THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 102, NO. 4 — MAY 15, 1956

Variation with Electron Energy of the Collision Cross Section of Helium for Slow Electrons*

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(Received December 16, 1955)

The method of interaction of microwaves in gaseous discharge plasmas has been applied to a determination of the variation of the effective collision cross section for momentum transfer, q_m, of helium atoms with electrons having energies from 0.04 to 0.4 ev. Under our experimental conditions with electron densities $\gtrsim 10^{10}$ electrons/cc, appropriate charge interactions have been considered. The mean fraction of excess energy loss of slow electrons upon collision, G, was found to be $\sim 2.7\times 10^{-4}$, the expected $2m/M$ value.

INTRODUCTION

THE phenomenon of interaction of guided microwaves in gaseous discharge plasmas, previously reported,[1] enables a determination of the "effective" electron collision cross section for momentum transfer in the collision of free electrons with other heavier constituents of the plasma. A preceding article[2] (referred to here after as I) reported a determination of the "effective" collision cross section of helium atoms and singly charged positive ions for slow electrons having a mean energy of 0.039 ev, corresponding to room temperature ($\sim$300°K). It was shown in I that the "effective" electron-molecule collision frequency, ν_m, for room temperature electrons with helium atoms was found to be 3.12×10^8/sec mm Hg, giving an effective collision cross section, q_m, equal to 6.8×10^{-16} cm^2 or a probability of collision for momentum transfer, P_m, equal to 24 cm^2/cm^3 at 0°C and 1 mm Hg. In the present discussion a controlled variation of the mean energy of the electrons in a decaying, initially room temperature, isothermal, discharge plasma established in helium permits the determination of the *variation* of q_m or P_m with electron energy. The geometries and assemblage of apparatus of the experiment are identical to those described in I. The momentum scattering of electrons by positive ions of the plasma was minimized in this case, however, by appropriate experimental conditions of relatively high gas pressures.

Briefly the conduct of the experiment is as follows: the discharge plasma, contained in a thin-wall Pyrex discharge tube located coaxially in square wave guide, is produced by application of a repetitious two-microsecond high-voltage dc pulse to a cathode and anode external to the guide. At appropriate times in the afterglow thus formed, two or more low-level microwaves are propagated. The higher power-level ($<$0.5 watt) "disturbing" microwave is a 1- to 100-microsecond duration square pulse of microwave energy at a frequency of 8600 Mc/sec. The "wanted" wave, lower in power level by $>$30 db, is a continuous wave at 9400 Mc/sec. Some fraction of the "disturbing" wave energy is absorbed by the plasma medium. The electromagnetic energy extracted increases the mean agitational energy of the free electrons of the plasma. This increase in electron energy alters the effective electron collision frequency with other heavier constituents of the plasma. The second "wanted" microwave is utilized to detect these small changes in collision frequency, its absorption being proportional to the product of electron collision frequency and electron number density.

Recent measurement of the microwave conductivity in the afterglow of a pulsed discharge in helium has been reported by Gould and Brown.[3] They utilize

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[1] Goldstein, Anderson, and Clark, Phys. Rev. **90**, 151 (1953).
[2] J. M. Anderson and L. Goldstein, Phys. Rev. **100**, 1037 (1955).
[3] L. Gould and S. C. Brown, Phys. Rev. **95**, 897 (1954).

cavity techniques[4] in a similar method of microwave interaction. The variation of the probability of collision for momentum transfer, P_m, with electron energy reported by Gould and Brown, is dissimilar to other earlier determinations in that it does not contain small variations in P_m for energies <1 ev as measured by Ramsauer and Kollath[5] and by Normand.[6] It, therefore, appears advisable to re-examine the variation of the helium cross section for electrons in this energy range utilizing the technique of cross modulation of propagating microwaves, in order to take advantage of the high degree of relative accuracy possible by this technique for small variations of electron energy.

PRELIMINARY CONSIDERATIONS

A determination of the variation of P_m as a function of electron energy follows immediately from an experimental determination of the variation of effective electron collision frequency, ν_m, as a function of mean electron energy. In an evaluation of ν_m, the following difficulties arise:

(1) Microwave energy is absorbed nonuniformly in the plasma, because of nonuniform applied microwave electric field strength and nonhomogeneous electrical conductivity of the plasma.

