Internal Pair Creation in Beta Decay*

KERSON HUANG[†]

Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

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A physical picture is presented for internal pair creation in beta decay, and the ratio of the total number of positrons to the total number of electrons emitted is calculated for the transition $P^{32} \rightarrow S^{32}$. The result is in agreement with the experiment of Greenberg and Deutsch.

COME beta-active nuclei emit a positron-electron ${f J}$ pair in addition to the beta particle during a radioactive transition. Arley and Møller¹ have investigated such a process in 1938 and calculated the ratio N^+/N^- of the total number of positrons to the total number of electrons emitted, as a function of the energy available for the beta transition. They based their considerations on the assumption that, in beta decay, one can consider a single free proton converting into a neutron inside the nucleus. While doing so, positronelectron pairs can also be emitted because of a Møller scattering between the proton and the beta particle in intermediate states with electrons in the Dirac negativeenergy sea. Their numerical results were obtained by making some radical approximations for the matrix element and were not in agreement with the experimental data existing then. They accordingly concluded that some other mechanism must be responsible for this phenomenon.

Recently, Greenberg and Deutsch² have measured the ratio N^+/N^- for the transition ${}_{15}P^{32} \rightarrow {}_{16}S^{32}$ and obtained the value

$$N^+/N^- = (0.75 \pm 0.25) \times 10^{-9},$$
 (1)

which is orders of magnitude different from previous measurements.² However, it still does not agree with the result of Arley and Møller. We wish to show here that the matrix elements for the process obtained by Arley and Møller are correct and can be derived from a more satisfactory starting point. However, the approximation they introduced in evaluating numerical answers was inadequate.

We consider a nucleus of Z protons and A-Zneutrons interacting with the electron-neutrino field, the electromagnetic field, and indirectly the Dirac electron field. The nucleus is assumed to be beta-unstable and can undergo a zero-order beta transition (no spin change). The interaction Hamiltonian is³

$$H_{\rm int} = H_{\beta} + ie \int d\mathbf{r} \bar{\psi}(\mathbf{r}) \gamma_{\nu} A_{\nu}(\mathbf{r}) \psi(\mathbf{r}) - \int d\mathbf{r} J_{\nu}(\mathbf{r}) A_{\nu}(\mathbf{r}), \quad (2)$$

where $ie\bar{\psi}\gamma_{\nu}\psi$ is the current of the Dirac electron field; $J_{\nu}(\mathbf{r})$ is the current of the nucleus, and H_{β} is the coupling that causes beta decay. Letting Ψ_Z denote a nuclear wave function, the only nonvanishing matrix elements of H_{β} are

$$\langle \Psi_{Z+1}, e^{-}, \nu | H_{\beta} | \Psi_{Z} \rangle = g \langle e^{-}, \nu | F | 0 \rangle \langle K \rangle, \qquad (3)$$

where g is a coupling constant, $\langle K \rangle$ is a nuclear matrix element, and F is some spin operator. The combination FK may mean $\sum F_i K_i$ or $\sum F_{\alpha\beta} K_{\alpha\beta}$, etc., depending on the type of coupling used. The matrix element $\langle e^{-\nu} | F | 0 \rangle$ will be explicitly written as follows:

$$\langle e^{-\nu} | F | 0 \rangle = V^{-1} \{ u^*(\mathbf{p}) Fv^*(\mathbf{p}) \} = V^{-1} \{ u^*(\mathbf{p}) F'w(\mathbf{p}) \},$$

where V is the normalization volume, $u(\mathbf{p}^{-})$ is an electron spinor of momentum \mathbf{p}^- , $v(\mathbf{p}_{\nu})$ is the spinor for an antineutrino of momentum \mathbf{p}_{ν} , and $w(\mathbf{p}_{\nu})$ is the spinor for a neutrino of momentum $-\mathbf{p}_{\nu}$. F' is related to \overline{F} by the relation F = F'B, where

