negligible we should find that

$$\lim_{L\to\infty}\frac{b_B{}^L}{b_M{}^L}\to 1.$$

We do not find this to be the case, either in the 14.3-Mev case where by the usual criteria the Coulomb effect is negligible or in the 3.6-Mev case where it is not. Thus it appears that even when the incident energy is well above the Coulomb barrier, the Coulomb interaction still may be strong enough to cause Bowcock's method to

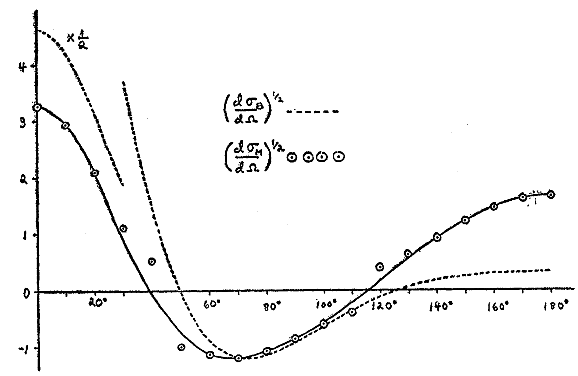

FIG. 2. Square roots of the calculated stripping cross sections plotted against angle. $d\sigma_B/d\Omega$ is the (d,p) cross section calculated from the Butler theory. $d\sigma_M/d\Omega$ is the result of modifying the Butler cross section by introducing Coulomb and nuclear interactions. $E_D=3.6$ Mev (lab), $Q=4.37$ Mev, $R=5.05\times 10^{-13}$ cm, $l=0$.

yield an incorrect result for the reduced width. However, in the 14.3-Mev case $b_B{}^L$ does tend to be about equal to $b_M{}^L$ in the first part of the exponential region.

It is rather curious that in these two cases one gets a rather good estimate of the reduced width ($\gamma=1$) by comparing the second peak of $d\sigma_M/d\Omega$ with that of $d\sigma_B/d\Omega$.

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Related jj-Coupling Configurations in K^{40} and Cl^{38}

S. GOLDSTEIN AND I. TALMI
Department of Physics, The Weizmann Institute of Science, Rehovoth, Israel
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THE magnetic moment as well as the spins and parities of the four lowest states[1] of K^{40} indicate jj-coupling and a $d_{3/2}{}^{-1}f_{7/2}$ configuration. The spins of these states are presumably 4− (ground state) 3− (0.032 Mev), 2− (0.80 Mev), and 5− (0.89 Mev), In doing various calculations in this mass number region,[2] it was of importance to examine the validity of the configuration assignment, i.e., how important are deviations from jj-coupling. To do this we investigated the Cl^{38} nucleus whose lowest configuration according to the shell model is $d_{3/2}f_{7/2}$.

Under the assumption that the radial functions in the two nuclei are the same, Racah's methods may be used to derive a linear relation between the energy levels of the $d_{3/2}{}^3f_{7/2}$ configuration and those of the configuration $d_{3/2}f_{7/2}$. As there is only one state, $J=\frac{3}{2}$, in the $d_{3/2}{}^3$ configuration, we need only consider the interaction between the $f_{7/2}$ neutron and the three $d_{3/2}$ protons. Thus the following conclusions do not depend on the assumption of charge independence.

Utilizing fractional parentage coefficients and the formula for change of coupling scheme,[3] we can rewrite the wave function of $d_{3/2}{}^3f_{7/2}$ as follows:

$$\begin{aligned}
&\psi(d_{3/2}{}^3(\tfrac{3}{2})f_{7/2}JM)\\
&\quad=(d_{3/2}{}^2(0)d_{3/2}\tfrac{3}{2}|\}d_{3/2}{}^3\tfrac{3}{2})\\
&\qquad\times\psi(d_{3/2}{}^2(0)d_{3/2}(\tfrac{3}{2})f_{7/2}JM)\\
&\qquad+(d_{3/2}{}^2(2)d_{3/2}\tfrac{3}{2}|\}d_{3/2}{}^3\tfrac{3}{2})\\
&\qquad\times\psi(d_{3/2}{}^2(2)d_{3/2}(\tfrac{3}{2})f_{7/2}JM)\\
&\quad=(d_{3/2}{}^2(0)d_{3/2}\tfrac{3}{2}|\}d_{3/2}{}^3\tfrac{3}{2})\\
&\qquad\times[4(2J+1)]^{\frac{1}{2}}W(0\tfrac{3}{2}J\,7/2;\tfrac{3}{2}J)\\
&\qquad\times\psi(d_{3/2}{}^2(0)d_{3/2}f_{7/2}(J)JM)\\
&\qquad+(d_{3/2}{}^2(2)d_{3/2}\tfrac{3}{2}|\}d_{3/2}{}^3\tfrac{3}{2})\\
&\qquad\times\textstyle\sum_{J'}[4(2J'+1)]^{\frac{1}{2}}W(2\tfrac{3}{2}J\,7/2;\tfrac{3}{2}J')\\
&\qquad\qquad\times\psi(d_{3/2}{}^2(2)d_{3/2}f_{7/2}(J')JM).
\end{aligned}$$

With this wave function the interaction energy in the case of any two-body forces can be readily expressed as

$$\begin{aligned}
&\langle d_{3/2}{}^3f_{7/2}JM|\sum_{j=2}^{4}V_{1j}|d_{3/2}{}^3f_{7/2}JM\rangle\\
&\quad=3\{(d_{3/2}{}^2(0)d_{3/2}\tfrac{3}{2}|\}d_{3/2}{}^3\tfrac{3}{2})^2\\
&\qquad\times 4(2J+1)W(0\tfrac{3}{2}J\,7/2;\tfrac{3}{2}J)^2\\
&\qquad\times\langle d_{3/2}f_{7/2}JM|V_{12}|d_{3/2}f_{7/2}\rangle\\
&\qquad+(d_{3/2}{}^2(2)d_{3/2}\tfrac{3}{2}|\}d_{3/2}{}^3\tfrac{3}{2})^2\\
&\qquad\times 4\textstyle\sum_{J'}(2J'+1)W(2\tfrac{3}{2}J\,7/2;\tfrac{3}{2}J')^2\\
&\qquad\qquad\times\langle d_{3/2}f_{7/2}J'M|V_{12}|d_{3/2}f_{7/2}J'M\rangle\}
\end{aligned}$$

(where $j=1$ is the $f_{7/2}$ neutron state, and $j=2, 3, 4$ are the $d_{3/2}$ proton states). Inserting in this formula the values $(d_{3/2}{}^2(0)d_{3/2}\frac{3}{2}|\}d_{3/2}{}^3\frac{3}{2})^2=\frac{1}{6}$, $(d_{3/2}{}^2(2)d_{3/2}\frac{3}{2}|\}d_{3/2}{}^3\frac{3}{2})^2=\frac{5}{6}$,[4] and the values of the W-functions, we obtain a linear relation between the energy levels of the two configurations.

TABLE I. Energy levels of Cl^{38} in Mev; spins and parities given when known.

Calculated from K^{40}	0(2−)	0.70(5−)	0.75(3−)	1.32(4−)
Experimental	0(2−)	0.672(5−)	0.762	1.312

It is now easy to see that a linear relation exists also between energy differences in the two configurations. If we add to the interactions V_{ij} a constant term $-E_0$ (E_0 might be chosen to be the energy of the ground state in the $d_{3/2}f_{7/2}$ configuration), it will contribute to all levels of the configuration $d_{3/2}f_{7/2}$ the same energy $-E_0$, and to all levels of the $d_{3/2}{}^3f_{7/2}$ configuration the same energy $-3E_0$. Thus we obtain the following relations, where $E_J\equiv E(d_{3/2}f_{7/2}J)$ and $E_J'\equiv E(d_{3/2}{}^3f_{7/2}J)$:

$$E_5-E_2=\frac{1}{210}[287(E_5'-E_4')-35(E_3'-E_4')-135(E_2'-E_4')],$$

$$E_4-E_2=\frac{1}{60}[77(E_5'-E_4')-35(E_3'-E_4')+15(E_2'-E_4')],$$

$$E_3-E_2=\frac{1}{84}[77(E_5'-E_4')+49(E_3'-E_4')-9(E_2'-E_4')].$$

Using the values of $E(d_{3/2}{}^3f_{7/2}J)-E(d_{3/2}{}^3f_{7/2}\,J=4)$ taken from K^{40}, we obtain the results given in Table I. At that time the only excited levels in Cl^{38} were believed to be 1.00 Mev and 1.92 Mev above the 2− ground state.[1] The only agreement existed for the spin of the ground state. However, experimental results[5] have very recently been reported which are in very good agreement with those predicted from the K^{40} levels (Table I). This agreement means that in the cases considered we deal with pure jj-coupling configurations. This result may be due to the rather weak interaction between the d- and f-nucleons.

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Attainment of Very High Energy by Means of Intersecting Beams of Particles

D. W. Kerst,* F. T. Cole,† H. R. Crane,‡ L. W. Jones,‡ L. J. Laslett,§ T. Ohkawa,|| A. M. Sessler,¶ K. R. Symon,** K. M. Terwilliger,‡ and Nils Vogt Nilsen††

Midwestern Universities Research Association,‡‡ University of Illinois, Champaign, Illinois

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In planning accelerators of higher and higher energy, it is well appreciated that the energy which will be available for interactions in the center-of-mass coordinate system will increase only as the square root of the energy of the accelerator. The possibility of producing interactions in stationary coordinates by directing beams against each other has often been considered, but the intensities of beams so far available have made the idea impractical. Fixed-field alternating-gradient accelerators[1] offer the possibility of obtaining sufficiently intense beams so that it may now be reasonable to reconsider directing two beams of approximately equal energy at each other. In this circumstance, two 21.6-Bev accelerators are equivalent to one machine of 1000 Bev.

The two fixed-field alternating-gradient accelerators could be arranged so that their high-energy beams circulate in opposite directions over a common path in a straight section which is common to the two accelerators, as shown in Fig. 1. The reaction yield is proportional to the product of the number of particles which can be accumulated in each machine. As an example, suppose we want 10^7 interactions per second from 10-Bev beams passing through a target volume 100 cm long and 1 cm^2 in cross section. Using 5×10^{-26} cm^2 for the nucleon interaction cross section, we find that we need 5×10^{14} particles circulating in machines of radius 10^4 cm.

There is a background from the residual gas proportional to the number of particles accelerated. With 10^{-6} mm nitrogen gas, we would have 15 times as many encounters with nitrogen nucleons in the target volume as we would have with beam protons. Since the products of the collisions with gas nuclei will be in a moving coordinate system, they will be largely confined to the orbital plane. Many of the desired p-p interaction products would come out at large angles to the orbital plane since their center of mass need not have high speed in the beam direction, thus helping to avoid background effects.

Multiple scattering at 10^{-6} mm pressure is not troublesome above one Bev; but beam life is limited by nuclear interaction with residual gas to $\sim$1300 seconds. Consequently, in about 1000 seconds the high-energy beam of 5×10^{14} particles must be established in each accelerator. The fixed-field nature of the accelerator allows it to contain beams of different energy simultaneously. It may be possible to obtain this high beam current in this time by using $\sim10^3$ successive frequency modulation cycles of radio-frequency acceleration, each cycle bringing up 5×10^{11} particles. It is encouraging to learn that Alvarez and Crawford[2] succeeded in building up a ring of protons by successively bringing up several groups of particles to the same final energy by frequency modulation in the 184-in. Berkeley cyclotron.

The number of particle groups which may be successively accelerated without leading to excessive beam spread can be estimated by means of Liouville's theorem.[3] One can readily convince himself that there is adequate phase space at high energy to accommodate