of supernovae of Type II do not show the 55-day half-life; the curves fall more steeply, suggesting that radioactive nuclei with decay periods longer than a few days are absent. There is considerable variety in the light curves of different Type II supernovae.

We therefore identify two cases, one in which the hydrogen concentration is deficient and the other in which it is present in excess concentration. These we tentatively associate with supernovae of types I and II, respectively.

CONCLUSION

In conclusion we wish to emphasize that the production of Cf^{254} in the November, 1952 thermonuclear test stands as clear evidence for the terrestrial production on a fast time-scale of heavy elements by neutron-capture processes. Our argument in this paper would indicate that this process is occurring on a large scale and has contributed to the synthesis of the heavy elements. In a similar manner, the existence of Tc in certain stars demonstrates that neutron-capture processes on a slow time-scale are occurring in stars. It is our point of view that neutron-capture processes on both a fast and a slow time-scale have been necessary to synthesize the heavy nuclei in their observed abundances.

We should like to express our thanks to Dr. W. Baade for many stimulating discussions, and for providing us with very valuable unpublished data on supernovae.

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Modification of the Brillouin-Wigner Perturbation Method

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By using the method of Goldhammer and Feenberg, a simple, generally valid prescription for improving the Brillouin-Wigner perturbation procedure is derived.

GOLDHAMMER and Feenberg have recently proposed an interesting modification of the Brillouin-Wigner perturbation method.[1] They illustrate their method with several examples, in each of which their refinement produces a correction factor of the same simple form. Its repeated appearance suggests that this simple correction factor may be rather generally applicable, and in this note we wish to show that this is indeed the case.

Assuming a familiarity with the material and notation of reference 1, we recall that the wave function has the form

$$\psi^{(n)}=\psi_0+G_1\sum{}'\psi_a\frac{V_{a0}}{E-E_a}+G_2\sum{}'\psi_b\frac{V_{ba}V_{a0}}{(E-E_b)(E-E_a)}+\cdots+G_n\sum{}'\frac{\psi_l V_{lk}\cdots V_{a0}}{(E-E_l)(E-E_k)\cdots(E-E_a)}. \quad (1)$$

The associated expression for the energy is

$$E=E_0+V_{00}+2\sum_{i=1}^{n}G_i\epsilon_{i+1}+\sum_{i,j=1}^{n}G_iG_j(\epsilon_{i+j+1}-\epsilon_{i+j}), \quad (2)$$

where

$$\epsilon_2=\sum_a{}'\frac{V_{0a}V_{a0}}{E-E_a},$$

$$\epsilon_3=\sum_{a,b}{}'\frac{V_{0a}V_{ab}V_{b0}}{(E-E_a)(E-E_b)},$$

and so on. In the Brillouin-Wigner method, all G_i's equal 1, whereas, according to Goldhammer and Feenberg, the G_i's in (2) are varied to make E a minimum. A formal discussion of the general case, for arbitrary n, is given in their paper.

It is instructive, however, to consider two special cases. In the first, we put all G_i's equal to 1 except G_n. The best choice of G_n is then given by

$$G_n=(1-\epsilon_{2n+1}/\epsilon_{2n})^{-1}, \quad (3)$$

while the energy becomes[2]

$$E=E_0+V_{00}+\sum_{i=2}^{2n}\epsilon_i+\frac{\epsilon_{2n+1}}{(1-\epsilon_{2n+1}/\epsilon_{2n})}. \quad (4)$$

Our second case is slightly more general: we put all G_i's equal to 1 except G_{n-1} and G_n. We then find, for the best choice of G_{n-1} and G_n:

$$G_{n-1}=\frac{\epsilon_{2n-2}(\epsilon_{2n}-\epsilon_{2n+1})-\epsilon_{2n-1}(\epsilon_{2n-1}-\epsilon_{2n})}{(\epsilon_{2n-2}-\epsilon_{2n-1})(\epsilon_{2n}-\epsilon_{2n+1})-(\epsilon_{2n-1}-\epsilon_{2n})^2},$$
$$G_n=\frac{\epsilon_{2n-2}\epsilon_{2n}-\epsilon_{2n-1}^2}{(\epsilon_{2n-2}-\epsilon_{2n-1})(\epsilon_{2n}-\epsilon_{2n+1})-(\epsilon_{2n-1}-\epsilon_{2n})^2}. \quad (5)$$

[1] P. Goldhammer and E. Feenberg, Phys. Rev. **101**, 1233 (1956).

[2] More generally, if all G_i's equal 1 except G_k, optimizing G_k leads to

$$G_k=1-\epsilon_{k+n+1}/(\epsilon_{2k+1}-\epsilon_{2k}),$$

$$E=E_0+V_{00}+\sum_{i=2}^{2n+1}\epsilon_i+\epsilon_{k+n+1}^2/(\epsilon_{2k}-\epsilon_{2k+1}).$$

Thus, regardless of k, the correction to the wave function is of order $n+1$ and the correction to the energy of order $2n+2$, as one would expect. Also, both corrections approach zero as n increases, again regardless of k.