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

For scalar coupling, for example, F'=1, and for tensor coupling $F' = \sigma$. We shall not go into these details. The whole calculations may be performed with scalar coupling because other types of coupling give rise to a difference only in the angular correlation of the emitted particles, and since we shall eventually integrate over all angles, these extra terms will vanish. The nuclear current is $J_{\mathbf{r}}(\mathbf{r}) = (\mathbf{J}(\mathbf{r}), i\rho(\mathbf{r}))$, with

$$\langle \Psi_{f} | \mathbf{J}(\mathbf{r}) | \Psi_{i} \rangle = \left(\frac{e}{2iM} \right) \sum_{n=1}^{Z} \int d\mathbf{r}_{1} \cdots d\mathbf{r}_{A} \{ \Psi_{f}^{*}(\nabla_{n}\Psi_{i}) - (\nabla_{n}\Psi_{f}^{*})\Psi_{i} \} \prod_{m=1}^{Z} \delta(\mathbf{r}-\mathbf{r}_{m}), \quad (4)$$

$$\langle \Psi_{f} | \rho(\mathbf{r}) | \Psi_{i} \rangle = e \int d\mathbf{r}_{1} \cdots d\mathbf{r}_{A} \Psi_{f}^{*} \Psi_{i} \prod_{m=1}^{Z} \delta(\mathbf{r}-\mathbf{r}_{m}),$$

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[†] Now at the Institute for Advanced Study, Princeton, New Jersey

¹N. Arley and C. Møller, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **15**, 9 (1938). See also L. Tisza, Physik. Z. Sowjetunion **11**, 245 (1937); J. K. Knipp and G. E. Uhlenbeck, Physica III, 425 (1936).

² J. Greenberg and M. Deutsch, preceding paper [Phys. Rev. **102**, 415 (1956)]. See the discussion there concerning the signifi-¹ cance of the experimental result (1). ³ Natural units $\hbar = c = 1$ will be employed throughout.

so that

$$\left\langle \Psi_{f} \middle| \int J_{\nu}(\mathbf{r}) A_{\nu}(\mathbf{r}) d\mathbf{r} \middle| \Psi_{i} \right\rangle$$

$$= e \sum_{n=1}^{Z} \int d\mathbf{r}_{1} \cdots d\mathbf{r}_{A} \left\{ \left(\frac{1}{2iM} \right) \left[\Psi_{f}^{*}(\nabla_{n} \Psi_{i}) - (\nabla_{n} \Psi_{f}^{*}) \Psi_{i} \right] \cdot \mathbf{A}(\mathbf{r}_{n}) - \Psi_{f}^{*} \Psi_{i} \phi(\mathbf{r}_{n}) \right\}, \quad (5)$$

where $A_{\nu} = (\mathbf{A}, i\phi)$. The first term in (5) involves transverse photon of pole order higher or equal to electric dipole. We can therefore neglect it compared to the second term which is predominantly monopole.

The process under consideration proceeds from the initial state $|\Psi_Z\rangle$ to the final state $|\Psi_{Z+1}, \nu, e^+, e_1^-, e_2^-\rangle$ and the matrix elements $\mathfrak{M}_1, \mathfrak{M}_2, \mathfrak{M}_3$ are represented by the Feynman diagrams in Fig. 1. We define two auxiliary operators M_N , M_β , whose matrix elements are given by the diagrams shown in Fig. 2. They are the operators for Møller scattering of vacuum electrons by the nucleus and by an electron. We shall need only the following matrix elements:

$$\langle \Psi_{Z}, e^{+}, e^{-} | M_{N} | \Psi_{Z} \rangle$$

$$= -Ze^{2} \{ u^{*}(\mathbf{p}_{-})u(\mathbf{p}_{+}) \}$$

$$\times \int d\mathbf{r} \left\langle \Psi_{Z} \left| \frac{\exp[-i(\mathbf{p}_{-} + \mathbf{p}_{+}) \cdot \mathbf{r}]}{|\mathbf{r} - \mathbf{r}_{p}|} \right| \Psi_{Z} \right\rangle,$$

where \mathbf{r}_{p} is the coordinate of a proton (any proton) in the nucleus. The exponential in the integrand can be approximated by unity for low-energy pairs. By expanding $|\mathbf{r} - \mathbf{r}_p|^{-1}$ in spherical harmonics and neglecting all multipoles except the monopole term, one obtains

$$\langle \Psi_{Z}, e^{+}, e^{-} | M_{N} | \Psi_{Z} \rangle$$

$$= -\frac{4\pi Z e^{2}}{V} \{ u^{*}(\mathbf{p}_{-}) u(\mathbf{p}_{+}) \} \left[\frac{1}{3} \langle \Psi_{Z} | r_{p}^{2} | \Psi_{Z} \rangle + \frac{1}{|\mathbf{p}_{+} + \mathbf{p}_{-}|^{2} - (E_{+} + E_{-})^{2}} \right], \quad (6)$$

where the first term represents the contribution from the interior volume of the nucleus and the other term, the contribution of the exterior volume. It can be seen that the first term is negligible for low-energy pair pro-

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duction:

$$\langle r_{p^{2}} \rangle / (E_{-} + E_{+})^{-2} \sim R^{2} / (\hbar/mc)^{2} = (m/\mu)^{2} A^{\frac{2}{3}} \leq 10^{-4},$$

where R is the nuclear radius, m is the electron mass, and μ is the π -meson mass. Hence, pairs are created by the nucleus almost exclusively in the exterior region, where the nucleus is effectively a Coulomb field of charge Ze.