(2) Heat may propagate through the plasma because of gradients of temperature and charge density.

(3) The electron gas loses energy in collisional contact with gas molecules and ions. The relaxation time for electron-molecule energy exchange is $\sim 1/G_m\nu_m$, where G_m is the mean fraction of excess electron-energy lost in each collision and is $\cong 2m/M$, where m/M is the ratio of electronic to atomic mass. The similar factor, G_i, for electron-ion collisions has not as yet been satisfactorily determined; based, however, on experimental results, shown in I, it will be neglected hereafter.

(4) The variation of microwave conductivity of the plasma as a function of mean electron energy is not known (precisely the quantity to be determined).

In the following it will be assumed that no change in electron density distribution occurs during periods of elevated electron temperature, T_e, in the presence of the disturbing wave (<20 microseconds). That is to say, it is expected that a steady state in electron temperature may establish in such short time intervals, but that the slower redistribution of electron density (as a result of T_e gradients) has not had time to take place. Such a "quasi" steady state is experimentally observed by negligible variation of the phase constant associated with wave propagation in the plasma-filled wave guide during the "heating" process. Indeed, the phase constant measured is sensitive to distribution as well as total electron density. Further, in any macroscopic volume, the Maxwellian distribution of velocities of the electrons is assumed to exist at all times. This appears to be justified, due primarily to electron-electron interactions at the electron densities here considered.

In each macroscopic volume of the plasma medium, the balance of mean agitational energy for each electron may be shown to be

$$\frac{dQ_e}{dt}+G_m\nu_m(Q_e-Q_m)=\frac{|E'|^2}{2n_e}\sigma_r+\frac{K}{n_e}\nabla^2T_e, \tag{1}$$

where σ_r is the real part of the complex high-frequency conductivity of the plasma and is assumed to be of the form

$$\sigma_r=(n_ee^2/m\omega^2)\nu_m; \tag{2}$$

Q_e and Q_m are the mean agitational energies of the electrons and atoms, respectively; $E'\sim e^{i\omega t}$ is the microwave electric field in the plasma medium; e and m are electronic charge and mass, respectively; n_e is the electron density, and K is the heat transfer coefficient. Since $Q_e=\frac{3}{2}kT_e$, where k is Boltzmann's constant, and in the "quasi" steady state, described above, $dT_e/dt=0$, the T_e may be written as

$$T_e=T_m+\frac{e^2|E'|^2}{3mkG\omega^2}+\frac{2K}{3kn_eG\nu_m}\nabla^2T_e. \tag{3}$$

The usually considered processes of heat propagation through the plasma medium, such as those due to ambipolar diffusion, emission and absorption of radiation, diffusion of hot electrons through the gas of neutral atoms, etc., and which do not take into account electron-electron interaction, may be shown to be negligible for the short time intervals of measurement and relatively small gradients of temperature in our experiment. A heat transfer coefficient for the completely ionized gas which does include the influence of electron-electron interaction has been given by Spitzer and Härm.[7] The evaluation of this coefficient for the charge densities of our experiment predicts appreciable heat transfer through the electron gas of the plasma, and the time constant associated with this heat transfer ($\sim 10^{-8}$ second) is calculated to be so small that some amount of temperature equalization may take place in the short time interval of measurement in the experiment. However, it is doubtful that the coefficient given by Spitzer and Härm is applicable in our case of relatively low percent ionization ($\sim 0.001\%$), since even if electron-electron interaction is taken into account, the presence of a large density of effective scattering centers (here the neutral atoms) would reduce heat transfer with respect to that of the completely ionized gas. Regardless of this latter statement, the total microwave energy dissipation in the plasma, with inclusion of this rapid heat propagation as given by

[4] L. Gould and S. C. Brown, J. Appl. Phys. **24**, 1053 (1953).
[5] C. Ramsauer and R. Kollath, Ann. Physik **3**, 536 (1929).
[6] C. E. Normand, Phys. Rev. **35**, 1217 (1930).
[7] L. Spitzer and R. Härm, Phys. Rev. **89**, 977 (1953).

