# Radiative $\pi-e$ Decay*

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(Received December 16, 1955)

Under the assumption that the pion decay into an electron and a neutrino is forbidden or partially forbidden, the probability for a pion to decay into an electron, a neutrino, and a photon is considered under general assumptions as to the intermediate state. The spectra of the electron energy in this radiative decay are obtained for the various types of the interaction between the particles in the intermediate state and the leptons in the final state. The total probability of the radiative decay is of the order of $10^{-5}$ per $\pi-\mu$ decay, except in the case of the scalar interaction where both the radiationless and radiative decays are forbidden.

IN their recent experiment Lokanathan and Steinberger[1] have found that the probability for a pion to decay into an electron and a neutrino,

$$\pi^+ \rightarrow e^+ + \nu, \tag{1}$$

is less than $5\times10^{-5}$ that for the ordinary decay into a muon and a neutrino,

$$\pi^+ \rightarrow \mu^+ + \nu. \tag{2}$$

This seems to indicate that (1) is forbidden or at least partially forbidden, whereas (2) is allowed. Theoretically it is known[2] that if we assume that the decays (1) and (2) take place via a virtual nucleon pair, then among five possible types of the interaction between nucleons and leptons, the scalar ($S$), vector ($V$) and tensor ($T$) interactions forbid those transitions, and the axial vector ($A$) interaction makes (1) partially forbidden by reducing the matrix element for (1) by a factor of $m_e/m_\mu$ compared to that of (2), if the same type of the interaction is assumed for those transitions. The latter result has also been obtained by Miyazawa and Oehme[3] as an application of their general theorem, without making any specific assumption as to the intermediate state.

In case (1) is forbidden or partially forbidden, the radiative decay,

$$\pi^+ \rightarrow e^+ + \nu + \gamma, \tag{3}$$

may be allowed and become significant in the electron decay of pions. The present note gives a general consideration of the probability of the radiative decay (3). The selection rules and the forms of the matrix elements for the radiationless and radiative decays (1) and (3) can be obtained from the invariance requirements which the matrix elements must satisfy under the Lorentz and the gauge transformations. The calculation shows that the spectrum of the electron energy in the radiative decay depends strongly upon the type of the interaction we assume, and that the total probability of the decay is of the order of $10^{-5}$ that of the ordinary decay (2). The results are compared with the previous estimates made with regard to this problem.

We assume that the radiationless decays (1) and (2) take place according to the Feynman diagram shown in Fig. 1: a positive pion with 4-momentum $q$ produces a virtual state which is represented by a circle in the diagram. The virtual state, which is not specified in detail, is subsequently replaced by a positron (or a positive muon) and an antineutrino, with 4-momenta $p$ and $l$, respectively, due to the interaction at a point $0$. The invariant matrix element for the transition can be regarded as a scalar product of two tensors, $\mathfrak{M}_1$ and $\mathfrak{M}_2$, where $\mathfrak{M}_1$ involves the spinor amplitudes $\bar{u}$ and $v$ of the positron (or muon) and the antineutrino, with an interaction at the point $0$, and $\mathfrak{M}_2$ consists of the pseudoscalar amplitude $\phi$ of the pion and the contribution from the intermediate state.

Among the five possible forms of $\mathfrak{M}_1$ ($S$, $V$, $T$, $P$, and $A$) only $P$ and $A$ can make the transition possible, because $\mathfrak{M}_1$ with $S$, $V$, or $T$ character has no corresponding $\mathfrak{M}_2$ which we can construct out of the given quantities of the pion and the intermediate state. The matrix element for the decay (1) should therefore have one of the forms[4]:

$S$, $V$, and $T$: forbidden,

$$P:\quad g_1(\bar{u}\gamma_5 v)\phi, \tag{4}$$

$$A:\quad \frac{g_2}{m}(\bar{u}\gamma_5\gamma_\lambda v)q_\lambda\phi = -\frac{m_e}{m}g_2(\bar{u}\gamma_5 v)\phi, \tag{5}$$

where $m$ and $m_e$ are the masses of the pion and electron. (The mass of the neutrino is neglected.) $g_1$ and $g_2$ are certain dimensionless constants. The right-hand side of the expression (5) follows from the conservation law,

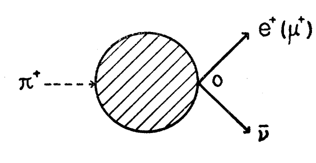


FIG. 1. Feynman diagram for the pion decays (1) and (2), which are possible only in the cases of the $P$ and $A$ interactions.

* Contribution No. 415.
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[1] S. Lokanathan and J. Steinberger, Phys. Rev. 98, 240(A) (1955).
[2] M. Ruderman and R. Finkelstein, Phys. Rev. 76, 1458 (1949).
[3] H. Miyazawa and R. Oehme, Phys. Rev. 99, 315 (1955).
[4] We use the units $c=\hbar=1$. $\gamma_\lambda$ ($\lambda=0, 1, 2, 3$) and $\gamma_5$ are ordinary Dirac matrices. $A\cdot B=A_\lambda B_\lambda=A_0B_0-(A_1B_1+A_2B_2+A_3B_3)$.

