# Interaction of $K$-Particles with Nuclear Matter*

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The possible acceleration in nuclear matter of the decay of a $K$-particle whose interaction with nuclei is intrinsically weak (the $K_1$ particle) is investigated. Many alternative decay channels are afforded a $K_1$ particle in nuclear matter in contrast to the single mode of free $K_1$ decay. Decay through these alternative channels is shown to accelerate the decay but not to rates more rapid than $\approx 10^1 - 10^2\ \tau_{\text{free}}^{-1}$. An interpretation of fragments whose total excitation exceeds that available in the case of $\Lambda$-hyperfragments in terms of a $K_1$ particle bound in nuclear matter seems, therefore, plausible. In addition, the absorption rates of $K$-particles whose interaction with nuclear matter is intrinsically strong (the $K_2$ particle) with and without pion emission are estimated phenomenologically.

## INTRODUCTION

$K$-PARTICLES may be separated into two classes according to the strength of interactions with nuclear matter. Following Sachs,[1] the $K_1$-particle interacts weakly with nuclear matter (for example, the reaction: $K_1$+nucleon→hyperon (or nucleon)+pion is "slow") and is anomalously stable against decay into pions.[2] The $K_2$-particle, on the other hand, interacts strongly with nuclear matter (for example, the reaction: $K_2$+nucleon→hyperon (but not nucleon)+pion is "fast") and also is anomalously stable against decay.[3]

Since the $K_1$-particle has an intrinsically weak interaction with nuclear matter, it may become bound to a nuclear fragment; the resulting fragment will exhibit the properties of a $\Lambda$ hyperfragment with the exception that the excitation energy of the disintegration products of the $K_1$-fragment will be considerably in excess of that characteristic of hyperfragments. The possible virtual emission and/or absorption of pions by the $K_1$-particle in a nucleus should not cause a breakdown of the selection rules inhibiting the free decay of the $K_1$. This pion emission and absorption can accelerate the decay by providing two decay channels not available for the decay of a free $K_1$. The decay channels possible for a bound $K_1$ exclusive of those which allow for transitions of the nucleons of the absorbing nucleus to hyperon states are as follows[4]:

$$K_1\ (\text{bound})+A\ \text{nucleons}\rightarrow\begin{cases}A\ \text{nucleons}+2\pi & \text{(1a)}\\ A\ \text{nucleons}+1\pi & \text{(1b)}\\ A\ \text{nucleons.} & \text{(1c)}\end{cases}$$

In addition to the decay channels of Eq. (1), the $K_1$(bound) may "decay" through "production" channels of Eq. (2)[5]:

$$K_1\ (\text{bound})+A\ \text{nucleons}\rightarrow\begin{cases}(A-1)\ \text{nucleons}+\text{hyperon}\ (Y)+\text{pion}\ (\pi) & \text{(2a)}\\ (A-1)\ \text{nucleons}+\text{hyperon}\ (Y). & \text{(2b)}\end{cases}$$

A purpose of this note is to investigate whether the channels for disappearance of a $K_1$-particle in nuclear matter summarized in Eqs. (1) and (2) will accelerate the $K_1$-decay sufficiently to make such fragments inherently unobservable in nuclear emulsions (i.e., $\tau_{\text{bound}}<10^{-14}-10^{-15}$ sec). This type of instability of a $K_1$-particle would be a violation of the selection rules[1,6] against free decay to the same extent that the instability of the free $K_1$-particle decay is a violation of these rules. This follows from the observation that the mean lives for the channels of Eq. (1) will be estimated in terms of the experimentally known lifetime for the free decay of the $K_1$-particle[7] and the mean lives for decay through channels of Eq. (2) will be estimated in terms of the relative strengths of the coupling (weak) of the $(\phi_K,\varphi_\pi^2)$ fields and the $(\phi_K\psi_N,\varphi_\pi\psi_Y)$ fields, where the subscript $N$ refers to a nucleon.

When the $K_2$-particle (we consider only the negative member of this charge family) is moderated in, say, nuclear emulsion, it presumably will be captured by a nucleus, finally cascading via Auger and radiative transitions to the lowest Bohr orbit, from which it will be absorbed. Such absorptions are known experimentally[8] to cause transitions of a nucleon in the ab-

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[1] R. Sachs, Phys. Rev. **99**, 1573 (1955).

[2] The $K_1$-particle family presumably contains as members the $\theta^0$ and $\chi$ mesons.

[3] The decay schemes for the $K_2$-particle are, at this writing, uncertain. The $K_2$ can energetically decay into as many as three pions.

