees.

URANIUM-238

Figure 1 shows a comparison of the neutron-induced fission cross section and the anisotropy of fission fragments. The fission cross section⁸ is that measured in this laboratory by relative measurements of a fission counter and a long counter⁹ while both were irradiated by the same neutron flux. Normalization of the relative curve¹⁰ was based on the absolute determination of the U²³⁵ fission cross section by Diven.¹¹



FIG. 2. Relative differential fission cross sections. On the curve for U²³⁸, " $E_n = 7.26$ Mev" should read " $E_n = 7.27$ Mev."

⁸D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, Brookhaven National Laboratory Report, BNL-325 (Super-intendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

⁹ A. O. Hansen and J. L. McKibben, Phys. Rev. 70, 673 (1947).

¹⁰ G. A. Jarvis *et al.* (unpublished).
¹¹ B. C. Diven, Los Alamos Scientific Laboratory Report LA-1336 (unpublished), and Phys. Rev. (to be published).

Substance	E_n	0°	15°	30°	60°	90°
$U^{238} \ Th^{232} \ Th^{232}$	7.27 Mev 7.15 Mev 1.60 Mev	1.58 ± 0.06 2.30 ± 0.16 0.33 ± 0.03	1.58 ± 0.06 2.15 ± 0.15 0.61 ± 0.08	1.24 ± 0.05 1.45 ± 0.10 0.86 ± 0.12	0.99 ± 0.04 1.03 ± 0.07 0.92 ± 0.12	1.00 ± 0.04 1.00 ± 0.08 1.00 ± 0.10

TABLE II. Angular distribution of fission fragments.

Conspicuous at the threshold, is a considerable forward peaking of the fission fragments. High-energy resolution investigation of the fission cross section near threshold¹² shows the effects of possible single energy levels of U²³⁸ at neutron energies of 0.9 and 1.1 Mev. No such effects were noticed at 1.5 Mev. The anomalous behavior of the anisotropy near threshold shows a greater change over a neutron energy interval of 0.25 Mev than that reported previously for any nucleus.

Figure 2 gives an angular distribution of the differential fission cross section for U²³⁸ at 7.27 Mev. The errors indicated are those arising from statistical effects only since other errors were small in comparison. The data taken at 0° and 15° to the neutron beam are plotted at effective angles of 7 and 16° respectively due to the finite resolution of the collimator and the angular distribution of neutrons from the source. The smooth curve shown for the U²³⁸ distribution is that obtained from a fit of the data with an expansion of Legendre polynomials of the form $W(\theta) = \sum_{1}^{3} A_{2n} P_{2n}(\cos\theta)$, including terms up to $P_6(\cos\theta)$. The earlier method of analysis using a simple expansion of $W(\theta) = 1 + A \cos^2 \theta$ $+B \cos^4\theta$ was abandoned since it was inadequate for fitting the thorium data and because it seemed to have no theoretical significance. The present analysis suggests what values of angular momentum may be involved.

THORIUM-232

Figure 3 shows the comparison of the fission cross section and the anisotropy of the differential fission cross section for Th²³². The Th²³² fission cross section¹³



FIG. 3. Comparison of total fission cross sections and relative differential fission cross sections for Th²³².

was measured in the same manner as described for U^{238} . The fluctuations of the fission cross section above threshold are larger than for any other reported nucleus and imply that levels having about hundred kev spacings are contributing. In the study of this interesting region near threshold, it was found that again large changes in the differential fission cross section occur near the threshold. In particular, there is a close correlation between a narrow maximum in the fission cross section and a minimum in the $0^{\circ}/90^{\circ}$ yield of fragments at 1.6-Mev neutron energy. This peaking of the differential fission cross section at 90° to the incident neutron beam was surprising and a number of changes in the instrumentation were made to verify that the effects were real.

An angular distribution was taken at 7.15 Mev and the results are shown in Fig. 2. The smooth curve was calculated from a Legendre polynomial expansion using terms up through $A_6P_6(\cos\theta)$. The peaking of the fragments along the neutron direction is greater for Th²³² at 7.15 Mev than for any other nucleus measured.

At 1.6 Mev, another angular distribution was measured which showed a definite minimum in the differential fission cross section along the neutron direction. The smooth curve shown is from an expansion of Legendre polynomials using terms up through $A_6P_6(\cos\theta)$. The improved fit using higher order terms does not seem justified considering the accuracy of the data. The accuracy of the data is not sufficient to infer a vanishing of the 0° cross section.