the coefficient for a completely ionized gas, was calculated by an approximate procedure. A comparison with the experimentally determined microwave energy dissipation shows considerable discrepancy, as will be discussed later. A further qualitative, but we believe significant, experimental observation by the phenomena of afterglow quenching[8] is also an argument in favor of a much smaller and probably negligible heat transfer coefficient. Viewing the emitted light of the afterglow with a photomultiplier free to move along the discharge tube, the quenching of the afterglow is observed to vary within experimental error as the expected exponential loss of microwave energy in propagation along the plasma-filled wave guide. Rapid heat propagation through the plasma with consequent temperature equalization would tend to destroy this agreement. The electron temperature distribution is, therefore, in what follows assumed to be little influenced by propagation of heat in the plasma.

PROBABILITY OF COLLISION AND ITS VARIATION WITH ELECTRON ENERGY

The calculation of the expected total microwave insertion loss in the plasma-filled portion of the wave guide is reasonably simple of solution if it is assumed initially that negligible heat propagates in the plasma and that the probability of collision for momentum transfer, P_m, is independent of electron energy. A procedure for calculation of this total microwave energy loss under these assumptions is given in Appendix I. When the results of this calculation are compared with the experimentally determined loss, and discrepancy is noted, such deviation could perhaps be due to a variation of P_m with electron energy. With this supposition and admitting the difficulty of a complete solution for the effective electron collision frequency, ν_m, as a function of electron temperature, and, what is the same thing, P_m as a function of electron velocity, it is further assumed that the discrepancy between theory and experiment is to a first approximation linearly related to a change in ν_m away from that value predicted by constant P_m. For the condition of little total microwave loss, some average value of the electric field in the plasma medium is assigned for each amplitude of the incident disturbing wave in order to determine the corresponding electron temperature. If the variation of ν_m is not appreciably different from $\nu_m \sim T_e^{\frac{1}{2}}$ the error in this procedure is probably not great. Finally, the measured ν_m will be expressed as a series of powers of T_e:

$$\nu_m = \sum_n b_n T_e^{\frac{1}{2}n+\frac{1}{2}} \tag{4}$$

from which the coefficients of a series expansion of P_m in terms of electron velocity, v, follow immediately,

$$P_m = \sum_n a_n v^n, \tag{5}$$

where

$$b_n = a_n p_0 \frac{8}{3\sqrt{\pi}} \left(\frac{2k}{m}\right)^{\frac{1}{2}n+\frac{1}{2}} \int_0^\infty u^{n+5} \exp(-u^2) du,$$

p_0 is the pressure normalized to 0°C, and $u = (m/2kT_e)^{\frac{1}{2}} v$.

EXCESS ELECTRON ENERGY LOSS FACTOR, G, IN HELIUM

Observation of the character of the pulse modulation transferred to the wanted wave under the conditions of experimentation previously described, will permit a determination of the mean fraction of excess energy lost, G, by an electron in colliding with heavier constituents of the plasma. In the simplest case of small deviation of the collision frequency of the electrons the relaxation time constant associated with the effect of cross-modulation is related to the effective electron collision frequency and the loss factor G by, $\tau \sim 1/G\nu_m$, as shown in I. More specifically, a measure of the rate of change of transmission of the wanted wave as a function of time, immediately after removal of the pulsed disturbing wave, and a knowledge of the extremes of electron collision frequency during the cross-modulation effect, suffice to calculate G. However, the precision with which this factor may be determined will be limited by the necessary following assumptions. As before, an average microwave electric field applicable for the entire length of the plasma is assumed for each incident disturbing wave amplitude, which is equivalent to assigning an average value of electron collision frequency to apply along the discharge tube and in the important central region of the waveguide transverse to microwave propagation. Further, the collision probability, P_m, is considered constant with electron energy, in the range here involved. Under these conditions the following expression may be written.