[4] The transition rates for these decay channels have also been estimated by A. Pais and R. Serber; Phys. Rev. **99**, 1551 (1955). For the purpose of this note, we shall assume that any charge-exchange scatterings of the type $K^{+,0}+A$ nucleons$\rightarrow K^{0,+}+A$ nucleons, "fast" reactions, are energetically prohibited.

[5] The author is indebted to H. Primakoff for comments on this mode of disappearance of $K_1$-particles in nuclear matter.

[6] See, for example, A. Pais, Physica **19**, 869 (1953); M. Gell-Mann, Phys. Rev. **92**, 833 (1953); K. Nishijima, Progr. Theoret. Phys. (Japan) **12**, 107 (1954).

[7] A similar situation is present in the analysis of the $\Lambda^0$ fragments in which the lifetime for nonmesonic decay was calculated in terms of the experimentally known lifetime for the mesonic decay of the free $\Lambda^0$. See W. Cheston and H. Primakoff, Phys. Rev. **92**, 1537 (1953).

[8] H. deStabler, Phys. Rev. **95**, 1110 (1954); Naugle, Ney, Freier, and Cheston, Phys. Rev. **96**, 1383 (1954); J. Hornbostel and E. Salant, Phys. Rev. **98**, 1202(A) (1955); Fry, Schneps, Snow, and Swami (private communication). (The author is indebted to Dr. Fry for making the data of his group available prior to publication.)

sorbing nucleus to a hyperon state via the reactions:

$$K_2^-\text{ (Bohr orbit)}+A\text{ nucleons}\rightarrow \begin{cases}(A-1)\text{ nucleons}\\ +\text{hyperon}+\text{pion}\end{cases} \quad (3a)$$

$$K_2^-\text{ (Bohr orbit)}+A\text{ nucleons}\rightarrow (A-1)\text{ nucleons}+\text{hyperon}. \quad (3b)$$

The hyperon of Eq. (3) may be either a $\Lambda$ or a $\Sigma$. A purpose of this note is to estimate the fraction of the time the absorption act should proceed without the emission of a real pion, making various assumptions concerning the nature of the produced hyperon. This estimate will be made considering the coupling of the nucleon absorbing the $K_2$ particle with the other nucleons of the absorbing nucleus via a static pion field.

## CALCULATION ON STABILITY OF BOUND $K_1$ PARTICLES

The appropriate Hamiltonian density $H$ for a system of nucleons, $K_1$-particles, and pions is postulated to be as follows:

$$H=H(\psi_N)+H(\psi_K)+H(\varphi_\alpha)+H^{\text{prod}}(\psi_K,\psi_N,\varphi_\alpha,\cdots)+\frac{g}{2\kappa_N}(\psi_N^*\boldsymbol{\sigma}\tau_\alpha\psi_N)\cdot\boldsymbol{\nabla}\varphi_\alpha+\{\eta\psi_K^*\theta\varphi^2+\text{h.c.}\}, \quad (4)$$

where $\psi_N$, $\psi_K$, $\varphi_\alpha\equiv$field amplitudes for the nucleon, $K_1$-particle, and pions, respectively; $H(\psi_N)$, $H(\psi_K)$, $H(\varphi_\alpha)$ $\equiv$corresponding free Hamiltonian densities; $\kappa_N\equiv$inverse Compton wavelength of the nucleon; $g\equiv$pion-nucleon coupling constant; $\eta\equiv$coupling constant between $K_1$-particle and pion pair fields ($\eta$ is adjusted to produce long mean life of a free $K_1$-particle against double pion decay); $H^{\text{prod}}\equiv$large interaction term describing copious $K_1$-particle production in nucleon-nucleon and/or pion-nucleon collisions; $\theta\equiv$operator coupling the $K_1$-particle and pion pair fields (for what follows, we shall assume a scalar $K_1$-particle and that $\theta$ is momentum independent, i.e., $\theta\rightarrow 1$)[9]; and h.c. refers to Hermitian conjugate.

We now assume, as in reference 7, that the meson field amplitude may be expanded as follows:

$$\varphi_\alpha=\varphi_{\alpha;\,\text{quantized}}+\varphi_{\alpha;\,\text{static}},$$

where

$$(\nabla^2-\kappa_\pi^2)\varphi_{\alpha;\,\text{static}}=4\pi\frac{g}{2\kappa_N}\boldsymbol{\nabla}\cdot(\psi_N^*\boldsymbol{\sigma}\tau_\alpha\psi_N), \quad (5)$$

or

$$\varphi_{\alpha;\,\text{static}}=-\frac{g}{2\kappa_N}\boldsymbol{\nabla}\cdot\int\psi_N^*(x')\boldsymbol{\sigma}'\tau_\alpha'\psi_N(x')Y(\mathbf{r}-\mathbf{r}')dx'$$

