PHYSICAL REVIEW VOLUME 103, NUMBER 4 AUGUST 15, 1956

Charge Conjugation and the τ^0 Meson

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(Received April 10, 1956)

The assumption of charge conjugation invariance, as used by Gell-Mann and Pais to discuss the decay of the θ^0 meson into two π mesons, has been applied to the decay of the τ^0 meson into three π mesons. Analogous to the θ^0-meson case, the τ^0 meson must be considered a "particle mixture" exhibiting two distinct lifetimes. There is no absolute selection rule against three-π-meson decay for either of these two τ^0 mesons, but their transition probabilities into three π mesons are quite different. For example, if the τ meson is a pseudoscalar particle, the ratio of these two transition probabilities is $\gg 100$.

A brief discussion is given of other modes of decay of neutral heavy mesons and of some experimental consequences of this picture.

I. INTRODUCTION

GELL-MANN and Pais[1] have applied to the θ^0 meson the assumption that the interaction Hamiltonian is invariant under charge conjugation even for weak interactions. They conclude that the θ^0 meson is a "particle mixture" of two types of mesons, θ_1^0 and θ_2^0, corresponding to the charge conjugation quantum numbers $+1$ and -1, each with different modes of decay and with different lifetimes. The purpose of this note is to consider the consequences of the conservation of the charge conjugation quantum number for the decay of the neutral τ^0 meson. Although there are many examples of anomalous V^0 events,[2] there is no positive experimental evidence for the existence of a τ^0 meson. However, its existence follows naturally from the assumption that the τ meson, just as the θ meson, has an isotopic spin of $\frac{1}{2}$. Following Gell-Mann[3] and Nishijima[4] we assume that the τ^0 meson has strangeness quantum number $+1$ and that the antiparticle $\bar{\tau}^0$ has strangeness quantum number -1.

If we denote the complex field operators that create the τ^0 and $\bar{\tau}^0$ mesons by the particle symbols themselves, and let C denote the charge conjugation operator, then

$$C\tau^0C^{-1}=\bar{\tau}^0,\qquad C\bar{\tau}^0C^{-1}=\tau^0. \tag{1}$$

The particles that belong to the eigenvalues $+1$ and -1 of C are τ_1^0 and τ_2^0, where the corresponding field operators satisfy

$$C\tau_1^0C^{-1}=\tau_1^0,$$

and

$$C\tau_2^0C^{-1}=-\tau_2^0. \tag{2}$$

Hence

$$\tau_1^0=(\tau^0+\bar{\tau}^0)/\sqrt{2},\quad \tau_2^0=(\tau^0-\bar{\tau}^0)/\sqrt{2}i. \tag{3}$$

The τ_1^0 (τ_2^0) particles can decay only into a state of lighter particles that is an eigenstate of C with the eigenvalue $+1$ (-1), since we have assumed that the interaction Hamiltonian is invariant under charge conjugation.

The most characteristic mode of decay of the τ^0 meson would be the decay into three π mesons:

$$\tau^0\rightarrow\begin{cases}\pi^++\pi^-+\pi^0 & \text{(4a)}\\ \pi^0+\pi^0+\pi^0. & \text{(4b)}\end{cases}$$

Other possible decay modes include

$$\tau^0\rightarrow\begin{cases}\mu^\pm+\pi^\mp+\nu & \text{(5a)}\\ e^\pm+\pi^\mp+\nu, & \text{(5b)}\end{cases}$$

$$\tau^0\rightarrow\gamma+\gamma, \tag{6}$$

and various other radiative decay modes.

The following discussion concerns the relative probabilities for the τ_1^0 and τ_2^0 mesons to decay into these decay modes.

II. 3π DECAY MODE OF THE τ^0 MESONS

We wish first to establish the properties of the three-π-meson system under charge conjugation. The π^0 meson is even under charge conjugation. The $3\pi^0$ state of (4b) is therefore an eigenstate of the charge conjugation operator with eigenvalue $+1$.[5] Hence the τ_1^0 meson, but not the τ_2^0 meson, can decay into the essentially invisible $3\pi^0$ mode of decay.