It was shown in I that the attenuation constant α is given by

$$\alpha = \frac{2e^2 n_0 \mu_0}{\pi^2 m\omega} \left(\frac{\lambda_g}{2\pi}\right) (1.14\nu_m + 0.765\nu_{i,\max}). \tag{6}$$

However, as already mentioned for the case here discussed, $\nu_i \ll \nu_m$. Hence,

$$\frac{\partial\alpha}{\partial t} = \frac{2.28e^2 n_0 \mu_0}{\pi^2 m\omega} \left(\frac{\lambda_g}{2\pi}\right) \frac{\partial \nu_m}{\partial t}. \tag{7}$$

But from Eq. (13) of I, $\partial\nu_m/\partial t$ is readily obtained, and finally

$$\left.\frac{\partial\alpha}{\partial t}\right|_{t=0} = \frac{2.28e^2 n_0 \mu_0}{\pi^2 m\omega} \left(\frac{\lambda_g}{2\pi}\right) \nu_0^2 G \left[\frac{-2F}{F^2 - 2F + 1}\right], \tag{8}$$

where $F = (\nu' + \nu_0)/(\nu' - \nu_0)$, ν_0 is the initial electron collision frequency in the isothermal plasma, and ν' is the elevated steady-state value of ν_m during the effect

[8] Goldstein, Anderson, and Clark, Phys. Rev. **90**, 486 (1953).

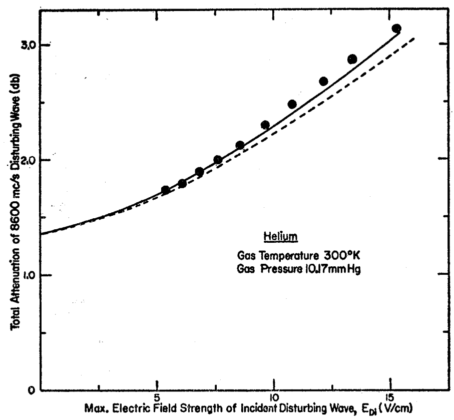

FIG. 1. Comparison of total attenuation of the 8600-Mc/sec wave by the plasma medium with that which would be expected by theory. (See text for description.)

of cross modulation. Time, $t=0$, coincides with removal of the disturbing wave pulse.

EXPERIMENTAL RESULTS AND CONCLUSIONS

A measurement was made of the variation of total loss of microwave energy in the plasma-filled portion of the wave guide as a function of incident electric field strength of the disturbing wave, which leads indirectly to the variation of the effective electron collision frequency with helium atoms as a function of the electron temperature. In a determination of microwave energy loss, the amplitude of the transmitted and detected continuous wanted wave, $\omega_W/2\pi=9400$ Mc/sec, was recorded photographically while simultaneously a higher power-level, (max$\sim\frac{1}{2}$ watt) 20-microsecond duration pulse of disturbing wave, $\omega_D/2\pi=8600$ Mc/sec, was propagated through the plasma. The measurements were made at a time when the electron density, n_0, reached the selected value of 2.5×10^{10} electrons/cc in the plasma decay, and the gas pressure in this experiment was 10.15 mm of Hg. These conditions were chosen to satisfy the assumptions previously stated. The disturbing wave was attenuated at its source in steps of 0, 1, 2, 3, 4, 6, and 10 db from a maximum incident power level of 410 milliwatts. At 8600 Mc/sec this maximum power implies a maximum electric field strength in the square wave guide of 15.3 volts/cm. After treatment according to calibration of equipment, the insertion loss of the wanted wave for various levels of disturbing wave is reproduced in Table I. Reflection of both microwave signals from the discontinuities due to the discharge tube proves to be small, allowing the insertion loss measured to be taken with negligible error as the true attenuation in the plasma.

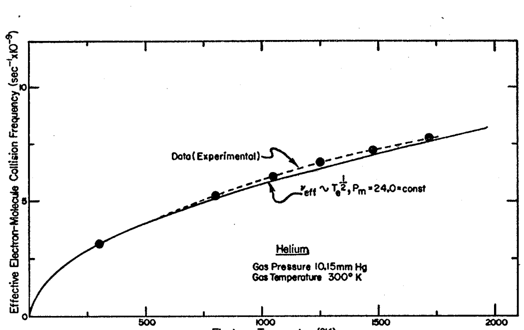

FIG. 2. Variation of "effective" electron-molecule collision frequency in the afterglow established in helium as a function of electron temperature (gas temperature $\sim$300°K).

