are predominantly E2 and that the lower energy transition is more intense than the high, are both predicted by this interpretation.

Level V could be a single particle excitation of the ground state configuration.

As the ground-state configuration of Kr<sup>82</sup> is subject to so many possible couplings to form excited states, many shell-model interpretations may be given.

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# Neutron Cross Section of Xenon-135 as a Function of Energy

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The neutron cross section of Xe<sup>135</sup> as a function of energy was measured, using as velocity selector a focusing-type singlecrystal spectrometer designed for transmission measurements of very small samples. The total cross section in the energy interval from 0.015 ev to 0.20 ev was measured. The samples, produced from neutron irradiated uranium metal, were in the form of PdI<sub>2</sub> and were contained in sealed Pyrex capillary tubing. The largest initial strength of the samples was 10 curies of I135 activity, corresponding to  $12 \times 10^{15}$  atoms of I<sup>35</sup>. The daughter Xe<sup>135</sup> grew from the I<sup>135</sup> as a known function of time, reaching a maximum value of about  $5 \times 10^{15}$  atoms of Xe<sup>135</sup> 11.3 hours after the  $I^{135}$  begins to decay. In the absolute assay of sample strengths, absolute  $\beta$  counting of pure I<sup>135</sup> samples, and  $\beta$ - $\gamma$  coincidence counting of pure Xe<sup>135</sup> samples served as primary standards. Hard gamma rays from I<sup>135</sup> served as a secondary standard. The total cross section of one entire sample of Xe was of the order of 1.5 square millimeters. The transmission of the sample was measured during the period of growth and decay of the Xe. The radioactive

### INTRODUCTION

THE cross section of Xe<sup>135</sup> is of interest for both fundamental and applied reasons. (1) It is of interest from the point of view of nuclear structure. The cross section of Xe<sup>135</sup> is the largest known neutron cross section. The value of  $\pi\lambda^2$  at the peak of the resonance, which was first characterized by this work, is approximately  $7.5 \times 10^{-18}$  cm<sup>2</sup>. The total cross section at resonance is about 0.1 of the maximum possible theoretical limit,  $4\pi\lambda^2$ . The capture cross section at resonance is about 0.4 of its maximum possible theoretical limit,  $\pi\lambda^2$ . If the spin of the target nucleus is taken into consideration, then the capture cross

sample was placed inside the shield of the ORNL graphite reactor. A thermal beam of neutrons from the reactor was allowed to pass longitudinally through the sample along the axis of the capillary tube onto the quartz crystal spectrometer. The desired energies were selected by use of the Bragg reflection law,  $\lambda = 2d \sin\theta$ . A resonance in the cross section of Xe<sup>135</sup> was discovered at 0.085 ev. The total cross section measurements were fitted to the singlelevel Breit-Wigner formula equally well with the following two sets of parameters:  $g = \frac{3}{8}$ ,  $E_0 = 0.0851 \pm 0.0011$  ev,  $\Gamma_n^0 = 0.0305$  $\pm 0.0008 \text{ ev}, \Gamma_{\gamma} = 0.0828 \pm 0.0031 \text{ ev}; g = \frac{5}{8}, E_0 = 0.0849 \pm 0.0010 \text{ ev},$  $\Gamma_n^0 = 0.0182 \pm 0.0005$  ev,  $\Gamma_{\gamma} = 0.0942 \pm 0.0032$  ev.  $E_0$  is the resonance energy,  $\Gamma_n^0$  is the neutron width at resonance,  $\Gamma_{\gamma}$  is the gamma ray width of the level, and g is the statistical weight factor. The factor g has two possible values because the spin of the compound state is not known. The capture cross section at resonance for state with  $g = \frac{5}{8}$  is 55% of the theoretical maximum possible, and the corresponding capture cross section for the state with  $g = \frac{3}{8}$  is 80% of the theoretical maximum value.

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section at resonance is very close, indeed, to its maximum possible theoretical limit  $g\pi\lambda^2$ , in which g is the statistical weight factor, since g for Xe<sup>135</sup> probably has the value either  $\frac{3}{8}$  or  $\frac{5}{8}$ . The cross section of Xe<sup>135</sup> is of interest, also, because the Xe<sup>135</sup> nucleus consists of 54 protons and 81 neutrons, needing one neutron to become a magic number nucleus. (2) It is a fission product poison of such large cross section that it affects markedly the operating characteristics of chain reactors. Knowledge of the cross section of Xe<sup>135</sup> as a function of energy is useful in the design of nuclear chain reactors to be operated at various temperatures.