with

$$x'\equiv\mathbf{r}',\,s',\,\tau';\quad Y(\rho)\equiv\rho^{-1}\exp\{-\kappa_\pi\rho\},$$

and

$$\varphi_{\alpha;\,\text{quantized}}(\mathbf{r})=\sum_{\mathbf{q}}[2\pi\hbar c\Omega(q^2+\kappa_\pi^2)^{-\frac{1}{2}}]^{\frac{1}{2}}\times\{a_\alpha(\mathbf{q})\exp(i\mathbf{q}\cdot\mathbf{r})+\text{h.c.}\},$$

with $\kappa_\pi\equiv$inverse Compton wavelength of the pion; $\mathbf{q}\equiv$wave number of pion in quantized field; $a(\mathbf{q})\equiv$destruction operator for mesons of wave number $\mathbf{q}$; and $\Omega\equiv$hohlraum volume.

To calculate the one-pion decay mode of the $K_1$ particle in nuclear matter, we employ the appropriate interaction term of Eq. (4), namely

$$H^{\text{int}}=\eta^*\psi_k\theta^\dagger\{\varphi_{\text{static}}^*\varphi_{\text{quantized}}^*+\varphi_{\text{quantized}}^*\varphi_{\text{static}}^*\}. \quad (6)$$

The reciprocal mean-life for the one-pion channel is given to lowest order in $\eta$ as

$$(\tau_{1\pi})^{-1}=\frac{2\pi}{\hbar}\sum_{\epsilon_f,\,\mathbf{q}}\left|\int\cdots dx_j\cdots\psi_f^*\mathcal{A}\psi_i\right|^2+\delta\{[(m\pi^2c^4+\hbar^2q^2c^2)^{\frac{1}{2}}+\epsilon_f+Am_Nc^2]-[\epsilon_i+\epsilon_k+m_Kc^2+Am_Nc^2]\}, \quad (7)$$

where

$$\mathcal{A}\approx\frac{\eta^*g}{\kappa_N}\left[\frac{2\pi\hbar^2c^2}{E_q\Omega}\right]^{\frac{1}{2}}\frac{4\pi i\hbar^2c^2}{E_q^2}\sum_{j=1}^{A}\psi_k(\mathbf{r}_j)\exp(-i\mathbf{q}\cdot\mathbf{r}_j)\boldsymbol{\sigma}_j\cdot\mathbf{q}.$$

Equation (6) is approximate in the sense that $q$ has been considered larger than $\kappa_\pi$; terms in the matrix element of higher order in $(\kappa_\pi/q)$ than those included in $\mathcal{A}$ have been omitted. This approximation is consistent with the plausible assumption that for most states $f$ of the residual fragment for which the one-pion matrix element is appreciable, most of the available energy is carried off by the emitted pion.[10] The summation over the final states of the residual fragment may then be carried out by applying closure with respect to $\psi_f$, yielding a reciprocal mean life for one-pion decay:

$$(\tau_{1\pi})^{-1}\approx\frac{32\pi^2}{3}\hbar^3c^4q^3E_q^{-4}\left(\frac{\eta g}{\kappa_N}\right)^2I(A),$$

where

$$I(A)\equiv\int d\mathbf{r}|\psi_K(\mathbf{r})|^2\{n(s_{\frac{1}{2}})\rho(s_{\frac{1}{2}};\mathbf{r})+n(p_{\frac{1}{2}})\rho(p_{\frac{1}{2}};\mathbf{r})+\cdots\},$$

with, for example, $\rho(s_{\frac{1}{2}};\mathbf{r})\equiv$space density at $\mathbf{r}$ due to a nucleon in the lowest $s_{\frac{1}{2}}$ orbital and $n(s_{\frac{1}{2}})\equiv$number of nucleons in the lowest $s_{\frac{1}{2}}$ orbital. Taking oscillator functions for the orbitals and assuming the $K_1$ to be bound in the lowest $s$ orbital of the harmonic well of the nucleons, we have

$$I(A)\approx[\pi(\alpha^2+\beta^2)]^{-\frac{3}{2}}A\quad(\text{for small }A)$$

with

$$\alpha^2\equiv(\hbar^2/m_N\mathfrak{X}_N)^{\frac{1}{2}}\quad\text{and}\quad\beta^2\equiv(\hbar^2/m_K\mathfrak{X}_K)^{\frac{1}{2}},$$

[9] Assuming spin higher than zero for the $K_1$ does not change the order of magnitude of the results. See reference 4.