The matrix elements for M_{β} are well known⁴:

$$\langle e_1^-, e_2^-, e^+ | M_\beta | e_0^- \rangle$$

= $-\frac{4\pi e^2}{V} \left\{ \frac{\{ \vec{u}(\mathbf{p}_1) \gamma_\nu u(\mathbf{p}_0) \} \{ \vec{u}(\mathbf{p}_2) \gamma_\nu u(-\mathbf{p}_+) \}}{|\mathbf{p}_2 + \mathbf{p}_+|^2 - (E_2 + E_+)^2} \right\}$

- (same but with 1 and 2 interchanged) $\left.\right\}$.

To write down the matrix elements \mathfrak{M}_1 and \mathfrak{M}_2 , we shall make the assumption that we need only consider one intermediate state $\Psi_{Z'}$ or $\Psi_{Z+1'}$ (see Fig. 1); namely, the intermediate state for the nucleus shall be the same as either the initial or the final state. i.e., $\Psi_Z' = \Psi_Z$, and $\Psi_{Z+1}' = \Psi_{Z+1}$. All other intermediate states are neglected by either or both of the following arguments: that they are connected to the initial or final states by radiation of pole orders higher than monopole, or that the overlap between their nuclear wave functions and the initial or final states is negligible. This is the basic assumption for our calculation.

With this approximation, it can be seen that the energy denominator for \mathfrak{M}_1 is $-(E_1+E_2)$, while for \mathfrak{M}_2 it is (E_1+E_2) . Hence \mathfrak{M}_1 and \mathfrak{M}_2 will have opposite signs, and they differ in magnitude only in the fact that \mathfrak{M}_1 is proportional to Z (creation of pair before beta decay), while \mathfrak{M}_2 is proportional to Z+1 (creation of pair after beta decay). Thus

$$\mathfrak{M}_{1} + \mathfrak{M}_{2} = -\frac{\langle \Psi_{Z+1}, e_{1}^{-}, \nu | H_{\beta} | \Psi_{Z} \rangle}{E_{+} + E_{2}} [\langle \Psi_{Z}, e_{2}^{-}, e^{+} | M_{N} | \Psi_{Z} \rangle - \langle \Psi_{Z+1}, e_{2}^{-}, e^{+} | M_{N} | \Psi_{Z+1} \rangle] + (\text{antisymmetric term in } e_{1}^{-} \text{ and } e_{2}^{-})$$

$$= -\frac{4\pi e^2 g\langle K \rangle}{V^2} \left\{ \frac{\{u^*(\mathbf{p}_1) F v^*(\mathbf{p}_\nu)\}\{u^*(\mathbf{p}_2)u(-\mathbf{p}_+)\}}{(E_+ + E_2)[|\mathbf{p}_+ + \mathbf{p}_2|^2 - (E_+ + E_2)^2]} - \text{(same but with 1 and 2 interchanged)} \right\},$$

⁴ See, for example, W. Heitler, Quantum Theory of Radiation (Oxford University Press, Oxford, 1954).

(7)

where we see now that it is not explicitly Z-dependent. We also have

$$\mathfrak{M}_{3} = -\frac{4\pi e^{2}g}{V^{2}} \langle K \rangle \left\{ \sum_{p_{0}} \frac{\{\bar{u}(\mathbf{p}_{2})\gamma_{\nu}u(-\mathbf{p}_{+})\}\{\bar{u}(\mathbf{p}_{1})\gamma_{\nu}u(\mathbf{p}_{0})\}\{u^{*}(\mathbf{p}_{0})Fv^{*}(\mathbf{p}_{\nu})\}}{[(U_{Z}-U_{Z+1})-E_{\nu}-E'][|\mathbf{p}_{2}+\mathbf{p}_{+}|^{2}-(E_{2}+E_{+})^{2}]} - (\text{same but with 1 and 2 interchanged}) \right\},$$