The existence of a fission product with a very large cross section was discovered by Fermi and Wheeler in the startup of the first Hanford reactor about the end of 1944. From the dynamic behavior of the pile, they deduced that the cross section for pile neutrons was roughly  $4 \times 10^{-18}$  cm<sup>2</sup>, and was associated with a half-life of about 9 hours. This value of the half-life suggested

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Xe<sup>135</sup> as the strong absorber. Following these observations an intensive effort was made by several groups of workers at different reactors to repeat the measurement of the average cross section from the dynamic pile behavior for the neutron energy spectrum of each of the respective reactors,1 while several other groups measured the average capture cross section for isolated samples of Xe<sup>135</sup> by the method of negative activation.<sup>1</sup> These efforts were followed by the measurements of the total cross section of isolated samples of  $Xe^{135}$  as a function of energy which are described in this report. This work was completed in 1948.<sup>2</sup>

In the work described in this report, a focusing-type single-crystal spectrometer designed for transmission measurements of very small samples was used. Each sample was produced from 7 kilograms of uranium metal which had been exposed to neutrons in the ORNL graphite reactor for 20 hours. The samples were in the form of  $PdI_2$ , and were contained in sealed Pyrex capillary tubing. The largest initial strength of the samples was 10 curies of I<sup>135</sup> activity, corresponding to  $12 \times 10^{15}$  atoms of I<sup>135</sup>. The daughter Xe<sup>135</sup> grew from the I<sup>135</sup> as a known function of the time, reaching a maximum value of about  $5 \times 10^{15}$  atoms of Xe<sup>135</sup> 11.3 hours after the I135 begins to decay. In the absolute calibration of the cross-section measurements, absolute  $\beta$  counting of pure I<sup>135</sup> samples, and  $\beta$ - $\gamma$  coincidence counting of pure Xe<sup>135</sup> samples served as primary standards. The transmission of the sample was measured during the period of growth and decay of the Xe. The radioactive sample was placed inside the shield of the ORNL graphite reactor. A thermal beam of neutrons from the reactor was allowed to pass longitudinally through the sample along the axis of the capillary tube onto the quartz crystal. The desired energies were selected by use of the Bragg reflection law,  $\lambda = 2d \sin\theta$ . The total cross section in the energy interval from 0.015 ev to 0.20 ev was measured. A resonance in the cross section of Xe<sup>135</sup> was discovered in the thermal energy region. This resonance was described by the Breit-Wigner constants given below.

Recently (1954),<sup>3</sup> the dependence of the cross section Xe<sup>135</sup> on energy was measured by another group of workers at the Oak Ridge National Laboratory using completely different chemical and physical techniques. The sample consisted of 500 curies of Xe<sup>135</sup> gas procured from a recently constructed homogeneous reactor. (These gaseous samples contained fully 50 times as many Xe135 atoms as the PdI2 samples in our experiment of 1948.) The transmission

measurements were taken using a newly constructed mechanical neutron chopper which permitted a quicker accumulation of data over a greater range of energies than did the crystal spectrometer. The absolute calibration of the cross section was based upon the number of long-lived Cs<sup>135</sup> atoms left in the sample after all the Xe<sup>135</sup> atoms had decayed. The results of this effort, described in a separate paper,<sup>3</sup> agree closely with the work described here.

### **PREPARATION OF XENON-135**

The iodide of palladium was chosen as the substance to be placed in the neutron beam as the source for xenon. Its advantage over other insoluble iodides, as was recognized by Borst<sup>4</sup> and others, consisted in that palladium has a lower neutron cross section than have other metallic ions. It was soon found that palladium iodide varied in its capacity to retain the xenon depending on its mode of formation. By studying the dependence of retentivity on various factors such as particle size, crystallite size, surface area, it was found possible so to adjust the concentrations, temperature, and rate of cooling that adequate retentivity could be achieved. Under the conditions finally adopted. the palladium iodide was anhydrous, free of hydrogenous material, in crystallites about 1000 A diameter, gathered into particles about  $1\mu$  in diameter. Curiously, the particle size could be made to vary over a considerable range without appreciable change in size of the crystallites. However, the retentivity improved markedly with increased particle size.