[10] Note that from this approximation it does not necessarily follow that stars (breakup of residual fragment) of small energy will result, since the pion produced in the decay has a high probability of interacting with the residual fragment.

where $\mathfrak{X}_N$ and $\mathfrak{X}_K$ are the elastic force constants for the nucleon and $K_1$-particle, respectively.

The lifetime for the one-pion decay mode of the $K_1$-fragment can be written in terms of the two-pion decay mode calculated from the appropriate interaction term of Eq. (4),

$$H^{\text{int}}=\eta^*\psi_K\theta^\dagger(\varphi_{\text{quantized}}^*)^2$$

yielding, to lowest order in $\eta$,

$$(\tau_{2\pi})^{-1}\approx 4\pi\hbar^2c^2\eta^2kE_k^{-1},$$

where $\mathbf{k}\equiv$wave number of the emitted mesons (assuming negligible energy taken off by the recoiling residual fragment). The lifetime for the two-pion decay is presumably closely approximated by the lifetime for the decay of the free $K_1$-particle. Consequently

$$\left(\frac{\tau_{1\pi}}{\tau_{2\pi}}\right)^{-1}\approx A\left(\frac{g^2}{\hbar c}\right)\frac{8}{3\pi^{\frac{1}{2}}}\left(\frac{q}{k}\right)\left(\frac{E_k}{E_q}\right)\left(\frac{m_\pi c^2}{E_q}\right)\times\left(\frac{\hbar qc}{E_q}\right)^2\left(\frac{m_\pi}{m_N}\right)^2(\kappa_\pi^2\alpha^2+\kappa_\pi^2\beta^2)^{-3/2}.$$

Taking the reasonable yet uncertain values $\alpha\approx\beta\approx 2\times10^{-13}$ cm, $(g^2/\hbar c)\approx 10$, $q\approx 3.5\kappa_\pi$, $k\approx 1.5\kappa_\pi$, we find (in essential agreement with Serber and Pais[4])

$$(\tau_{1\pi}/\tau_{2\pi})^{-1}\approx 0.1A. \tag{8}$$

It is evident that the one-pion decay mode is ineffective competition to the two-pion decay mode. This reflects the fact that only those nucleons which are relatively close together (within $\approx q^{-1}$) are effective participants (i.e., contribute to the static pion field responsible for the decay) in the one-pion decay mode. It is to be remembered in the above estimate that the coupling operator of Eq. (4) has been assumed momentum independent. If, on the other hand, $\theta\sim p_\pi{}^n$, then

$$(\tau_{1\pi}/\tau_{2\pi})^{-1}\approx(q/k)^{2n}[(\tau_{1\pi}/\tau_{2\pi})^{-1}\text{ of Eq. (8)}].$$

Since $q/k\approx 2$–$3$, this will not affect the general conclusion that the one-pion decay mode does not accelerate the decay of a $K_1$-particle in nuclear matter beyond the limit of detection in photographic emulsions.

The lifetime for decay through the totally nonmesonic channels is calculated from the appropriate interaction term of Eq. (4), namely,

$$H^{\text{int}}=\eta^*\psi_K\theta^\dagger(\varphi_{\text{static}}^*)^2.$$

The bilinear static pion field amplitude is written in configuration-space representation as

$$\varphi_{\alpha;\,\text{static}}\varphi_{\alpha';\,\text{static}}=\left(\frac{g}{2\kappa_N}\right)^2\int\cdots dx_j\cdots\sum_{l,j=1}^{A}\times\psi_f^*(\cdots x_j\cdots)[\boldsymbol{\sigma}_j\cdot\nabla Y(\mathbf{r}-\mathbf{r}_j)]\times[\boldsymbol{\sigma}_l\cdot\nabla Y(\mathbf{r}-\mathbf{r}_l)]\tau_{\alpha;\,j}\tau_{\alpha';\,l}\psi_i(\cdots x_j\cdots). \tag{9}$$

In the calculation of the matrix element for the nonmesonic decay mode, we shall omit antisymmetrization of the initial and final state nuclear wave functions. In addition, we shall drop all dependence of the nuclear wave functions on the isotopic spin variables. Considering only those terms of Eq. (9) in which $j\neq l$:[11]

$$H_{fi}{}^{\text{int}}=\eta^*\left(\frac{g}{2\kappa_N}\right)^2\sum_{j\neq l}\int[\boldsymbol{\sigma}_j\cdot\nabla_j u_f^*(y_j)u_i(y_j)]\times[\boldsymbol{\sigma}_l\cdot\nabla_l w_f^*(y_l)w_i(y_l)]\psi^*(\mathbf{r}_K)Y(\mathbf{r}-\mathbf{r}_j)\times Y(\mathbf{r}-\mathbf{r}_l)dy_l dy_j d\mathbf{r}_K\mathfrak{D}, \tag{10}$$