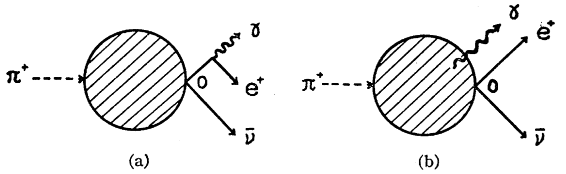


FIG. 2. Diagrams for the radiative decay (3). (a) is allowed in the cases of the $P$ and $A$ interactions, and (b) in the cases of the $V$, $T$, and $A$ interactions.

$q=p+l$, and is an example of the equivalence theorem[5] and the theorem of Miyazawa and Oehme.[3]

For the radiative decay we have the two diagrams shown in Fig. 2. Figure 2(a) represents the inner bremsstrahlung by the positron, the matrix element of which is obtained by the perturbation method, and Fig. 2(b) is the radiation by one of the particles in the virtual state, the corresponding matrix element being obtained by considerations similar to those of the radiationless decay, except for the requirement of the gauge invariance. After the gauge of the electromagnetic field is chosen such that $A\cdot q=0$, where $A$ is the amplitude of the photon emitted, the matrix element should have one of the forms:

$S$: forbidden,

$$V:\quad \frac{f_1}{m^3}(\bar{u}\gamma_5\gamma_\lambda\gamma_\mu\gamma_\nu v)A_\lambda k_\mu q_\nu \phi F_1(q\cdot k) \tag{6}$$

$$T:\quad \frac{f_2}{m^2}(\bar{u}\gamma_5\gamma_\lambda\gamma_\mu v)A_\lambda k_\mu \phi F_2(q\cdot k) \tag{7}$$

$$P:\quad eg_1\left(\bar{u}\gamma_\lambda\frac{1}{(p_\mu+k_\mu)\gamma_\mu-m_e}\gamma_5 v\right)A_\lambda\phi, \tag{8}$$

$$A:\quad \frac{eg_2}{m}\left(\bar{u}\gamma_\lambda\frac{1}{(p_\mu+k_\mu)\gamma_\mu-m_e}\gamma_5\gamma_\nu v\right)A_\lambda q_\nu\phi + \frac{f_3}{m}(\bar{u}\gamma_5\gamma_\lambda v)A_\lambda\phi F_3(q\cdot k), \tag{9}$$

where $k$ is the 4-momentum of the photon, and $f_1$, $f_2$ and $f_3$ are certain constants. $F_1(q\cdot k)$, $F_2(q\cdot k)$ and $F_3(q\cdot k)$ are certain scalar functions of $q\cdot k$, which can be expressed by expansions of the form

$$F_i(q\cdot k)=1+\alpha_{i1}\frac{(q\cdot k)}{m^2}+\alpha_{i2}\frac{(q\cdot k)^2}{m^4}+\cdots.\quad (i=1,2,3).$$

If we regard the above expressions as the result of the expansion of the electromagnetic wave into the power series of the ratio between the pion radius, which is assumed to be $1/m$, and the wavelength of the radiation, we can reasonably assume that the numerical coefficients $\alpha$ are of the order of unity. Thus the functions $F_i(q\cdot k)$ are of the order of unity and henceforth we shall replace them by one, except in the case of the $A$ interaction where we take up to the second term of the expansion for the following reason.

In the limit as $k\rightarrow 0$, the second term of the expression (9) should be identical with the correction term of (5) which we obtain by replacing $q$ in the left-hand side of (5) by $q-eA$. We therefore get $f_3=-eg_2/m$. The expression (9) now reduces to

$$-\frac{m_e}{m}eg_2\left(\bar{u}\gamma_\lambda\frac{1}{(p_\mu+k_\mu)\gamma_\mu-m_e}\gamma_5 v\right)A_\lambda\phi - \frac{\alpha_1 eg_2}{m^3}(\bar{u}\gamma_5\gamma_\lambda v)A_\lambda(q\cdot k)\phi+\cdots, \tag{10}$$

by the use of the conservation law $q=p+l+k$. We note that the radiative decay (3) with the $A$ interaction is partially forbidden to the extent that the first term of (10) applies, as in the case of the radiationless decay with the same interaction.

In view of the fact that the interaction constant between nucleons and electron and neutrino is of the same order as that between nucleons and muon and neutrino,[6] it seems natural to assume that

$$g_1,g_2\sim G \quad\text{and}\quad f_1,f_2\sim eG,$$

where $G$ is the effective coupling constant of the $\pi-\mu$ decay (2) which is assumed to have the matrix element of the type (4).