The π^+ and π^- mesons are interchanged under charge conjugation,[5] so that the classification of the (π^+, π^-, π^0) state of (4a) depends upon its symmetry character.[6] Let L denote the orbital angular momentum

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[2] A summary of most of the published anomalous V^0 events is given by R. W. Thompson, *Progress in Cosmic Rays* (Interscience Publishers, Inc., New York, to be published), Vol. 3, Chap. 5. See also the discussion of these anomalous V^0 events by Ballam, Crisaru, and Treiman, Phys. Rev. **101**, 1438 (1956).

[3] M. Gell-Mann Phys. Rev. **92**, 833 (1953); M. Gell-Mann and A. Pais, *Proceedings of the International Conference, Glasgow, 1954* (Pergamon Press, London, 1955).

[4] T. Nakano and K. Nishijima, Progr. Theoret. Phys. (Japan) **10**, 581 (1953); K. Nishijima, Progr. Theoret. Phys. (Japan) **13**, 285 (1955).

[5] For some detailed discussions of the consequences of charge conjugation invariance see A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952); L. Wolfenstein and D. G. Ravenhall, Phys. Rev. **88**, 279 (1952); L. Michel, Nuovo cimento **10**, 319 (1953).

[6] See, for example, the excellent discussion of D. Amati and B. Vitale, Nuovo cimento **2**, 719 (1955). Earlier references are given there.

TABLE I. Possible combinations of the two independent orbital angular momenta of the three-π-meson system for a few values of the spin and parity of the τ^0 meson. The τ_1^0 and τ_2^0 mesons have charge conjugation quantum numbers $+1$ and -1 respectively.

Spin	Parity	τ_1^0 (L even)	τ_2^0 (L odd)
0	$-$	$(0,0),(2,2),\cdots$	$(1,1),(3,3),\cdots$
1	$+$	$(0,1),(2,1),\cdots$	$(1,0),(1,2),\cdots$
2	$-$	$(0,2),(2,0),\cdots$	$(1,1),(1,3),\cdots$

of the relative coordinates between the π^+ and π^- mesons, and l the orbital angular momentum of the π^0 meson relative to the π^+, π^- center of mass. Then the eigenvalue of the (π^+, π^-, π^0) state under charge conjugation is $(-1)^L$, independent of l. Even and odd L's correspond to eigenvalues $+1$ and -1 respectively. The allowed values of L and l for the τ_1^0 and τ_2^0 meson decays are determined by the spin and partity of the τ meson. Table I lists these possibilities for the first few possible values of spin and parity of the τ. (Only those values that are not consistent with the decay into two π mesons are considered.[7])

Since the (π^+, π^-, π^0) spatial states available to the τ_2^0 meson and to the τ_1^0 meson are quite different, the transition probabilities of the τ_2^0 meson and τ_1^0 meson into this mode will also be different. The forbiddenness of the $3\pi^0$ mode of decay for the τ_2^0 meson clearly has the effect of lengthening its lifetime relative to that of the τ_1^0. In the discussion that follows it will be shown that the (π^+, π^-, π^0) mode of decay also tends to be faster for the τ_1^0 meson than for the τ_2^0 meson.

Dalitz[8] and Fabri[9] have given arguments to justify the assumption that for a given spin and parity of the τ meson, the three-π-meson state with the lowest allowed value of $L+l$ dominates the τ-meson decay. They estimate that the transition probability decreases by factors $\gtrsim 100$ for each successive increment of $L+l\,[\Delta(L+l)=+2]$. If the τ meson is a 0^- particle, the lowest value of $(L+l)$ is 2 for the τ_2^0 meson but 0 for the τ_1^0 meson, so that the relative transition probability of τ_2^0 and τ_1^0 into (π^+, π^-, π^0) would be expected to be $\lesssim 1/100$.