where

$$\mathfrak{D}\equiv\int dy_i\cdots dy_{j-1}dy_{j+1}\cdots dy_{l-1}dy_{l+1}\cdots dy_A\times\psi_f^*(\cdots y_{j-1}y_{j+1}\cdots y_{l-1}y_{l+1}\cdots)\psi_i(\cdots),$$

and $u_i$, $u_f$; $w_i$, $w_f$ are the single-particle wave functions for the $j$th and $l$th nucleons in the initial and final state. We shall now assume that the energy available in nonmesonic decay is shared between two nucleons.[12] Under this assumption, the wave number $q_0$ of an outgoing nucleon $\approx 7\kappa_\pi$. Consequently, the terms in Eq. (10) which are dominant are those for which $j$ and $l$ correspond to the nucleons sharing the available energy. In addition, $\nabla u_f^*u_i\approx u_i\nabla u_f^*=i\mathbf{q}_0u_f^*u_i$. As additional consequences of $q_0>\kappa_\pi$, $\psi_K(\mathbf{r}_K)\approx\psi_K(\mathbf{r}_j)$, and $w(\mathbf{r}_l)\approx w(\mathbf{r}_j)$, i.e., $q_0$ is large enough so that nonmesonic decay proceeds only when the nucleons sharing the available energy are close together. The mean life for nonmesonic decay is calculated then to be

$$(\tau_{0\pi})^{-1}=\frac{2\pi}{\hbar}\sum_{\epsilon_f,\,\mathbf{q}_0}|H_{fi}{}^{\text{int}}|^2\times\delta\{[(\hbar^2q_0{}^2/m_N)+\epsilon_f+Am_Nc^2]-[\epsilon_K+m_Kc^2+\epsilon_i+Am_Nc^2]\}.$$

Once again, the application of closure with respect to $\psi_f$ (of the $A-2$ nucleons remaining in the fragment) yields

$$(\tau_{0\pi})^{-1}\approx\frac{2\pi}{\hbar}\eta^2\left(\frac{g}{2\kappa_N}\right)^4\frac{4\pi q_0{}^2dq_0}{(2\pi)^3dE}B(A),$$

[11] This is then the two-nucleon absorption model of Pais and Serber. However, we have elected to calculate $H_{fi}{}^{\text{int}}$ with the form of $\varphi^2$ implied by Eq. (9) rather than assuming some phenomenological value taken from other nuclear absorption data involving high-energy transfer to the absorbing nucleus. Both approaches rest on a somewhat insecure base, since both involve a detailed knowledge of the nucleon-nucleon interaction for separations $\approx(7\kappa_\pi)^{-1}$. In addition, although the terms where $j=l$ are small if Eq. (9) is assumed to hold only down to distances of the order of $(\kappa_N)^{-1}$, these terms give a divergent contribution to $H_{fi}{}^{\text{int}}$ if Eq. (9) is assumed to hold rigorously.

[12] The assumption is, at first sight, in contradiction with the available evidence on pion absorption by nuclei. In the cases of pion absorption, the available energy is shared among more than two nucleons [see S. Tamor, Phys. Rev. **77**, 412 (1950), and W. Cheston and L. Goldfarb, Phys. Rev. **78**, 683 (1950)]. However, since $\kappa_K\approx 3\kappa_\pi$, one would expect that relatively fewer nucleons would participate in the $K_1$-fragment disintegration.

where

$$B(A)\equiv\int d\mathbf{r}|\psi_K(\mathbf{r})|^2|n(s_{\frac{1}{2}})\rho(s_{\frac{1}{2}};\mathbf{r})+\cdots|^2$$
$$\approx A^2[\pi(2\beta^2+\alpha^2)\alpha^2]^{-\frac{3}{2}}\text{ (for light } A).$$

We obtain, finally,

$$\left(\frac{\tau_{0\pi}}{\tau_{2\pi}}\right)^{-1}\approx 2\pi^{\frac{1}{2}}A^2\left(\frac{g^2}{\hbar c}\right)^2\left(\frac{m_\pi}{m_N}\right)^2\left(\frac{E_k}{m_\pi c^2}\right)\left(\frac{\kappa_\pi}{q_0}\right)^4\left(\frac{q_0}{k}\right)$$
$$\times(\alpha\kappa_\pi)^{-3}[\kappa_\pi^2(2\beta^2+\alpha^2)]^{-\frac{3}{2}}\approx 0.01A^2.$$

Although this value is somewhat larger than that obtained via the two-nucleon absorption model of Pais and Serber, the conclusion remains that the nonmesonic decay mode is also ineffective as a competing mechanism for the decay of a $K_1$-fragment.