Again following Borst, but with considerably greater degree of control, it was decided to separate fission iodine from the metallic uranium slugs as gaseous elementary iodine. The uranium irradiated in the pile twenty hours was quickly decoated and dissolved in hydrochloric acid. At this stage, the iodine was in the form of iodide and the uranium in trivalent and quadrivalent forms. By adding oxidizing agent, a solution of lithium dichromate, in potentiometrically controlled quantities in the presence of ferrous and ferric sulfate, a gradual and almost complete transformation of iodide into elementary iodine was effected. The iron sulfates acted as catalysts for the oxidation of the uranium ions and also as an oxidation buffer which guaranteed under easy potentiometric control that the iodine itself would not be oxidized further and would thus appear in the gaseous state. The iodine was then distilled over into a solution of palladium chloride containing acid and sulfur dioxide. The latter reduced the iodine to iodide and by forming a complex with the palladium iodide kept it in solution. Upon boiling off the sulfur dioxide and cooling slowly, the palladium iodide precipitated and acquired the property of

<sup>&</sup>lt;sup>1</sup> Borst, Jones, Nordheim, Slotin, and Soodak (unpublished); Elliot, Knight, Novey, and Shapiro (unpublished); Freedman, Turkevitch, Adams, Sugarman, Raynor, and Stang, J. Inorg. Nuclear Chem. (to be published); Pardue, Moak, Levy, Wollan, and Meiners (unpublished). <sup>2</sup> A detailed report of this work is given in the Physics Division Quarterly Report of December, 1948, Oak Ridge National Labora-tory report ORNL-325 (unpublished), pp. 6–71. <sup>3</sup> Smith, Pawlicki, and Thurlow (to be published).

<sup>&</sup>lt;sup>4</sup> The first preliminary work to prepare a sample of Xe<sup>135</sup> for spectrometer transmission studies was done by L. B. Borst at the Oak Ridge National Laboratory in 1947 (private communication).





retaining xenon. The precipitate was centrifuged, filtered into the capillary tube, washed free of water with alcohol and ether, and finally washed free of these hydrogenous substances with perfluorotriethylamine.

All these operations were carried out in a concrete cell under remote control to eliminate health hazard. During the dissolution of the uranium slugs, the radioactivity of all the fission products amounted to several thousand curies. A detailed description of the technology may be published elsewhere.

### EXPERIMENTAL ARRANGEMENT

The entire experiment was designed on the basis of a sample only one square millimeter in cross-sectional area. Such a sample was very small, indeed, even for the "intense" neutron beams available from the ORNL graphite reactor, the first chain reactor operating at appreciable power. The crystal spectrometer used for the work was an old x-ray spectrometer which had been adapted for use with neutrons. It was of the (approximately) focusing type. Focusing action at a given energy was obtained by varying the curvature of the crystal by exerting a torque upon it by means of adjusting screws. A focusing-type spectrometer is well suited to the use of very small samples, since the sample may be placed at the position of the "point" source of neutrons, and the detector at the "point" image. The design and operating characteristics of the spectrometer have been described previously.5

A schematic diagram of the arrangement of neutronbeam collimator, sample, and detector is shown in Fig. 1. The point source was procured by means of the long hour-glass-like graphite collimator shown. The beam originates deep inside the pile where the neutron flux was about  $10^{12}$  neutrons per cm<sup>2</sup> per second. The collimator converges uniformly from an initial cross section of 4 inches×4 inches to a diameter of  $\frac{3}{8}$  inch at the point *S*, four feet inside the outermost face of the pile shield. A 1.52-mm bore sealed Pyrex capillary tube containing the sample is placed at S so that its axis coincides with that of the collimator. The minimum cross-sectional area of the neutron beam is defined at the sample by the three-inch length of the Pyrex capillary wall. After passing through the sample, the beam diverges again until it strikes the crystal, which focuses it upon the detector slit. The crystal is a 5 inch×1 inch×1 mm quartz crystal whose 5 inch×1 inch surface was ground parallel to the 100 planes.

Actually two identical capillaries were used. A very accurately constructed sample-shifting device, operated at the pile face, was capable of alternately placing first the sample containing tube, then an equivalent empty tube, in identically the same position in the neutron beam. The transmission of the sample is given by the ratio of the counting rate through the sample-containing tube to the counting rate through the empty tube.

The energy spectrum of the neutrons being emitted from the pile is roughly that of a Maxwell distribution corresponding to a temperature of about 520°K. A beam of a given energy was selected by use of Bragg's law,  $\lambda = 2d \sin\theta$ , which, for reflections from the 100 planes of quartz becomes:

$$E = (1.136 \times 10^{-3}) / \sin^2\theta, \tag{1}$$

in which E is the neutron energy in ev, and  $\theta$  is the glancing angle in degrees. The detector used was a BF<sub>3</sub> proportional counter enriched in the isotope B<sup>10</sup>. The maximum counting rate was about 6000 counts per minute at about 0.06 ev. The most favorable count to background ratio of 180 was achieved at 0.05 ev. The lower energy limit of usefulness of the instrument of about 0.02 ev was determined by the increasing amount of higher order reflection as the energy decreases. The upper energy limit of usefulness is determined by the rapid decrease of the number of neutrons in the Maxwell distribution at energies greater than that of the most probable energy, and by the increase of the background at smaller glancing angles, or greater energy.