The total attenuation of the 8600-Mc/sec disturbing wave in the plasma (self-distortion, see I) is linearly related to the right column of Table I, according to Eq. (19), of Appendix I. In passing it may be noted that an average of measurements of the effective electron collision frequency in the isothermal room temperature plasma established in helium made at a microwave frequency of 9400 mc/sec, agrees within 2% with measurements at 8600 Mc/sec.

From the earlier development with the viewpoint of negligible heat propagation in the plasma, the calculated total microwave energy in the plasma in decibels for the 8600-Mc/sec signal is given in Fig. 1 as the solid curve. The dotted curve is for a similar calculation assuming rapid heat propagation (equalization of T_e) over the cross section of the wave guide but negligible heat propagation along the discharge tube. Such assumptions are admittedly an oversimplification of fact but will permit a reasonable simple calculation for the purpose of comparison. The data are seen to fall above both calculated curves at the higher temperatures but to agree fairly well with the solid curve at low temperatures. The discrepancy between experiment and the solid curve is, at most, $\sim$3.5%, and therefore any significance of the discrepancy may be shrouded in experimental error. However, the experimental points possess no appreciable random nature, but rather follow a well-described pattern. Further, this type of data depends strongly upon relative measurements which are inherently more accurate. The discrepancy be-

TABLE I. Attenuation of the transmitted (9400-Mc/sec) wanted wave through the plasma in the presence of simultaneously propagated (8600-Mc/sec) disturbing waves of various levels.

Attenuation of incident 8600-Mc/sec disturbing wave power source (decibels)	Maximum incident electric field strength of the 8600-Mc/sec wave (volts/cm)	Attenuation of propagating 9400-Mc/sec wanted wave by the plasma (decibels)
0	15.3	2.41
1	13.5	2.245
2	12.15	2.06
3	10.84	1.904
4	9.65	1.767
6	7.77	1.536
8	6.09	1.377
10	4.83	1.256
∞	0	1.045

tween the dotted curve and experiment is appreciable, which does not lend strong support to the hypothesis of strong electron temperature equalization in the plasma.

If the discrepancy between the solid curve and the experiment is interpreted as a variation of the effective electron collision frequency with temperature of the electrons in the manner described earlier, the result obtained is recorded in Fig. 2. The solid curve presumes that ν_m is dependent upon $T_e^{\frac{1}{2}}$, or, in other words, that P_m, is a constant and equal to 24.0 cm^2/cm^3. The experimental points are seen to fall above the solid curve for $T_e > 700$°K, and at the highest temperature there is a tendency to return or cross over the solid curve. This "hump" in ν_m predicts, therefore, a "hump" in the curve of P_m *versus* electron energy. To obtain the course of P_m, proceeding as shown in Eqs. (4) and (5), ν_m was expressed in terms of T_e, and the coefficients a_0 through a_4 solved from five simultaneous linear equations. Finally

$$P_m = 54.8 - 4.975\times10^{-6}v + 2.77\times10^{-13}v^2 - 6.00\times10^{-21}v^3 + 4.53\times10^{-29}v^4 \text{ (cgs units).} \quad (9)$$

In the above, four significant figures were carried only to help remove arithmetic errors. The values of P_m given by Eq. (9) are plotted as the curve labeled (1) of Fig. 3. The transcribed results of Ramsauer and Kollath[5] for the total probability of collision, P_c, obtained from monoenergetic beam type experiments are given as the crosses and circles, (2). It is doubtful that P_c differs by more than a few percent from P_m under the conditions of our experiment, since for lower energies, electrons generally tend toward isotropic scattering in helium.[9] The lower solid curve (3) is that recently found by Gould and Brown.[3]