Finally, we must estimate the lifetime associated with the "production" channels of Eq. (2). Consider the decay channel:

$$K_1\text{ (bound)}+A\text{ nucleons}\rightarrow$$
$$(A-1)\text{ nucleons}+\text{hyperon }(Y)+\text{pion.}\quad(2a)$$

Consistent with our previous assumption concerning the scalar nature of the $K_1$, we postulate an interaction Hamiltonian density leading to decay via Eq. (2a) of the form

$$H^{\text{int}}=\{[\kappa_\pi^5(m_\pi c^2)]^{-\frac{1}{2}}\eta'\psi_N^*\boldsymbol{\sigma}\tau_\alpha\psi_Y\cdot\nabla(\varphi_K^*\varphi_{\pi;\alpha})+\text{h.c.}\},\quad(11)$$

where $\eta'$ is the coupling constant for the four fields under consideration and has the same dimensions as $\eta$. The reciprocal lifetime for decay through the channel of Eq. (2a) is then

$$(\tau_{Y\pi})^{-1}=\frac{2\pi}{\hbar}[\kappa_\pi^5(m_\pi c^2)]^{-1}\eta'^2\frac{2\pi\hbar c}{\Omega}\sum_{f,\mathbf{q}_Y}[q_Y^2+\kappa_\pi^2]^{-\frac{1}{2}}$$
$$\times\sum_{j,l}\langle i|\mathbf{U}_j^\dagger\cdot\mathbf{q}_Y|f\rangle\langle f|\mathbf{U}_l\cdot\mathbf{q}_Y|i\rangle$$
$$\times\delta\{[(m\pi^2c^4+\hbar^2q_Y^2c^2)^{\frac{1}{2}}+E_Y$$
$$+(A-1)m_Nc^2+\epsilon_f$$
$$-[\epsilon_i+\epsilon_K+m_Kc^2+Am_Nc^2]\},\quad(12)$$

where $\mathbf{q}_Y\equiv$ wave number of emitted pion, and

$$\mathbf{U}_j^\dagger\cdot\mathbf{q}_Y=(\boldsymbol{\sigma}_j\cdot\mathbf{q}_Y)\varphi_K(\mathbf{r}_j)\exp\{-i\mathbf{q}_Y\cdot\mathbf{r}_j\}.$$

(The sum over $f$ in Eq. (12) is over all final states of the $A$ fermions.) Assuming that the matrix element of $H^{\text{int}}$ is small except for an emitted pion momentum corresponding to the "production" reaction from an unbound nucleon, the sums in Eq. (12) can be performed in the closure approximation yielding

$$(\tau_{Y\pi})^{-1}\approx\frac{16}{3\pi^{\frac{1}{2}}}\left(\frac{q_Y}{\alpha}\right)^3\hbar^{-1}[m_\pi c^2(\kappa_\pi)^5]^{-1}\eta'^2$$

or finally:

$$\left(\frac{\tau_{Y\pi}}{\tau_{2\pi}}\right)^{-1}\approx\frac{4}{3\pi^{\frac{3}{2}}}\left(\frac{q_Y}{\kappa_\pi}\right)^3\left(\frac{\kappa_\pi}{k}\right)\left(\frac{E_k}{m_\pi c^2}\right)(\alpha\kappa_\pi)^{-3}\left(\frac{\eta'}{\eta}\right)^2.$$

Taking $q_Y\approx 2\kappa_\pi$, we have

$$(\tau_{Y\pi}/\tau_{2\pi})^{-1}\approx 3(\eta'/\eta)^2.\quad(13)$$

It is probably premature to speculate concerning the approximate value of $(\eta'/\eta)$.[13]

Finally, the decay of the $K_1$ through production channel 2(b) can be estimated in a manner similar to the previously discussed nonmesonic decays of slightly unstable particles in nuclear matter. The result is

$$(\tau_{Y,0\pi}/\tau_{2\pi})^{-1}\approx 0.3A(\eta'/\eta)^2,\quad\text{for light }A.\quad(14)$$

The above phenomenological discussion of the apparent stability of $K_1$-particles in nuclear matter has demonstrated that the alternative decay modes afforded a bound $K_1$-particle do not appreciably accelerate the decay. Another accelerating mechanism is present in the possible dependence of the coupling constants $\eta$ and $\eta'$ on the $K_1$-particle environment, the magnitudes of $\eta$ and $\eta'$ being a measure of the degree of breakdown of the absolute selection rules forbidding free $K_1$-particle decay. Such breakdown may be attributed to, for example, interactions such as that of $\beta$ decay which do do not conserve isotopic spin. However, no such environmental dependence of the $\Lambda^0$-decay constant is apparent in the experimental evidence on hyperfragments, and it seems plausible that a similar situation exists in the case of the $K_1$-particle. Such a plausibility statement must await the ultimate test of an analytic theory of the decay mechanisms of the heavy, unstable particles.