Transmission measurements were made as a function

<sup>&</sup>lt;sup>5</sup> Bernstein, Borst, Standford, Stephenson, and Dial, Phys. Rev. 87, 487 (1952).

of energy from 0.015 ev to 0.20 ev. In addition, the average total cross section for the neutrons of the continuous energy distribution having energies below the energy of the Cd cutoff was measured with the fission chamber in the direct beam (shown in Fig. 1) by taking fission counts with and without cadmium surrounding the fission chamber.

The energy scale of the spectrometer was calibrated in terms of its angle readings by the following procedure. The transmission of a B<sup>10</sup>F<sub>3</sub> absorber was measured as a function of energy on both sides of the direct beam. The reading of the spectrometer angle scale corresponding to zero scattering angle is then given by the average of two scale readings corresponding to equal values of the transmission measured on opposite sides of the direct beam. The energy scale calibration using the boron absorber was done for each individual xenon sample independently. The "zero" determination was checked also by repeating the procedure using the peak of the 0.176 ev resonance in Cd on both sides of the beam, and also scale readings corresponding to equal values on both sides of the direct beam of the transmission of Cd on the rapidly descending portion of the Cd cross section vs energy curve at energies greater than the resonance energy.

Previous experience with the spectrometer<sup>5</sup> had demonstrated that the resolving power of the instrument was very adequate for the determination of line shapes in the thermal energy region.

# ANALYSIS OF THE DATA

The samples were in the form of anhydrous palladium iodide, with  $Pd(I^{135})_2$  contained in  $PdI_2$  carrier material. The Xe<sup>135</sup> grows from the I<sup>135</sup> according to the relation

 $N_2 = N_1^0 f(t),$ 

where

$$f(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$
(3)

(2)

Here  $N_2$  is the number of Xe<sup>135</sup> atoms present in the sample at time t,  $N_1^0$  is the number of I<sup>135</sup> atoms present



FIG. 2. Calculated growth and decay of Xe<sup>135</sup> from I<sup>135</sup>.

in the sample at time zero. (There are no Xe atoms present at time zero.)  $\lambda_1, \lambda_2$  are the transformation constants of I<sup>135</sup> and Xe<sup>135</sup>, respectively. The relation (2) describes the growth of the daughter substance from the parent by simple decay. In this case, about 25 percent of the I<sup>135</sup> atoms decay to the 9.2-hour Xe<sup>135</sup> level via an isomeric state of about 15 minutes half-life. Since 15 minutes  $\ll 9.2$  hours, the fraction of Xe<sup>135</sup> atoms existing in the isomeric state at any given time is negligibly small. A plot of  $(N_2/N_1^0)$  vs t, calculated from (2), is given in Fig. 2, taking the half-lives of I<sup>135</sup> and Xe<sup>135</sup> to be 6.7 hours and 9.2 hours, respectively.

The transmission, T, of the sample is given by

$$T = (T_1)[T_2(t)], (4)$$

in which  $T_1$  is the transmission of the carrier material and is constant in time, and  $T_2(t)$  is the transmission of the Xe<sup>135</sup> which is given by

$$T_2 = \exp\left[-N_1^0 f(t)\sigma_{\mathbf{X}\mathbf{e}}\right],\tag{5}$$

in which  $\sigma_{Xe}$  is the cross section per atom of Xe<sup>135</sup>. In (5),  $N_1^0$  should, of course, enter as the number of atoms per cm<sup>2</sup> of sample.

If, then, the natural logarithm of the transmission is plotted as a function of f(t), a straight line should be obtained whose slope is proportional to the product of the strength of the sample and the cross section of Xe<sup>135</sup>. For a given sample the ratio of the slopes of the lines for the various energies gives the relative values of the cross sections at these energies. For the same energy value but for different samples, the ratio of the slopes is proportional to the relative strengths of the samples. The intercept of the straight line plot  $(\ln T)$  vs f(t) on the vertical axis is proportional to the total cross section of carrier material. The values of f(t) were calculated on the assumption that the half-lives of I<sup>135</sup> and Xe<sup>135</sup> were 6.7 hours and 9.2 hours, respectively. and that the "zero" time of Xe growth, or the time of "entrapment" of the Xe gas in the sample was well defined and known with sufficient accuracy. The halflives of I135 and Xe135 were measured during the course of the work, and found to have the values given above to an accuracy of 1%. Auxiliary experiments showed, also, that an "effective" zero time could be defined in the program of chemical operations which served very well in describing the growth and decay of the Xe<sup>135</sup> in accordance with (2).