This simple criterion does not suffice to distinguish the τ_1^0 and τ_2^0 transition probabilities if the spin of the τ meson is different from zero. There are some essential differences, however, between the spatial wave functions of the 3 π mesons for the two types of τ^0 mesons. These differences become evident when the symmetry properties of the meson wave functions are examined in more detail. For this discussion it is necessary to consider the isotopic spin wave functions, as well as the spatial wave functions, of the three-π-meson system.

The complete (π^+, π^-, π^0) wave function, ψ, must be totally symmetric since the π mesons are Bose particles. ψ can be written as a linear combination of products of an isotopic spin wave function, $\chi_T^{T_z=0}$ (1,2,3), and a spatial wave function $\varphi(r_1,r_2,r_3)$. The total isotopic spin of the three $(t=1)$ π mesons can take the values $T=0$, 1, 2, or 3. The χ_T wave functions can be constructed in a straightforward manner using the method of Van Hove.[10] An isotopic spin wave function, χ, of arbitrary symmetry characteristics, can be combined with a general spatial wave function, $\varphi(r_1,r_2,r_3)$, to form a totally symmetric three-π meson wave function;

$$\psi=\tfrac{1}{6}\sum P\chi(1,2,3)\,\varphi(r_1,r_2,r_3).$$

The sum is carried out over all six permutations of (1,2,3). Only those parts of φ that have the same symmetry character as χ will contribute nonvanishing terms to this sum.

We can write χ_T^0 as a linear combination of products of $\alpha_+^{(i)}$, $\alpha_0^{(j)}$, and $\alpha_-^{(k)}$, where these α's represent the three independent isotopic spin wave functions for a π meson and (i,j,k) represent permutations of (1,2,3) (see Appendix). Charge conjugation applied to the wave function χ_T^0 is equivalent to the transposition $\alpha_+\leftrightarrow\alpha_-$, with α_0 unchanged. This transposition of α_+ and α_- changes the wave functions $\chi_{T=0}^0$ and $\chi_{T=2}^0$ into $-\chi_{T=0}^0$ and $-\chi_{T=2}^0$, while it leaves the wave functions $\chi_{T=1}^0$ and $\chi_{T=3}^0$ unaltered. Hence the $T=0$ and $T=2$ states have the eigenvalue -1 and the $T=1$ and $T=3$ states have the eigenvalue $+1$ with respect to the charge conjugation operator. It follows that the τ_1^0 meson can decay only into the states with $T=1$ or $T=3$, while the τ_2^0 meson can decay only into the states with $T=0$ or $T=2$.

This selection rule, which follows from our assumption of charge conjugation invariance, restricts the types of spatial symmetry that are allowed for the three-π-meson wave function since the symmetry properties of the isotopic spin wave functions belonging to different values of T are different. There is one wave function with $T=0$ which is totally antisymmetric; there are three wave functions with $T=1$, one totally symmetric and two of intermediate symmetry[11]; two wave functions with $T=2$, of intermediate symmetry; and finally one wave function with $T=3$, which is totally symmetric.

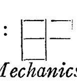
If we assume that there exists an isotopic spin selection rule for slow decays, namely $\Delta T=\pm\frac{1}{2}$,[12] then

[7] The most recent analysis of τ^+-meson decays, as reported by J. Orear, Bull. Am. Phys. Soc. Ser. II, **1**, 52 (1956), shows that the parity of the τ meson is almost certainly $-(-1)^J$. Furthermore the assignment 1^+ for the spin and parity of the τ was reported to be much less probable than either 0^- or 2^-.

[8] R. H. Dalitz, Phil. Mag. **44**, 1068 (1953); Phys. Rev. **94**, 1046 (1954).

[9] E. Fabri, Nuovo cimento **11**, 479 (1954).

[10] Van Hove, Marshak, and Pais, Phys. Rev. **88**, 1211 (1952); L. Van Hove, U. S. Atomic Energy Commission Report NYO-3074 (unpublished).

[11] These are usually denoted by the Young symbol: .
See H. Weyl, *The Theory of Groups and Quantum Mechanics* (Methuen and Company, Ltd., London, 1931).

