It is now easy to see that a linear relation exists also between energy differences in the two configurations. If we add to the interactions $V_{ij}$ a constant term $-E_0$ ($E_0$ might be chosen to be the energy of the ground state in the $d_{3/2}f_{7/2}$ configuration), it will contribute to all levels of the configuration $d_{3/2}f_{7/2}$ the same energy $-E_0$, and to all levels of the $d_{3/2}{}^3f_{7/2}$ configuration the same energy $-3E_0$. Thus we obtain the following relations, where $E_J\equiv E(d_{3/2}f_{7/2}J)$ and $E_J'\equiv E(d_{3/2}{}^3f_{7/2}J)$:

$$E_5-E_2=\frac{1}{210}[287(E_5'-E_4')-35(E_3'-E_4')-135(E_2'-E_4')],$$

$$E_4-E_2=\frac{1}{60}[77(E_5'-E_4')-35(E_3'-E_4')+15(E_2'-E_4')],$$

$$E_3-E_2=\frac{1}{84}[77(E_5'-E_4')+49(E_3'-E_4')-9(E_2'-E_4')].$$

Using the values of $E(d_{3/2}{}^3f_{7/2}J)-E(d_{3/2}{}^3f_{7/2}\,J=4)$ taken from $K^{40}$, we obtain the results given in Table I. At that time the only excited levels in $Cl^{38}$ were believed to be 1.00 Mev and 1.92 Mev above the 2− ground state.[1] The only agreement existed for the spin of the ground state. However, experimental results[5] have very recently been reported which are in very good agreement with those predicted from the $K^{40}$ levels (Table I). This agreement means that in the cases considered we deal with pure *jj*-coupling configurations. This result may be due to the rather weak interaction between the *d*- and *f*-nucleons.

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## Attainment of Very High Energy by Means of Intersecting Beams of Particles

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In planning accelerators of higher and higher energy, it is well appreciated that the energy which will be available for interactions in the center-of-mass coordinate system will increase only as the square root of the energy of the accelerator. The possibility of producing interactions in stationary coordinates by directing beams against each other has often been considered, but the intensities of beams so far available have made the idea impractical. Fixed-field alternating-gradient accelerators[1] offer the possibility of obtaining sufficiently intense beams so that it may now be reasonable to reconsider directing two beams of approximately equal energy at each other. In this circumstance, two 21.6-Bev accelerators are equivalent to one machine of 1000 Bev.

The two fixed-field alternating-gradient accelerators could be arranged so that their high-energy beams circulate in opposite directions over a common path in a straight section which is common to the two accelerators, as shown in Fig. 1. The reaction yield is proportional to the product of the number of particles which can be accumulated in each machine. As an example, suppose we want $10^7$ interactions per second from 10-Bev beams passing through a target volume 100 cm long and 1 $cm^2$ in cross section. Using $5\times10^{-26}$ $cm^2$ for the nucleon interaction cross section, we find that we need $5\times10^{14}$ particles circulating in machines of radius $10^4$ cm.

There is a background from the residual gas proportional to the number of particles accelerated. With $10^{-6}$ mm nitrogen gas, we would have 15 times as many encounters with nitrogen nucleons in the target volume as we would have with beam protons. Since the products of the collisions with gas nuclei will be in a moving coordinate system, they will be largely confined to the orbital plane. Many of the desired *p-p* interaction products would come out at large angles to the orbital plane since their center of mass need not have high speed in the beam direction, thus helping to avoid background effects.

Multiple scattering at $10^{-6}$ mm pressure is not troublesome above one Bev; but beam life is limited by nuclear interaction with residual gas to $\sim$1300 seconds. Consequently, in about 1000 seconds the high-energy beam of $5\times10^{14}$ particles must be established in each accelerator. The fixed-field nature of the accelerator allows it to contain beams of different energy simultaneously. It may be possible to obtain this high beam current in this time by using $\sim10^3$ successive frequency modulation cycles of radio-frequency acceleration, each cycle bringing up $5\times10^{11}$ particles. It is encouraging to learn that Alvarez and Crawford[2] succeeded in building up a ring of protons by successively bringing up several groups of particles to the same final energy by frequency modulation in the 184-in. Berkeley cyclotron.