### SAMPLE ASSAY

Seven samples of Xe<sup>135</sup> were used for transmission measurements. Measurements of the absolute number of Xe<sup>135</sup> atoms were made on two of the samples. The relative strengths of the remaining five samples were determined in terms of the two from the neutron measurements, in the manner described in the preceding section, and from least squares fitting of the data to the Breit-Wigner formula, as discussed below. The number



FIG. 3. The transmission of  $Xe^{135}$  for thermal neutrons as a function of time, show-to decay of the  $Xe^{135}$  from its parent I<sup>135</sup> (Sample III).

of Xe<sup>135</sup> atoms in each of two samples was determined from an ionization chamber measurement of the gamma radiation emitted by the source which penetrated a 4-inch lead absorber. These measurements were started approximately 40 hours after precipitation of PdI<sub>2</sub> after all neutron transmission measurements had been made, and, hence, essentially all radioactive species except Xe<sup>135</sup>, Xe<sup>133</sup>, Xe<sup>133m</sup>, Xe<sup>131m</sup>, I<sup>135</sup>, I<sup>133</sup>, and I<sup>131</sup> had decayed. Of these isotopes, only I<sup>135</sup> and I<sup>133</sup> decay with the emission of appreciable amounts of energetic gamma radiation and, hence, the decay curves had only two components corresponding to the half-lives of I<sup>135</sup> and I<sup>133</sup>.

The relation between ionization current which decayed with the 6.7-hour half-life of  $I^{135}$  and the number of  $I^{135}$  atoms was determined by two independent methods. In one method, a high purity  $I^{135}$  sample was measured in the standard ion chamber position and an aliquot was beta counted in a counter with calibrated geometry. In the second method, after a definite growth time all of the xenon daughter was collected from a pure  $I^{135}$  source which had been measured in the standard ion chamber position. The absolute disintegration rate of the Xe<sup>135</sup> sample was determined by beta-gamma coincidence counting.

The two calibration procedures agreed within counting statistics to give a standard deviation of 1%in the disintegration rate-ionization current ratio. Combination of this calibration error with other errors resulting from uncertainties in the I<sup>135</sup> and Xe<sup>135</sup> decay constants, in the PdI<sub>2</sub> precipitation time and in the correction for nonlinearity of the ionization chamber response gives a standard deviation of about 2.0% in the number of Xe<sup>135</sup> atoms at the time of the neutron absorption measurements.

The results of these measurements were that  $N_1^0$  for

Sample I was  $(5.85\pm0.12)\times10^{15}$ , and  $N_1^0$  for Sample IV was  $(12.6\pm0.2)\times10^{15}$ .

# EXPERIMENTAL RESULTS

The transmission of the sample, of course, decreases as the amount of xenon increases. The growth and decay of the Xe is clearly shown by the data represented in Fig. 3, the transmission of Sample III for Cd absorbable neutrons as a function of time. Similar plots of transmission vs time for several discrete energies are given in Fig. 4 for Sample III. The value of the asymptote to each of the curves in Fig. 4 is the transmission of the carrier. The total transmission of carrier plus xenon at any time, the transmission of the xenon only, and the carrier transmission are related by expression (4), from which the transmission of the xenon alone can be procured. From Fig. 4, it can be seen immediately that the cross section of Xe<sup>135</sup> at 0.03 ev, 0.08 ev, and



FIG. 4. The transmission of xenon Sample III vs time for several values of the neutron energy.



FIG. 5. Natural logarithm of the transmission of xenon Sample VI for thermal neutrons vs relative number of Xe<sup>135</sup> atoms.

0.12 ev are very roughly equal, and that the cross section at 0.20 ev is considerably smaller. The minima in Fig. 3 and Fig. 4 come at values of the time close to the time at which the number of Xe atoms is a minimum.

A plot of the natural logarithm of the transmission vs the calculated relative number of xenon atoms is shown in Fig. 5, for Sample VI, using the Cd absorbable thermal neutron beam detected by the fission chamber. The solid straight line is a least-squares fit to the data using expressions (4) and (5). The good fit of the calculated line to the data suggests that the zero time used for the Xe growth and the half-lives given above are correct, and that the sample did not leak appreciably.

Plots of  $\ln T$ , vs relative number of Xe<sup>135</sup> atoms, f(t), are shown in Fig. 6 for Sample III at 0.08, 0.03, 0.12, and 0.20 ev. The straight lines drawn through the data are again the least-square fits to the data using (4)



FIG. 6. Logarithm of the transmission vs relative number of Xe<sup>135</sup> atoms for several values of energy. The relative cross sections per atoms of Xe<sup>135</sup> are given by the relative slopes of the lines. (Sample III.)

and (5). It can be seen from qualitative inspection of the slopes of the lines of Fig. 6 that the cross section per atom of  $Xe^{135}$  decreases in the order of energies 0.08 ev, 0.03 ev, 0.12 ev, and 0.20 ev.