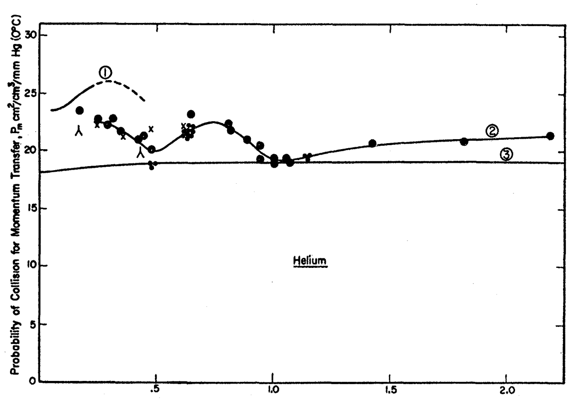

FIG. 3. Comparison of experimentally determined collision probabilities for slow electrons in helium. (1) indicates the result of this work, Eq. (9), (2) is the total probability of collision from the measurements of Ramsauer and Kollath, and (3) is the momentum transfer collision probability recently determined by Gould and Brown. (See text for further discussion.)

[9] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Oxford University Press, New York, 1952), p. 92.

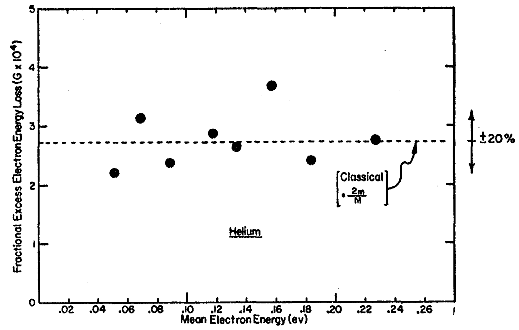

FIG. 4. Experimentally determined fractional excess electron energy loss, G, in a decaying plasma established in helium.

The results of calculation of the excess electron energy loss factor, G, according to Eq. (8), are given on Fig. 4, along with the expected value of $G=2m/M \cong 2.7\times10^{-4}$ for helium (dashed curve). The spread of the data is a reflection of the difficulty of accurate determination of the rate of change in amplitude of the wanted wave at precisely $t=0$. However, agreement within the expected precision with the theoretical value of G is noted.

The experimental results reported here seem to lend support to a *slight* variation of P_m with electron energy in the range of energies investigated, which is in some agreement with the original work of Ramsauer and Kollath. In view of the difficulties involved in these determinations, the above results in the unfortunately narrow energy range should not be considered final.

APPENDIX I. CALCULATION OF TOTAL MICROWAVE POWER LOSS IN THE PLASMA

At any point in the cross section of the wave guide where free electron density exists, the temperature of the electron gas in the previously discussed "quasi" steady state is determined by the prevailing microwave field strength and constants of the experiment according to

$$T_e = T_m + \frac{e^2}{3m\omega^2 Gk}|E'|^2. \quad (10)$$

Assume initially that the probability of collision of electrons with molecules for momentum transfer, P_m, around 300°K, is a constant = 24.0. The effective electron collision frequency is then $\sim T_e^{\frac{1}{2}}$, and under our conditions of experiment, $T_m = 300$°K and pressure $p = 10.15$ mm of Hg, the effective electron collision frequency becomes

$$\nu_m = 1.83\times10^8 T_e^{\frac{1}{2}}. \quad (11)$$

The microwave power dissipated, P_L, in a volume of the discharge tube contained by the two transverse dimensions of the guide ($a = 2.07$ cm) and Δz in the propagating z-axis is

$$P_L=4\int_0^{a'/2}\int_0^{a'/2}\int_0^{\Delta z}\frac{|E'|^2}{2}\cos^2\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi x}{a'}\right)\times\cos\left(\frac{\pi y}{a'}\right)\frac{n_0e^2}{m\omega_D{}^2}\nu_m e^{-2\alpha_D z}dxdydz, \quad (12)$$

where the electron density distribution, determined primarily be ambipolar diffusion, is approximated by

$$n_e=n_0\cos(\pi x/a')\cos(\pi y/a'),$$

where a' is the internal diameter of the discharge tube and x and y are the transverse axes with origin of coordinates at the central axis of the guide. Treating α as a slowly varying function of z and of E', Eq. (12) may further be written

$$P_L=|E'|^2\frac{1}{\alpha_D}(1-e^{-2\alpha_D\Delta z})\int_0^{a'/2}\cos\left(\frac{\pi x}{a'}\right)dy$$