Figure 4 shows a comparison of all the angular distributions made to date. At 7-Mev the anisotropy, $\sigma_F(0^\circ)/\sigma_F(90^\circ)$, seems to be a linear function of Z^2/A . Th²³² has about the lowest value of this parameter which can be easily measured with the neutron flux intensities available at present. The 1.6-Mev distribution stands out as being unusual in this comparison.

A summary curve of the measurements for U²³⁵, U²³⁸, and Th²³² as a function of neutron energy is given in Fig. 5. The anomalies near threshold, the maxima above 6 Mev, and apparent variations as high as 20 Mev are shown by the comparison.

DISCUSSION

At first, it seems curious that variations showing resonance-like structure should appear in neutroninduced reactions with nuclei as heavy as uranium and thorium. However, it has been suggested^{14,15} that a

 ¹² Malcolm E. Ennis (private communication).
¹³ R. L. Henkel and R. K. Smith, in D. J. Hughes and J. A. Harvey, Neutron Cross Sections, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

¹⁴ D. M. Chase, and L. C. Wilets (private communication). ¹⁵ N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939); N. Bohr, Phys. Rev. 58, 864 (1940).

large proportion of the energy available from the neutron binding and kinetic energy is probably used up in distorting the nucleus before fission and the excitation energy of the transition state at the time of fission is very low. Another way of saying this is that after the neutron interacts with the nucleus, the excited nucleus "cools off" by using energy for deformation before fission can occur. Since this effect might leave the excitation energy close to zero, level spacings similar to those of levels near the ground state might appear in fission excitation curves. Indeed, this suggestion is not in disagreement with existing measurements since the

FIG. 4. Comparison of existing angular distributions of fission fragment anisotropy. In the top righthand section of the figure, " $E_n = 7.4$ Mev" should read " $E_n = 7.2$ Mev."



fission "levels" have about 100-kev spacing, low-lying levels of even-even nuclei have about 40-kev spacing, and not every level would be expected to contribute heavily to the fission cross section.

The fact that the fission cross section of Th²³² rises about 6 Mev is probably due to inelastic scattering of a neutron which can leave more than 6 Mev of excitation energy in the nucleus and cause it to fission. Bohr and Wheeler¹⁵ predicted in 1940 that such effects are probable from (n,n'), (n,2n), etc., processes. Thus, below about 6 Mev, fission would probably be from a compound nucleus in the following manner: $A+n\rightarrow(A+1)^*\rightarrow$ fission. Above 6 Mev, another component of fission yield is possible from the original excited nucleus $(A+n\rightarrow A^*\rightarrow$ fission). Since above 6 Mev a "quasi-threshold" appears for fission of the excited nucleus, one might expect more resonance structure if one investigated with high enough neutron resolution. In fact, similar effects might appear for (n,2n), (n,3n),



FIG. 5. Summary of anisotropy measurements for U²³⁵, U²³⁸, Th²³².

etc., processes at higher energies, perhaps diluted by the fission contribution of the heavier components.

The fact that expansions of Legendre functions up to at least $P_6(\cos\theta)$ are required, suggests that neutrons having angular momentum of at least l=3 participate in the process. At 1.6 Mev, this may seem high; however, measurements of the angular distribution of scattered neutrons in this energy range also require relatively high l values¹⁶ to fit the data.

Anisotropic yields of fission fragments have been found in 22-Mev proton-induced fission^{17,18} for a number of nuclei with yields greatest along the direction of the bombarding particle. At energies between 60 and 600 Mev,¹⁹ 90° peaking has been observed for protonand deuteron-induced fission. Bohr⁷ and Hill and Wheeler^{4,11} have attempted to interpret this phenomenon in fission by using the collective model of the nucleus. At the present time, no quantitative predictions of anisotropies have been made. Each of these theoretical approaches looks promising to explain the peaking along the neutron direction.

In addition, Bohr's considerations seem to be able to account for lemniscatal distributions provided the nucleus bombarded be even-even and the neutron energy be sufficiently near threshold that only one channel is available for fission. Thus, at the saddle point, the nucleus is in the lowest rotational state and is partially aligned normal to the incident neutron direction.¹⁴ The curves for Th²³² show such a distribution at 1.60-Mev neutron energy.

No. 3, 407 (1955).

¹⁶ M. Walt and J. R. Beyster (private communication).

¹⁷ Cohen, Jones, McCormick, and Ferrell, Phys. Rev. 94, 625 (1954).

¹⁸ Cohen, Ferrel-Bryan, Coombe, and Hullings, Phys. Rev. 98, 685 (1955). ¹⁹ Loshkin, Perfilov, and Shamov, Proc. Acad. Sci. U.S.S.R. 103,