The number of particle groups which may be successively accelerated without leading to excessive beam spread can be estimated by means of Liouville's theorem.[3] One can readily convince himself that there is adequate phase space at high energy to accommodate

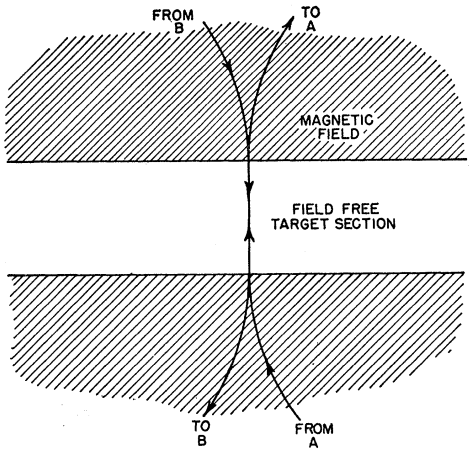


FIG. 1. The target straight section. $B$ and $A$ can be adjacent or concentric fixed-field alternating-gradient accelerators.

the necessary number, $N$, of particle groups. Assume for simplicity that synchrotron and betatron phase space are separately conserved, so that for the former

$$(\Delta p)_f(\Delta S)_f = N(\Delta p)_i(\Delta S)_i,$$

where $\Delta S$ and $\Delta p$ are the arc length and momentum spread at injection and final energy. Then, employing the fact that $P\sim R^{k+1}$, where $R$ is the radius and $k$ is the field index, one obtains

$$N = 2(k+1)(\Delta R/R)(p_f/p_i)(\Delta S_f/\Delta S_i)(E_i/\Delta E_i).$$

Using typical numbers such as

$$(p_f/p_i)\sim 100,\quad k\sim 100,\quad R\sim 0.5 \text{ cm},$$
$$R\sim 10^4 \text{ cm},\quad (\Delta E_i/E_i)\sim 10^{-3},$$

one finds that there is room for $N\sim 10^3$ frequency-modulation cycles.

The betatron phase space available is so large that it cannot be filled in one turn by the type of injectors used in the past which can inject $10^{11}$ particles. Thus there is the possibility of attaining and exceeding the yield used for this example by improving injection.

The more difficult problem of whether one can, in fact, use all of the synchrotron and betatron phase space depends in detail upon the dynamics of the proposed scheme and this is presently under study.

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[3] We are indebted to Professor E. Wigner who pointed out to us the importance of this consideration.

## Nuclear Spins of $Mo^{95}$ and $Mo^{97}$

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In a recent publication[1] Murakawa concludes from optical hfs data that the nuclear spins of $Mo^{95}$ and $Mo^{97}$ are each $I=7/2$. On the other hand, previous optical data[2,3] and nuclear resonance data[4] have led to the value $I=5/2$, in agreement with the predictions of the nuclear shell model. All of these estimates depend on relative intensity measurements and may be to some extent uncertain. We wish to confirm that the nuclear spins of these isotopes are in fact $I=5/2$, as measured by the paramagnetic resonance method.[5] There is no uncertainty since one has only to count the $2I+1=6$ hyperfine lines in the spectrum (Fig. 1).

The measurements were made on crystals grown from an HCl solution containing the colorless diamagnetic salt $K_3(InCl_6)\cdot 2H_2O$ and the pink paramagnetic salt $K_3(MoCl_6)$ with Mo:In$\sim$1:100. Under suitable conditions it could be arranged that the molybdenum in the mixed crystal was either in a trivalent state ($Mo^{3+}$, $4d^3$, $S=\frac{3}{2}$, color pink), or in a five-valent state ($Mo^{5+}$, $4d^1$, $S=\frac{1}{2}$, color green). It is probable that the latter state resulted from oxidation and the formation of molybdyl ions, $(MoO)^{3+}$. Both types of ion gave a similar six-line hfs in its paramagnetic resonance spectrum, and below we give details of the simpler spectrum from $Mo^{5+}$. In this case, there is a single unpaired electron, $4d^1$, and the lowest energy levels can be described by the spin-Hamiltonian

$$\mathcal{H} = g_{\parallel}\beta H_z S_z + g_{\perp}\beta(H_x S_x + H_y S_y) + A_{\parallel} S_z I_z + A_{\perp}(S_x I_x + S_y I_y);$$

where $S=\frac{1}{2}$, and $I=0$ for the even isotopes of Mo (relative abundance 74.9%), $I=5/2$ for the odd isotopes $Mo^{95}$(15.7%) and $Mo^{97}$(9.45%). The spectrum showed that there are two inequivalent types of $Mo^{5+}$ ion (I and II) present with slightly differently oriented axes of symmetry. The values of the constants in the Hamiltonian measured at 20°K using wavelengths $\lambda\approx 3.0$ cm and 1.2 cm were found to be

$$\text{I:}\quad g_{\parallel}=1.951\pm 0.005,\quad g_{\perp}=1.939\pm 0.006,$$
$$A_{\parallel}=0.0079\pm 0.0002 \text{ cm}^{-1},$$
$$A_{\perp}=0.00385\pm 0.0002 \text{ cm}^{-1},$$

$$\text{II:}\quad g_{\parallel}=1.959\pm 0.004,\quad g_{\perp}=1.939\pm 0.006,$$
$$A_{\parallel}=0.0077\pm 0.0002 \text{ cm}^{-1},$$
$$A_{\perp}=0.00385\pm 0.0002 \text{ cm}^{-1}.$$

The tracing of the spectrum shown in Fig. 1 is for the magnetic field $H$ directed fairly close to the $z$-axes of both ions, and using $\lambda=3.0$ cm. The spectrum of I is