[12] See references 3 and 4; G. Wentzel, Phys. Rev. **101**, 1215 (1956); and R. Gatto, Nuovo cimento **3**, 318 (1956). The fact

the spatial symmetry requirements on the τ_2^0 decay are considerably sharpened while those for the τ_1^0 decay are not changed.[13] The τ_2^0 meson can then only decay into the $T=0$, three-π-meson state. Hence the three-π-meson spatial wave function must be totally antisymmetric. This condition does not yield an absolute selection rule against three-π-meson emission for any of the possible values of spin and parity of the τ_2^0 meson. However, it does increase the number of nodes of the three-π-meson wave function within the small region of overlap with the τ_2^0 meson wave function, thus tending to reduce the τ_2^0-meson decay rate.

For example, in terms of the two independent relative coordinate vectors, $\mathbf{r}=\mathbf{r}_+-\mathbf{r}_-$ and $\mathbf{r}'=\mathbf{r}_++\mathbf{r}_--2\mathbf{r}_0$, the simplest totally antisymmetric meson wave function having zero spin and odd parity (including the intrinsic odd parity of the three π mesons), is[14]

$$\sim(\mathbf{r}\cdot\mathbf{r}')[(\mathbf{r}\cdot\mathbf{r}')^2-\tfrac{1}{4}(\mathbf{r}'^2-3\mathbf{r}^2)^2].$$

The complexity of these wave functions suggests that the very simple estimate of relative transition probabilities for different (L,l) combinations consistent with a given J, as given by Dalitz[8] and Fabri[9] in terms of a momentum to the lowest possible $(l+L)$th power, is not reliable for these antisymmetric meson wave functions. Fabri explicitly points out that in order to justify Dalitz's approximation, one must be able to write the 3-meson wave function as a totally *symmetric* function of the variables r and r'. It is probable therefore that the τ_2^0 transition probabilities for spins >0 are smaller than the τ_1^0 transition probabilities even for identical values of $(L+l)$. This is a qualitative argument that is hard to make quantitative without some very detailed assumptions about the meson decay mechanism.

In summary, it is likely that the τ_2^0 transition probability into 3π mesons is substantially less than the corresponding τ_1^0 transition probability for all values of spin and parity of the τ meson. If the simplest hypothesis prevails, namely that the τ meson is a 0^- particle, then the τ_2^0 decay probability should be particularly small, $\ll(1/100)$ of the τ_1^0 decay probability.

III. OTHER DECAY MODES OF THE τ^0 MESON

Dalitz[15] has extensively discussed various probabilities for radiative τ^+ decay, such as

$$\tau^+\rightarrow\pi^++\pi^++\pi^-+\gamma \tag{7}$$

and

$$\tau^+\rightarrow\pi^++\pi^0+\gamma. \tag{8}$$

One might expect that such decays would be less probable by a factor of $\alpha=1/137$ and Dalitz's conclusions in most cases confirm this.[16] For pseudoscalar τ mesons, the probabilities of these radiative modes are particularly small. His arguments should apply equally well to neutral τ mesons. These decay modes are available to both the τ_1^0 and the τ_2^0 mesons, but they would be relatively more important for the τ_2^0 meson since a transition probability that is $\sim\alpha$ times smaller than the transition probability of a τ_1^0 into three π mesons, could be of comparable magnitude to the allowed three-π-meson transition probability of the τ_2^0 meson. (The γ ray has an eigenvalue -1 with respect to charge conjugation so that the decay $\tau_2^0\rightarrow\pi^++\pi^-+\pi^0+\gamma$ demands that the three-π-meson state belong to the eigenvalue $+1$ with respect to charge conjugation. This is the same requirement as that for the 3-π-meson state in τ_1^0 decay.)

If the τ meson is a 0^- particle, the essentially invisible decay, $\tau^0\rightarrow\gamma+\gamma$, is allowed for the τ_1^0. It is never allowed for the τ_2^0. It is difficult to estimate the relative transition probability of this decay mode relative to the three-π-meson mode of decay of the τ_1^0, but again one might venture that it is smaller by a factor $\sim\alpha$. In any case this essentially invisible mode does not contribute to the τ_2^0 lifetime.