All told, transmission measurements were made at 4 or 5 values of the energy on seven samples of Xe. All of the data for each of the runs were plotted as shown in Fig. 5 and Fig. 6, and fitted by an established least squares routine to obtain the most probable values of the slopes of the straight lines, their intercepts with the y-axis, and the standard deviations of each of the slopes and intercepts.

## FITTING TO BREIT-WIGNER FORMULA

The data were fitted by the theory of least squares to the Breit-Wigner single level resonance formula for the total cross section for slow neutrons:

$$\sigma_t = g 4\pi \lambda^2 \frac{\Gamma_n \Gamma}{\Gamma^2 + 4(E - E_0)^2},\tag{6}$$

in which  $\Gamma_n$  is the neutron width of the level,  $\Gamma$  the total width,  $E_0$  the resonance energy, g the statistical weight factor,  $\lambda$  the neutron wavelength divided by  $2\pi$ , E the neutron energy. We have

$$\Gamma_n = (E/E_0)^{\frac{1}{2}} \Gamma_n^{\ 0}, \tag{7}$$

$$\Gamma = \Gamma_n + \Gamma_\gamma, \tag{8}$$

$$g = (2J+1)/[2(2I+1)],$$
 (9)

where  $\Gamma_n^0$  is the value of  $\Gamma_n$  at  $E=E_0$ , and  $\Gamma_{\gamma}$  is the gamma-ray width of the level. I is the total angular momentum quantum number of the target nucleus. J is the total angular momentum quantum number of the compound nucleus, and has the two possible values  $J=I+\frac{1}{2}, J=I-\frac{1}{2}$ . Available information<sup>6</sup> suggests that

<sup>&</sup>lt;sup>6</sup> S. Thulin, Phys. Rev. 94, 734 (1954).

Sample	E, ev	a	$\sigma \times 10^{18}, \text{ cm}^2$ $(g = \frac{3}{8})$	$\sigma \times 10^{18}, \text{ cm}^2$ $(g = \frac{5}{8})$
I	0.0293	$0.877 \pm 0.026$	$2.72 \pm 0.10$	$2.72 \pm 0.10$
	0.0578	$1.025 \pm 0.014$	$3.18 \pm 0.08$	$3.18 \pm 0.08$
	0.141	$0.378 \pm 0.017$	$1.17 \pm 0.06$	$1.17 \pm 0.06$
	0.188	$0.104 \pm 0.023$	$0.324 \pm 0.073$	$0.324 \pm 0.073$
II	0.0292	$0.665 \pm 0.017$	$2.61 \pm 0.17$	$2.61 \pm 0.17$
	0.0576	$0.774 \pm 0.015$	$3.03 \pm 0.20$	$3.04 \pm 0.19$
	0.0951	$0.750 \pm 0.014$	$2.94 \pm 0.19$	$2.95 \pm 0.18$
III	0.0294	$1.80 \pm 0.02$	$2.66 \pm 0.12$	$2.67 \pm 0.11$
	0.0770	$1.97 \pm 0.02$	$2.92 \pm 0.13$	$2.93 \pm 0.12$
	0.115	$1.45 \pm 0.03$	$2.14 \pm 0.10$	$2.15 \pm 0.10$
	0.189	$0.398 \pm 0.023$	$0.588 \pm 0.043$	$0.590 \pm 0.041$
IV	0.0304	$1.87 \pm 0.03$	$2.69 \pm 0.07$	$2.69 \pm 0.07$
	0.0609	$2.20 \pm 0.03$	$3.17 \pm 0.07$	$3.17 \pm 0.07$
	0.0814	$2.21 \pm 0.04$	$3.18 \pm 0.09$	$3.18 \pm 0.09$
	0.102	$1.75 \pm 0.05$	$2.52 \pm 0.09$	$2.52 \pm 0.09$
	0.123	$1.23 \pm 0.05$	$1.77 \pm 0.08$	$1.77 \pm 0.08$
v	0.0301	$1.04 \pm 0.02$	$2.76 \pm 0.11$	$2.78 \pm 0.11$
	0.0802	$1.17 \pm 0.02$	$3.09 \pm 0.12$	$3.11 \pm 0.12$
	0.101	$1.000 \pm 0.02$	$2.65 \pm 0.11$	$2.66 \pm 0.10$
	0.121	$0.693 \pm 0.021$	$1.83 \pm 0.09$	$1.84 \pm 0.09$
	0.150	$0.344 \pm 0.019$	$0.911 \pm 0.061$	$0.915 \pm 0.061$
VI	0.0303	$1.40 \pm 0.03$	$2.56 \pm 0.19$	$2.56 \pm 0.18$
	0.0606	$1.72 \pm 0.02$	$3.15 \pm 0.23$	$3.14 \pm 0.22$
	0.153	$0.541 \pm 0.033$	$0.989 \pm 0.093$	$0.986 \pm 0.090$
	0.205	$0.253 \pm 0.040$	$0.462 \pm 0.081$	$0.461 \pm 0.080$
VII	0.0153	$0.765 {\pm} 0.015$	$2.75 \pm 0.13$	$2.78 \pm 0.12$
	0.0308	$0.719 \pm 0.017$	$2.59 \pm 0.12$	$2.61 \pm 0.12$
	0.0453	$0.779 \pm 0.013$	$2.81 \pm 0.12$	$2.83 \pm 0.12$
	0.105	$0.721 \pm 0.017$	$2.60 \pm 0.12$	$2.62 \pm 0.12$