Finally, there is an unexpected paucity of experimental evidence on $K_1$-particle fragments if it is noted that the phenomenological production schemes so far suggested predict the production of $\Lambda^0$ particles and $K_1$-particles in association. One possible explanation of this fact is a $K_1$-nucleon interaction relatively weaker than the $\Lambda^0$-nucleon interaction, the latter already much weaker than the nucleon-nucleon interaction.

## CALCULATION ON THE ABSORPTION OF $K_2$ PARTICLES

We postulate an interaction Hamiltonian density of the simplest form to describe the absorption of a $K_2$-particle by a nucleon:

$$H^{\text{int}}=\{\gamma(\psi_N^*\tau_\alpha\psi_Y)(\phi_K^*\varphi_\alpha)+\text{h.c.}\},$$

where $\psi_Y$, $\phi_K\equiv$ quantized wave amplitudes for the hyperons and $K_2$ fields, respectively; $\gamma\equiv$ coupling constant describing the "strong" interaction among the four fields. There is experimental evidence[8,14] that the

[13] In the language of Sachs, the two-pion decay is nonconserving of one unit of the "attribute," whereas the "production" decay channel is nonconserving of attribute by 2 units.

[14] Haskin, Williams, Goodman, and Schein, Phys. Rev. **100**, 1263(A) (1955).

hyperon ($Y$) produced in the absorption can be either a $\Lambda$ or a $\Sigma$; we shall treat both cases assuming an interaction Hamiltonian similar in form in both cases ($\gamma$ need not be the same, however). We shall assume that the $K_2^-$ is absorbed from the lowest Bohr orbit of the capturing atom. The mean life for $K_2$-absorption accompanied by the emission of a real pion is written to lowest order in $\gamma$ as

$$(\tau_{Y\pi})^{-1}\approx\frac{2\pi}{\hbar}\gamma^2\sum_{\epsilon_f,\,\mathbf{q}_Y}\sum_{j,l}^{Z}\langle i|U_l^\dagger|f\rangle\langle f|U_j|i\rangle \times\delta\{[(m_\pi^2c^4+\hbar^2q_Y^2c^2)^{\frac{1}{2}}+\epsilon_f+(A-1)m_Nc^2+m_Yc^2]-[\epsilon_i+Am_Nc^2+m_Kc^2]\},\quad(15)$$

where the summation index $\epsilon_f$ refers to a summation over all energy conserving states of the $A$ fermions in the final state and

$$U_j=\phi_K(\mathbf{r}_j)[\Omega^{-1}(q_Y^2+\kappa_\pi^2)^{-\frac{1}{2}}2\pi\hbar c]^{\frac{1}{2}}\exp\{i\mathbf{q}_Y\cdot\mathbf{r}_j\}.$$

$\langle i|$ and $\langle f|$ are the state vectors for the initial and final $A$ fermions, respectively. Assuming that the absorption matrix element is large only for pion momenta approximately equal to the pion momentum obtained when the $K_2$ is absorbed by a free nucleon and performing the closure procedure to Eq. (15), we discover that

$$(\tau_{Y\pi})^{-1}\approx\frac{2\pi}{\hbar}\gamma^2\langle i|\sum_{l,j}U_l^\dagger U_j|i\rangle\frac{4\pi q_Y(q_Y^2+\kappa_\pi^2)^{\frac{1}{2}}\Omega}{(2\pi)^3\hbar c}.\quad(16)$$

The mean life for absorption of the $K_2$ without pion emission can be written to lowest order in $\gamma$ by applying arguments previously advanced that lead to the mean life for one pion decay of the $K_1$-particle in nuclear matter, yielding

$$(\tau_{Y,0\pi})^{-1}\approx\frac{2\pi}{\hbar}\gamma^2\left(\frac{g}{2\kappa_N}\right)^2\left(\frac{4\pi}{k_Y^2+\kappa_\pi^2}\right)^2\int\frac{d\omega}{4\pi}\sum_{j,l}\langle i|\theta_l^\dagger\theta_j|i\rangle \times\frac{4\pi k_Y(k_Y^2+\kappa_Y^2)^{\frac{1}{2}}\Omega}{(2\pi)^3\hbar c},$$