So far we have considered modes of decay of the τ_2^0 meson which imply that its lifetime is much longer than the lifetime of the τ_1^0 meson. (This factor is $\gg 100$ if the τ meson is a 0^- particle.) Such large *a priori* differences of decay probabilities between the τ_1^0 and τ_2^0 mesons do not exist for the $K_{\mu 3}^0$ or K_{e3}^0 modes of decay[17]; that is,

$$\tau^0\rightarrow\pi^\pm+\begin{Bmatrix}\mu^\mp\\ e^\mp\end{Bmatrix}+\nu \text{ or } \bar{\nu}. \tag{9}$$

By forming appropriate linear combinations of the two charge combinations on the right-hand side of Eq. (9), one can form eigenstates with eigenvalues $+1$ and -1 of the charge conjugation operator. For either of these two modes, the τ_2^0 and τ_1^0 meson decay rates are equal. Of course it is still an open question as to whether it is the θ^+ meson or the τ^+ meson or both that contribute to the observed $K_{\mu 3}^+$ and K_{e3}^+ events. If these modes of decay are available to the τ^0 meson, then it is probable that this is the dominant mode of decay of the τ_2^0 meson, particularly if the τ meson has spin 0. It might also be noted that if the $K_{\mu 3}$ and K_{e3} modes of decay are available to the θ^0 meson, they, rather than the radiative decay $(\theta_2^0\rightarrow\pi^++\pi^-+\gamma)$ that was discussed by Gell-

that the ratio $(\tau^+\rightarrow\pi^++\pi^++\pi^-)/(\tau^+\rightarrow\pi^++\pi^0+\pi^0)$ lies between 1 and 4 is consistent with this selection rule. If the experimental data yield a result of 4 for the ratio, it would imply that only the totally symmetric $T=1$ wave function plays a role in the τ^+ decay.

[13] If the τ_1^0 decays into a pure $T=1$ state, it follows that $0\leq(\tau_1^0\rightarrow3\pi^0)/(\tau_1^0\rightarrow\pi^++\pi^-+\pi^0)\leq\frac{3}{2}$. See R. H. Dalitz, Proc. Roy. Soc. (London) **66**, 710 (1953). I am indebted to R. Glasser for pointing out a misprint for this ratio in Dalitz's paper.

[14] In the (L,l) notation, this wave function is a combination of (1,1) and (3,3) states. There does not exist a totally antisymmetric pure (1,1) wave function. [See R. G. Sachs, *Nuclear Theory*, (Addison-Wesley Press, Cambridge, 1953) for a discussion of this point.] Similarly the, simplest $J=2^-$ wave function that is totally antisymmetric involves combinations of (1,1) and (1,3) wave functions.

[15] R. H. Dalitz, Phys. Rev. **99**, 915 (1955).

[16] See also S. B. Treiman, Phys. Rev. **95**, 1360 (1954) on the radiative θ^0 meson decay.

[17] Yekutieli, Kaplon, and Hoang, Phys. Rev. **101**, 506 (1956); Bull. Am. Phys. Soc. Ser. II, **1**, 64 (1956).

Mann and Pais,[1] could form the dominant modes of decay for the θ_2^0 meson.

IV. DISCUSSION AND CONCLUSIONS

The basic idea of invariance of the Hamiltonian with respect to charge conjugation of Gell-Mann and Pais has been applied to the τ^0 meson. Corresponding to the existence of two distinct types of θ^0 mesons, θ_1^0 and θ_2^0, there are two distinct types of τ^0 mesons, τ_1^0 and τ_2^0, with different lifetimes and different modes of decay. From the experimental point of view, the ease of observation and clarification of this complicated state of affairs depends upon the quantitative lifetimes of these mesons and on the relative transition probabilities into the different modes of decay for each particle. A few qualitative remarks on these questions seem appropriate.