TABLE I. Cross section vs energy.

the spin of  $Xe^{135}$  is  $\frac{3}{2}$ . Therefore, g has the two possible values,  $\frac{3}{8}$  or  $\frac{5}{8}$ .

In the least square analysis, the values of  $E_0$ ,  $\Gamma_n^0$ ,  $\Gamma_\gamma$ , and  $N_1^0$  for Samples II, III, V, VI, and VII, were determined by minimizing the function:

$$S = \sum_{r=1}^{7} \sum_{i=1}^{K_r} w_i \left[ \frac{Z(E_i)}{I_r} - \sigma(E_i, E^0, \Gamma_n^0, \Gamma_\gamma) \right]^2, \quad (10)$$

in which  $Z(E_i)$  is the experimentally determined relative cross section at energy  $E_i$ ,  $w_i$  is a weighting factor,  $I_r$  is the value of  $N_1^0$  for Sample r, and  $\sigma$  is the Breit-Wigner function (6). The minimizing equations of the function  $S(E_0, \Gamma_n^0, \Gamma_\gamma, I_{II}, I_{III}, I_V, I_{VI}, I_{VII})$  were solved by a standard iterative procedure.7 The minimizing equations were expanded in a Taylor series. Second derivative and higher terms were neglected. The resulting set of eight equations in eight unknowns were solved for corrections to the initial estimates of the set of constants to be determined. Since the weights,  $w_i$ , depend upon the variances of the  $I_r$ 's, new estimates of the weights were calculated along with corrections to the constants. The procedure was repeated until further iteration produced no further change in the constants or their variances. The procedure was programmed and computations were carried out on the ORACLE, under the direction of N. M. Dismuke.

The data and some results of the least square analysis are given in Table I. Column 1 is the sample number.

Column 2 gives the energies at which the transmission was measured. Column 3 gives the values, a, of the slopes of the straight lines,  $\ln T vs f(t)$ , which are proportional to  $(\sigma N_1^0)$ . Column 4 gives the values of the cross section  $\sigma$ , for  $g=\frac{3}{8}$ , and column 5 gives the values of  $\sigma$  for  $g=\frac{5}{8}$ . The uncertainties in all the entries in Table I are the standard deviations determined from the last squares analysis.

The values of  $N_1^{0}$ , the total number of  $I^{185}$  atoms initially present in the samples, are given in Table II. The values of  $N_1^{0}$  for Samples I and IV are derived from the absolute assay measurements. The values of  $N_1^{0}$  for the remaining five samples, determined from the least square analysis, are given for  $g=\frac{3}{8}$  and  $g=\frac{5}{8}$ . The uncertainties given are again the standard deviations. The separate sets of  $\sigma$ 's for  $g=\frac{3}{8}$  and  $g=\frac{5}{8}$  in Table I, come about because the values of  $N_1^{0}$  for Samples II, III, V, VI, and VII depend upon the value of g used in the analysis. The values of  $\sigma$  for Samples I and IV are, of course independent of the value of g.

The least-squares analysis showed that the data could be fit equally well with either of the two following sets of parameters:

$$q = \frac{3}{2}$$
:

$$E_{0} = (0.0851 \pm 0.0011) \text{ ev},$$

$$\Gamma_{n}^{0} = (0.0305 \pm 0.0008) \text{ ev},$$

$$r_{\gamma} = (0.0828 \pm 0.0031) \text{ ev},$$

$$E_{0} = (0.0849 \pm 0.0010) \text{ ev},$$
(11)

$$\Gamma_n^0 = (0.0182 \pm 0.0005) \text{ ev},$$
 (12)  
 $\Gamma_n = (0.0942 \pm 0.0032) \text{ ev}.$ 

The sums of the squares of the deviations of the data from the analytic functions were closely the same for both sets of parameters. The uncertainties given for the parameters are the standard deviations.