where $k_Y\equiv$ wave number of ejected hyperon, and where

$$\theta_j\approx\phi_K(\mathbf{r}_j)\sum_{t\neq j}^{Z}u_t(\mathbf{r}_j)(\boldsymbol{\sigma}_j\cdot\mathbf{k}_Y)\Omega^{-\frac{1}{2}}\exp\{i\mathbf{k}_Y\cdot\mathbf{r}_j\},$$

and $u_t(\mathbf{r}_j)$ is the amplitude at $\mathbf{r}_j$ of the single-particle orbital of the absorbing nucleon. Since $k_\Sigma\approx5.7\kappa_\pi$ and $k_Y\approx6.5\kappa_\pi$, the only terms giving a significant contribution to the mean life for the nonmesonic absorption are those for which $j=l$. Finally,

$$\left(\frac{\tau_{Y,0\pi}}{\tau_{Y\pi}}\right)^{-1}\approx2\pi A\left(\frac{g^2}{\hbar c}\right)\left(\frac{k_Y}{q_Y}\right)\left(\frac{k_Y}{\kappa_\pi}\right)^2\left(\frac{k_Y^2+\kappa_Y^2}{q_Y^2+\kappa_\pi^2}\right)^{\frac{1}{2}} \times\left[\frac{k_Y^2}{\kappa_Y^2}+1\right]^{-\frac{3}{2}}\left(\frac{m_\pi}{m_N}\right)^2R,\quad(17)$$

where

$$A\kappa_\pi^6R\equiv\sum_{j,l}^{Z}\int|\phi_K(\mathbf{r})|^2\{n(s_{\frac{1}{2}})\rho(s_{\frac{1}{2}};\mathbf{r})+\cdots\}u_j^*(\mathbf{r}) \times u_l(\mathbf{r})d\mathbf{r}\left[\sum_{j,l}\left|\int\phi_K(\mathbf{r})u_j(\mathbf{r})\exp\{i\mathbf{q}_Y\cdot\mathbf{r}\}d\mathbf{r}\right| \cdot\left|\int\phi_K(\mathbf{r})u_l(\mathbf{r})\exp\{i\mathbf{q}_Y\cdot\mathbf{r}\}d\mathbf{r}\right|\right]^{-1}.\quad(17a)$$

For light nuclei, $R\approx2\times10^{-3}$, $R$ decreasing somewhat with increasing $A$. A decrease of 5% in $R$ is apparent over the range $12\leqslant A\leqslant16$.

For the lighter elements of a nuclear emulsion (C, N, O), the relative number of absorption occurring without pion emission is

$$\left(\frac{\tau_{Y,0\pi}}{\tau_{Y\pi}}\right)^{-1}\approx\begin{cases}1.4\times10^{-3}(g^2/\hbar c)A & \text{for } Y=\Sigma\\ 7.6\times10^{-3}(g^2/\hbar c)A & \text{for } Y=\Lambda.\end{cases}\quad(18)$$

For these elements, nonmesonic absorption should occur in approximately 20% of the cases, when the associated hyperon is a $\Sigma$ but in approximately 55% of the cases when the produced hyperon is a $\Lambda$. These figures reflect qualitatively the trend of the preliminary experimental data on $K_2^-$-absorptions.[8,14]

The increase in the relative number of nonmesonic absorptions accompanying an increase in the energy release in the absorption can be understood in the following manner. The wave number of the emitted pion in the absorptions with pion emission increases by $\approx40\%$ when the mass of the emitted hyperon decreases from that of the $\Sigma$ to that of the $\Lambda$ ($q_\Sigma\approx1.24\kappa_\pi$; $q_\Lambda\approx1.78\kappa_\pi$) resulting in a suppression of the coherent effect of the absorbing nucleons. The absorbing nucleons in the absorptions unaccompanied by pion emission act incoherently in both the $\Sigma$ and $\Lambda$ emission since the wave number of the emitted hyperons is, in both cases, much greater than $\kappa_\pi$. The phase space factors are actually less favorable for pion emission in the case of $\Sigma$ production since $k_\Sigma/q_\Sigma\approx4.6$ whereas $k_\Lambda/q_\Lambda\approx3.7$.

To compare the estimates quoted above with experiment, the absorption by the nucleus of any real pions produced in the $K_2$-absorption must be taken into account. This, of course, will decrease the relative number of absorptions with observed pion emission (correcting for the unobserved neutral pions by charge independence arguments) under that estimated above. In addition, many of the $K_2$-absorptions will occur in the Ag and Br in the emulsions for which it is difficult to perform the estimation procedures carried out for the (C, N, O) group. Finally, any momentum dependence in the interaction Hamiltonian density describing the $K_2$-absorption will tend to increase the relative number of absorptions without pion emission since $(k_Y/q_Y)>1$.