The best known neutral heavy meson is the θ^0 meson that decays into a π^+ and a π^-. (We shall call it the θ_1^0.[18]) It is a striking experimental fact that the lifetime of the θ_1^0 meson is less by a factor of 100 than the lifetime of its isotopic brother the θ^+. An attractive explanation of this effect in terms of the approximate validity of the $\Delta T=\pm(\frac{1}{2})$ selection rule has been proposed.[12] The application of this selection rule to the τ meson decay into 3 π mesons does *not* yield any speed-up of the τ_1^0 decay as compared to the τ^+ decay. This follows from the fact that the same $(T=1)$, three-π-meson wave functions in isotopic spin space are available to the τ_1^0 as to the τ^+, and that from our previous arguments the $T=0$ state that is available to the τ_2^0 meson will have a much smaller three-π-meson decay probability than the τ_1^0 meson. The absence of any evidence of a neutral particle that decays into (π^+, π^-, π^0) with a lifetime comparable to that of the θ^0 is in agreement with this conclusion.[19] As for the $K_{\mu3}$ and K_{e3} modes of decay, there is no *a priori* reason why these should be faster for neutral particles than for charged particles. However, our ignorance of this whole subject of slow decays certainly allows such a speed-up as a distinct possibility.

Neglecting any speeding-up factors for the neutral τ's, one might then try to estimate the τ_1^0 and τ_2^0 lifetimes from the observations on τ^+ mesons. Unfortenately, the absolute transition probability for a τ^+ meson to decay into three π mesons or into $(\mu+\pi+\nu)$ is not uniquely determined by the measured τ^+ lifetime and the branching ratios of the K^+ meson into its various modes. A further assumption is necessary concerning the breakdown of the $K_{\mu2}^+$ and $K_{\mu3}^+$ (K_{e3}^+) decays among the two candidates θ^+ and τ^+.[19] Roughly, the reciprocal of the transition rate of the τ^+ meson into 3 π mesons is $\sim 10^{-7}$ sec. The reciprocal of the transition rate of some mixture of τ^+ and θ^+ mesons into the $K_{\mu3}^+$ and K_{e3}^+ modes is about twice as long. Equating the transition probabilities of the neutral τ mesons to that of the charged τ^+ meson, in a way consistent with our previous discussions, one obtains for the lifetimes of the τ_1^0, $\tau_{(\tau_1^0)}\approx 10^{-7}$ sec, and for the ratio of the lifetimes of the τ_1^0 and τ_2^0 mesons $\tau_{(\tau_1^0)}/\tau_{(\tau_2^0)}\lesssim\frac{1}{2}$. If the $K_{\mu3}$ and K_{e3} modes of decay were dominant for the θ_2^0 meson, then again one would estimate a lifetime $\tau_{(\theta_2^0)}\lesssim 10^{-7}$ sec. None of these estimates take into account any enhancement of the neutral meson decay probability over that of the charged meson decay probability.[20]

If enhancement is ignored, the estimated lifetimes for the τ_1^0, τ_2^0, and θ_2^0 mesons are indeed long compared to the time needed to cross a typical cloud chamber. Hence experimental observations of these particles would be very difficult. Such observations would be further complicated by the possibility that a single decay mode, such as the $K_{\mu3}^0$ or K_{e3}^0, could arise from three different particles, the τ_1^0, τ_2^0, and θ_2^0 mesons, with distinct lifetimes.

On the other hand, the observations of a large number of anomalous V_0 events could only be consistent with lifetimes much shorter than $\sim 10^{-7}$ sec. This would necessarily imply some enhancement of the neutral meson decays, the most probable being the $K_{\mu3}^0$ and K_{e3}^0 modes.[21] Such an enhancement would also have the effect of making our discussion of the 3π modes of decay of the τ_1^0 and τ_2^0 mesons somewhat superfluous since both particles would have the same transition rate into the $K_{\mu3}^0$ or K_{e3}^0 modes of decay, and this transition rate would essentially determine their lifetimes.