The corresponding value of E is obtained from (2):

$$E=E_0+V_{00}+\sum_{i=2}^{2n-1}\epsilon_i+\epsilon_{2n}\bigg/\left\{1-\frac{(\epsilon_{2n-2}-\epsilon_{2n-1})\epsilon_{2n+1}+\epsilon_{2n}^2}{(\epsilon_{2n-2}+\epsilon_{2n-1})\epsilon_{2n}-\epsilon_{2n-1}^2}\right\}$$
$$+\epsilon_{2n+1}\bigg/\left\{1-\frac{(\epsilon_{2n-1}+\epsilon_{2n-2})\epsilon_{2n}+(\epsilon_{2n-1}-\epsilon_{2n-2})\epsilon_{2n+1}-\epsilon_{2n}^2}{\epsilon_{2n-1}^2}\right\}. \quad (6)$$

For the special circumstance, considered by Goldhammer and Feenberg, where

$$\epsilon_{2l+1}=0 \quad \text{for all } l, \quad (7)$$

we can allow such ϵ_i to approach zero in the preceding formulas. Equations (3) and (4) then reduce to the Brillouin-Wigner form for this case:

$$G_n=1, \quad (8)$$

$$E=E_0+V_{00}+\sum_{i=2}^{n}\epsilon_{2i}. \quad (9)$$

Thus, if (7) holds, the Brillouin-Wigner scheme cannot be improved by varying G_n alone.

However, in our second case, if (7) holds, (5) and (6) are replaced by

$$G_{n-1}=G_n=(1-\epsilon_{2n}/\epsilon_{2n-2})^{-1}, \quad (10)$$

$$E=E_0+V_{00}+\sum_{i=1}^{n-1}\epsilon_{2i}+\frac{\epsilon_{2n}}{(1-\epsilon_{2n}/\epsilon_{2n-2})}. \quad (11)$$

In this case, then, by varying G_{n-1} and G_n an improvement on the Brillouin-Wigner procedure is obtained.

Equations (4) and (11) are clearly of the same form. Together, they provide a simple, generally valid prescription for improving the Brillouin-Wigner expansion for the energy: namely, divide the highest order term in the Brillouin-Wigner expansion by 1 minus the ratio of the highest order term to the term of next lower order.[3]

A numerical example illustrating the improvement resulting from this prescription, relative to the usual Brillouin-Wigner procedure, is given in reference 1.

[3] My attention has been called to the following proof that the improved formulas actually reduce the energy: Since the last two terms of Eq. (4) are $(\epsilon_{2n}+\epsilon_{2n+1})(1-\epsilon_{2n+1}^2/\epsilon_{2n}^2)^{-1}$, whereas the corresponding terms of Eq. (2) (with all G's$=1$) are just $\epsilon_{2n}+\epsilon_{2n+1}$, these terms are greater, in absolute value, in Eq. (4) than in Eq. (2). Therefore, if the energy is reduced by the inclusion of these terms in Eq. (2), a greater reduction follows by using Eq. (4).

I am also indebted to P. Goldhammer for the observation that Eq. (4) is exact, in any order n, if the ϵ_i form a geometric progression. A similar remark applies to Eq. (11) and the equation in reference 2.

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Helmholtz Instability of a Plasma*

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A plasma having infinite electrical conductivity and no viscosity is assumed to be in contact with a uniform magnetic field along a plane boundary which is parallel to the field. The behavior of small perturbations of this boundary when the plasma is flowing at velocity v_0 perpendicular to the magnetic field is calculated by linearized theory. Perturbations which only move lines of force parallel to themselves are unstable; for small v_0/c the motion is incompressible and the rate of growth of the perturbation can be obtained from the incompressible hydrodynamic expression by replacing the mass density of each fluid in the hydrodynamic case by the sum of twice the magnetic energy density divided by c^2 and the mass density of each magnetohydrodynamic fluid. The magnetic field is to be considered as a "fluid" having only magnetic mass. It is shown that this analogy holds even in the nonlinear equations for two-dimensional incompressible flow. Perturbations which only bend lines of force are stable, while those which both move lines parallel to themselves and bend them are stable if the bending wavelength is short enough.

INTRODUCTION

HELMHOLTZ instability will be observed in hydrodynamics if two fluids are in relative tangential motion at a sharp plane boundary. Perturbations of the plane boundary are unstable and lead to mixing of the fluids. Another type of instability (Rayleigh instability) occurs if a denser fluid lies in a layer over a less dense one in a gravitational field. An analysis of combined Rayleigh-Helmholtz instability for incompressible fluids is given by Lamb,[1] while

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[1] H. Lamb, *Hydrodynamics* (Dover Publications, New York, 1945), sixth edition, p. 373.