Under these assumptions we can calculate the probability of the decay (3) by the usual perturbation method. The results for the probability of this decay with the energy of the positron between $E$ and $E+dE$ are

$$V:\quad W_V(E)dE=\frac{G^2e^2}{192\pi^3m^4}(3m^2-12mE+14E^2)E^2dE,$$

$$T:\quad W_T(E)dE=\frac{G^2e^2}{16\pi^3m^3}(m-2E)E^2dE,$$

$$P:\quad W_P(E)dE=\frac{G^2e^2}{16\pi^3m}\frac{1}{m-2E}\left\{(m^2+4E^2)\log\frac{2E}{m_e}-2mE\log\frac{m}{m-2E}-2mE\right\}dE, \tag{11}$$

$$A:\quad W_A(E)dE=\left(\frac{m_e}{m}\right)^2W_P(E)dE+\alpha_1^2W_V(E)dE+\frac{\alpha_1G^2e^2}{64\pi^3m^2}\left(\frac{m_e}{m}\right)^2(m-2E)\times\left\{(m-2E)\log\frac{2E}{m_e}+m\log\frac{m}{m-2E}-E\right\}dE.$$

[5] K. M. Case, Phys. Rev. 76, 14 (1949).

[6] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. 21, 153 (1949).

The total probabilities given by the integral of the above expressions, relative to that of the $\pi-\mu$ decay (2), which is $G^2m/8\pi$, are $\sim 10^{-5}$ for the $V$, $T$ and $A$ assumptions, and $\sim 10^{-2}$ for the $P$ assumption, where for the latter two cases ($A$ and $P$) the positron energy up to 65 Mev has been taken because of the infrared divergence at $E=m/2\sim 70$ Mev. It is well known that the $P$ interaction must be excluded because it gives a higher probability for (1) than for (2). The distributions of the positron energy given by $W_V(E)$, $W_T(E)$ and $W_A(E)$ are shown in Fig. 3. The $V$ interaction gives a monotonic increase of the probability up to the maximum energy. For the $T$ interaction the probability is the maximum at $E\sim 50$ Mev and zero at 70 Mev, whereas the $A$ interaction predicts that the radiationless decay (1), which has the probability $10^{-4}$ relative to the $\pi-\mu$ decay, is more likely than the radiative decay. It is hoped that future experiments will reveal which interaction is actually effective.

Ruderman[7] calculated the probability of (3) to the lowest order of the perturbation, by assuming that the process takes place by the virtual creation and annihilation of a nucleon pair, assuming a pseudoscalar interaction between a pion and a nucleon and various beta-interactions between nucleons and leptons. The integral over the momentum of the virtual nucleon is divergent in all cases except the $V$ interaction, for which he obtained $\sim 10^{-4}$ sec$^{-1}$ as the total probability for the radiative decay, a slightly higher value than our present result. For the $A$ interaction, he considered only the diagram of the type of Fig. 2(a) and obtained the ratio of the probabilities of the radiative and the radiationless decays (3) and (1) as $\sim 10$, which is probably incorrect because the contribution of the diagram Fig. 2(b) gives a matrix element which is of the same order, but of the opposite sign, as that of Fig. (2a). Miyazawa and Oehme[3] expected that the partial forbiddenness of the process (1) with the $A$ interaction would in some cases be removed in the process (3), but our general consideration shows that this still exists in the radiative process to the first approximation, and that the radiative process can never exceed the radiationless decay. Our result for the total probability is in essential agreement with that obtained by Iwata *et al.*[8] who have calculated the matrix element to the first order of the pion-nucleon interaction constant. Finally we note that the observation of the distribution of the positron energy in the radiative decay (3), especially in the region where $E$ is close to 70 Mev, would give us a check on the choice of the beta interaction.

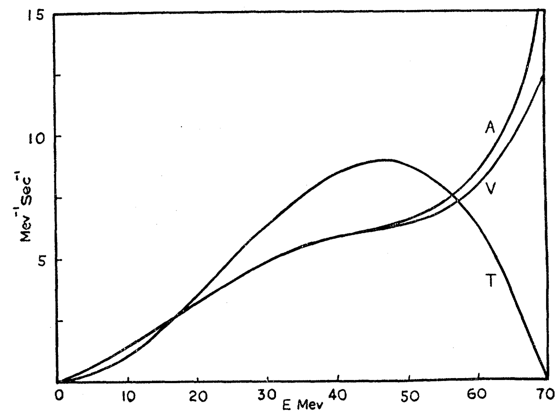


FIG. 3. Distributions of the energy of the positron in the radiative decay (3). $V$, $T$, and $A$ in the above graph represent $W_V(E)$, $W_T(E)/5$ and $W_A(E)$ of (11), respectively, with $\alpha_1=1$.

The author wishes to express his thanks to the Institute for Atomic Research, Iowa State College, for the privilege of studying there.

[7] M. Ruderman, Phys. Rev. **85**, 157 (1952).

[8] Iwata, Ogawa, Okonogi, Sakita, and Oneda, Progr. Theoret. Phys. (Japan) **13**, 330 (1955).