If τ^0 mesons and θ^0 mesons are produced with equal probabilities, as has been suggested by Lee and Yang,[22] then in three-fourths of the production events of neutral heavy mesons, the neutral meson would be either a θ_2^0, τ_1^0, or τ_2^0. All of these particles are sufficiently long-lived so as to have only a small probability of being detected in association with a Λ^0 or Σ hyperon.[23]

The above discussion on the possibility of experimental observation of the different types of neutral heavy mesons has been predicted upon the assumption

[18] The θ_1^0, as we have defined it heretofore, is the particle with eigenvalue +1 under charge conjugation. If the parity of the θ^0 meson is even, then this is the particle that decays into the π^+ and π^- mesons.

[19] A. H. Rosenfeld and M. L. Stevenson, University of California Radiation Laboratory Report UCRL-3314 (unpublished), come to a similar conclusion as to the predicted absence of fast $\tau^0\rightarrow 3\pi$ decays. They argue that the difference in lifetimes between the $\theta^0\rightarrow\pi^++\pi^-$ decay and the $\tau^+\rightarrow 3\pi$ decay is due primarily to the difference in available phase space. See also M. L. Stevenson, University of California Radiation Laboratory Report UCRL-3275 (unpublished). These papers contain references to the measurements of the lifetimes and branching ratios of K^+ mesons.

[20] As mentioned previously, such an enhancement could not reasonably be expected for the three-π-meson mode of decav.

[21] This point has been made by S. Treiman (private communication). See also Ballam, Grisaru, and Treiman, Phys. Rev. **101**, 1438 (1956).

[22] T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956).

[23] The observations of the Columbia cloud chamber group: Blumenfeld, Booth, Lederman, and Chinowsky, Bull. Am. Phys. Soc. Ser. II, **1**, 63 (1956). Is consistent with this ratio of observable to nonobservable V^0 mesons.

that the θ^+ meson and τ^+ meson are different particles that each have a long lifetime ($\approx 10^{-8}$ sec). Some of the conclusions would be changed if the cascade scheme of Lee and Orear[24] prevailed. There is then a bewildering number of alternative combinations of decays of θ's into τ's or vice versa, and we shall not engage in an exhaustive discussion of all the possibilities. We merely note that, if the θ^0 and τ^0 mesons both have zero spin, the possibile cascade decay schemes that are consistent with charge conjugation invariance are

$$\begin{aligned}\tau_1^0 &\rightarrow \theta_1^0+\gamma+\gamma,\\ \tau_2^0 &\rightarrow \theta_2^0+\gamma+\gamma,\end{aligned} \tag{10}$$

if $M_{\tau^0} > M_{\theta^0}$, or conversely, $\theta^0 \rightarrow \tau^0$ if $M_{\theta^0} > M_{\tau^0}$. If both particles do not have zero spin, then for $M_{\tau^0} > M_{\theta^0}$ we have

$$\begin{aligned}\tau_1^0 &\rightarrow \theta_2^0+\gamma,\\ \tau_2^0 &\rightarrow \theta_1^0+\gamma,\end{aligned} \tag{11}$$

and conversely for $M_{\theta^0} > M_{\tau^0}$.

These cascade possibilities can obscure to some extent an experiment that has been proposed by Pais and Piccioni[25] as a direct test of the theoretical suggestion of Pais and Gell-Mann that the θ^0 meson is a "particle mixture." Essentially they propose that one look for (π^+, π^-) decays of neutral mesons just below an absorber that is located sufficiently far from the production source of the neutral mesons so as to ensure that the elapsed time is long compared to the lifetime of the ordinary $\theta^0 \rightarrow \pi^+ + \pi^-$ decays ($\sim 10^{-10}$ sec). The point is that a τ^0 meson that has a lifetime $>10^{-10}$ sec and that can decay into a θ^0 meson will act as a source of θ^0 mesons that appear to have lived a long time. The possibility of the existence of a long-lived τ^0 meson is clearly independent of the theoretical idea of "particle mixtures," which follows from the assumption of charge conjugation invariance. It is true, however, that this source of θ^0 mesons would not be distributed more heavily below the absorber than above the absorber, as is predicted by Pais and Piccioni in their θ^0-meson discussion. The proposal of the Columbia cloud chamber group[23] to look for Λ^0 or $\Sigma^\pm$ particles after the neutral mesons have traversed the absorber would not be affected by such long-lived τ^0 mesons, provided that the production of the neutral mesons took place at an energy below the threshold for direct $\bar{\tau}^0$ production. [Analogous to the decays depicted in Eqs. (10) or (11), the $\bar{\tau}^0$ mesons could act as a source of $\bar{\theta}^0$ mesons via γ emission.]