$$\times 3.17\times 10^9\frac{n_0e^2}{m\omega_D{}^2}\int_0^{a'/2}\cos^3\left(\frac{\pi x}{a}\right)\left[1+b^2\cos^2\left(\frac{\pi x}{a}\right)\right]^{\frac{1}{2}}dx,$$

where

$$b^2=\frac{e^2|E'|^2}{3m\omega_D{}^2GkT_m},$$

or, finally, replacing a' by a in the x integration for ease of handling (a small error enters in this procedure),

$$P_L=\frac{|E'|^2}{\alpha_D}(1-e^{-2\alpha_D\Delta z})\left(\frac{a'}{\pi}\right)\left(\frac{a}{\pi}\right)3.17\times 10^9\frac{n_0e^2}{m\omega_D{}^2}\times\left[\frac{3}{8}+\frac{1}{8b^2}+\left(\frac{3b}{8}+\frac{2}{8b}-\frac{1}{8b^3}\right)\text{arc sin}\frac{b}{(1+b^2)^{\frac{1}{2}}}\right]. \quad (13)$$

From wave-guide considerations and the definition of insertion loss, the loss is also

$$P_L=|E'|^2\left(\frac{2\pi}{\lambda_g}\right)\left(\frac{a^2}{4}\right)\frac{1}{\omega_D\mu_0}(1-e^{-2\alpha_D\Delta z}). \quad (14)$$

Equating Eqs. (13) and (14), the attenuation constant, α_D, is found to be

$$\alpha_D=\left(\frac{\lambda_g}{2\pi}\right)\left(\frac{4}{a^2}\right)\omega\mu_0\left(\frac{a'}{\pi}\right)\left(\frac{a}{\pi}\right)\frac{n_0e^2}{m\omega_D{}^2}(3.17\times 10^9)\times\left[\frac{3}{8}+\frac{1}{8b^2}+\left(\frac{3b}{8}+\frac{2}{8b}-\frac{1}{8b^3}\right)\text{arc sin}\frac{b}{(1+b^2)^{\frac{1}{2}}}\right]. \quad (15)$$

The bracketed portion of Eq. (15) is difficult to handle in further intergrations and is, therefore, approximated by the simpler function,

$$\left[5.90b+\frac{2}{3.96b+3}\right].$$

Evaluating α_D for this experiment at $n_0=2.5\times 10^{10}$ elec/cc, one obtains (in mks rational units)

$$\alpha_D=0.1580\times\left[\frac{6.62\times 10^{-6}|E'|^2+2.98\times 10^{-3}|E'|+2}{6.67\times 10^{-3}|E'|+3}\right]. \quad (16)$$

The desired result is, however, the total attenuation in the plasma filled portion of wave guide, which follows immediately from the known magnitude of the incident electric field, $|E_i'|$, and a calculation of the microwave electric field emerging from the plasma, $|E_t|$, after propagation of the wave a distance z, appropriate to the experiment. By definition of the attenuation constant, α_D, we have

$$\frac{d|E'|}{|E'|}=-\alpha_D dz. \quad (17)$$

Integrating Eq. (17) and evaluating the constant of integration at $z=0$ where $|E'|$ becomes the incident field strength, $|E_i'|$, and for $z=1.52$ meters, the length of the discharge tube, one obtains

$$0.4423[\text{arc tan}(1.99\times 10^{-3}|E_t'|+0.4475)-\text{arc tan}(1.99\times 10^{-3}|E_i'|+0.4475)]+0.250\ln\left[\frac{|E_t'|^2}{|E_i'|^2}\left(\frac{6.62\times 10^{-6}|E_i'|^2+2.98\times 10^{-3}|E_i'|+2}{6.62\times 10^{-6}|E_t'|^2+2.98\times 10^{-3}|E_t'|+2}\right)\right]=-0.0800. \quad (18)$$

For known values of $|E_i'|$, the attenuated magnitude of the disturbing wave field strength emerging from the discharge is calculated by graphical methods and finally the desired attenuation in decibels is obtained. In the experiment as performed, the attenuation constant, α_W, of the "wanted" wave is measured, but α_D and α_W are related for these experimental conditions by

$$\alpha_D=1.30\alpha_W. \quad (19)$$