The data for all seven samples are summarized in the form of a plot of the total cross section,  $\sigma$ , vs the energy, E, in Fig. 7. Some average relative standard errors in the values of the cross section derived from a single sample, taking into account the neutron counting data only, are: 2% at 0.03 ev, 2% at 0.1 ev, 5% at 0.15 ev, and 14% at 0.20 ev. The increase in the error at the two higher energies is due to the great decrease in the number of incident neutrons in the Maxwell distribution at these energies and the limited amount of

TABLE II. Sample strengths.

Sample	$N_{1^0}(g=\tfrac{3}{8})$	$N_1^0(g=\frac{5}{8})$
I III IV V VI VI	$\begin{array}{c} (5.85 \pm 0.12) \times 10^{15} \\ (4.62 \pm 0.27) \times 10^{15} \\ (12.3 \ \pm 0.5) \times 10^{15} \\ (12.6 \ \pm 0.2) \times 10^{15} \\ (6.85 \pm 0.26) \times 10^{15} \\ (9.91 \pm 0.68) \times 10^{15} \\ (5.03 \pm 0.20) \times 10^{15} \end{array}$	$\begin{array}{c} (5.85\pm0.12)\times10^{15}\\ (4.61\pm0.25)\times10^{15}\\ (12.9\ \pm0.5)\times10^{15}\\ (12.6\ \pm0.2)\times10^{15}\\ (7.17\pm0.25)\times10^{15}\\ (9.95\pm0.65)\times10^{15}\\ (5.00\pm0.19)\times10^{15} \end{array}$

<sup>&</sup>lt;sup>7</sup> F. Garwood, Biometrika 33, 46-58 (1941, 1942).

time available for measurements during the life of a single xenon sample. The error in  $\sigma$  is increased because of the error in the sample assay. Average relative standard deviations due to both the neutron measurements and the assay are: 4% at 0.33 ev, 5% at 0.10 ev, 6% at 0.15 ev, and 15% at 0.2 ev. These average standard deviations are shown in Fig. 7. The solid curve of Fig. 7 represents the least squares Breit-Wigner fit to the data, using the set of parameters (11) for  $g = \frac{3}{8}$ .

A plot of  $\sigma E^{\frac{1}{2}}$  vs E is shown in Fig. 8. The points of Fig. 8 represent all of the transmission measurements made at the various energies for all seven samples. The absolute values of  $\sigma$ , the cross section per Xe<sup>135</sup> atom, used in Fig. 7 and Fig. 8, are those listed in Table I, which are based upon the absolute assays of Samples I and IV, and the sample strengths of the remaining five samples as determined from the neutron transmission measurements in the least squares fitting of the data to the Breit-Wigner curve. The solid curve again represents the least-squares fit of the Breit-Wigner single level resonance formula to the data, using the set of parameters (11) for  $g = \frac{3}{2}$ .

The mean value of the xenon total cross section for the Cd-absorbable neutrons of the thermal energy distribution of the reactor as determined from the fission chamber measurements is  $2.6 \times 10^6$  barns. This average value of the thermal cross section should be good statistically to several percent. The energy distribution of neutrons in the Oak Ridge graphite reactor corresponds, below the Cd cutoff energy, to a Maxwell distribution of temperature of about 520°K.



FIG. 7. Total cross section of xenon vs energy for all seven samples. The typical standard deviations shown include errors due to both neutron measurements and sample assay measurements. The values shown are based on the absolute assays of Samples I and IV.



FIG. 8.  $\sigma E^{\frac{1}{2}}$  vs E for all seven xenon samples. The absolute values are based on the absolute assays of Samples I and IV and the relative sample strengths as determined from the neutron transmission measurements.

Using the Breit-Wigner parameters derived from our data, the average total cross section for such a distribution is calculated to be  $2.4 \times 10^{-18}$  cm<sup>2</sup>.

The capture cross section,  $\sigma_c$ , is given in terms of the total cross section,  $\sigma_t$ , by the relation

$$\sigma_c = \sigma_t \Gamma_\gamma / \Gamma. \tag{13}$$

Using the sets of parameters (11) and (12), for  $g=\frac{5}{8}$  the capture cross section at resonance is 55% of its maximum possible value,  $g\pi\lambda_0^2$ , where  $\lambda_0$  is the value of  $\lambda$  at  $E=E_0$ . For  $g=\frac{3}{8}$ , the capture cross section is 80% of its maximum possible theoretical value.

The potential scattering,  $4\pi R^2$ , is quite negligible in all of the above considerations. Also, in the analysis of the data described above, no corrections were made for Doppler broadening of the resonance line.

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