[24] T. D. Lee and J. Orear, Phys. Rev. **100**, 932 (1955).
[25] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

ACKNOWLEDGMENTS

It is a pleasure to thank R. Glasser and S. Treiman for stimulating discussions.

APPENDIX

As an illustration we list the isotopic spin wave functions belonging to $T=0$ and $T=2$ with $T_z=0$:

$$\chi_{T=0}{}^0(1,2,3) = (1/\sqrt{6})\sum_P \epsilon_p \alpha_+{}^{(i)}\alpha_0{}^{(j)}\alpha_-{}^{(k)}, \tag{12}$$

where P denotes a permutation (i,j,k) of the numbers (1,2,3), and ϵ_p is $+1$ or -1 depending on whether P is an even or odd permutation;

$$\begin{aligned}{}_2\chi_{T=2}{}^0 = \tfrac{1}{2}\{&\alpha_0{}^{(1)}(\alpha_+{}^{(2)}\alpha_-{}^{(3)} - \alpha_-{}^{(2)}\alpha_+{}^{(3)})\\ &+\alpha_0{}^{(2)}(\alpha_+{}^{(1)}\alpha_-{}^{(3)} - \alpha_-{}^{(1)}\alpha_+{}^{(3)})\},\end{aligned} \tag{13}$$

$$\begin{aligned}{}_1\chi_{T=2}{}^0 = &(1/2\sqrt{3})(\alpha_0{}^{(1)}\alpha_-{}^{(2)} - \alpha_-{}^{(1)}\alpha_0{}^{(2)})\alpha_+{}^{(3)}\\ &+(1/\sqrt{3})(\alpha_+{}^{(1)}\alpha_-{}^{(2)} - \alpha_-{}^{(1)}\alpha_+{}^{(2)})\alpha_0{}^{(3)}\\ &+(1/2\sqrt{3})(\alpha_+{}^{(1)}\alpha_0{}^{(2)} - \alpha_0{}^{(1)}\alpha_+{}^{(2)})\alpha_-{}^{(3)}.\end{aligned} \tag{14}$$

The subscripts 2 and 1 on the left side of $\chi_{T=2}$ denote the total isotopic spin state of the first two π mesons. This is then combined with the third π meson. It is clear that

$$P_{(+\leftrightarrow-)}\begin{Bmatrix}\chi_{T=0}{}^0\\ {}_2\chi_{T=2}{}^0\\ {}_1\chi_{T=2}{}^0\end{Bmatrix} = -\begin{Bmatrix}\chi_{T=0}{}^0\\ {}_2\chi_{T=2}{}^0\\ {}_1\chi_{T=2}{}^0\end{Bmatrix}. \tag{15}$$

Hence all three of these functions belong to the eigenvalue -1 with respect to charge conjugation. Similarly the $T=1$ and $T=3$ wave functions with $T_z=0$ satisfy

$$P_{(+\leftrightarrow-)}\begin{Bmatrix}\chi_1{}^0\\ \chi_3{}^0\end{Bmatrix} = +\begin{Bmatrix}\chi_1{}^0\\ \chi_3{}^0\end{Bmatrix}.$$

Under charge conjugation, the $T=0$ and 2 isotopic spin wave functions belong to the eigenvalue -1, and the $T=1$ and 3 wave functions belong to the eigenvalue $+1$.