which can be simplified by performing the sum over p_0 by the standard Casimir method⁴:

$$\mathfrak{M}_{3} = -\frac{4\pi e^{2}g}{V^{2}} \langle K \rangle \left\{ \frac{\{\tilde{u}(\mathbf{p}_{1})\gamma_{\nu}(U_{Z}-U_{Z+1}-E_{\nu}+\alpha\cdot\mathbf{p}'+\beta m)Fv^{*}(\mathbf{p}_{\nu})\}\{\tilde{u}(\mathbf{p}_{2})\gamma_{\nu}u(-\mathbf{p}_{+})\}}{[(U_{Z}-U_{Z+1}-E_{\nu})^{2}-E'^{2}][|\mathbf{p}_{+}+\mathbf{p}_{2}|^{2}-(E_{+}+E_{2})^{2}]} - (\text{same but with 1 and 2 interchanged}) \right\}, \quad (8)$$

where U_Z is the nuclear energy for the state Ψ_Z , and

$$\mathbf{p}' = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_+$$

 $\Xi'^2 = \mathbf{p}'^2 + m^2$.

The total matrix element

$$\mathfrak{M} = \mathfrak{M}_1 + \mathfrak{M}_2 + \mathfrak{M}_3 \tag{9}$$

agrees with that derived by Arley and Møller.¹

The physical picture for the pair creation considered can therefore be described as follows: The pairs are created either by the electron from beta decay or by the nucleus. In the former case, the process is just a Møller scattering between an electron in the positive energy region and one in the negative energy sea. There is no explicit Z-dependence aside from that arising from the electron Coulomb wave functions. In the case of pair creation by the nucleus, the mechanism is mainly that of creation by the Coulomb field of the nucleus. Now the nuclear Coulomb field has a charge of Ze before and (Z+1)e after the beta decay. It turns out that the two matrix elements subtract, and the explicit Z-dependence cancels out. The Coulomb field of the nucleus can be imagined to be constantly emitting virtual pairs. However, it normally must reabsorb these pairs (vacuum polarization) because the required energy of $2mc^2$ for real pair production is not available. However, if the nucleus in question undergoes beta decay, it would be possible to "tap" part of the available energy off to release these virtual pairs. As the process is "triggered" by a change of charge of one unit in the nucleus, it does not depend explicitly on Z.

Still another physical way to describe the nuclear creation of pairs would be to say that, because of the beta decay, Z is changed by one unit, and the wave functions for electrons about the nucleus changes. Consequently, the Dirac negative-energy sea needs redefinition, and the Dirac vacuum before the beta decay fails to remain a vacuum afterwards.



The remaining calculations will be outlined briefly. The probability per second of observing one electron of energy E_1 , one electron of energy E_2 and one position of energy E_+ in the intervals dE_1 , dE_2 , dE_+ , respectively, is

$$P(E_1, E_2, E_+) dE_1 dE_2 dE_+$$

$$= 2\pi \int d\Omega_1 d\Omega_2 d\Omega_+ d\Omega_\nu$$

$$\times \int dE_\nu \sum_{\substack{\text{all} \\ \text{spins}}} |\mathfrak{M}|^2 \rho(E_1 E_2 E_+ E_\nu) dE_1 dE_2 dE_+,$$

where \mathfrak{M} is given by (9), $\rho(E_1E_2E_+E_{\nu})$ is the usual density of energy states, and $d\Omega_1$ is a solid angle element for electron 1, etc. Converting from natural units, we have

$$P(E_{1}, E_{2}, E_{+}) dE_{1} dE_{2} dE_{3}$$

$$= \frac{2}{\pi^{5}} \eta^{2} \langle K \rangle^{2} \left(\frac{mc^{2}}{\hbar}\right) \frac{(E_{0} - E)^{2}}{(mc^{2})^{2}} \frac{E_{1}E_{2}E_{+}}{(mc^{2})^{3}}$$

$$\times \frac{p_{1}p_{2}p_{+}}{(mc)^{3}} \frac{dE_{1} dE_{2} dE_{+}}{(mc^{2})^{3}} f(E_{1}E_{2}E_{+}), \quad (10)$$

where

$$\eta = \left(\frac{e^2}{\hbar c}\right) \left(\frac{g}{\hbar c}\right) \left(\frac{mc}{\hbar}\right)^2,$$

 $E_0 = U_Z - U_{Z+1} = mc^2 + (\text{upper limit of beta spectrum}),$ and

$$f(E_{1}E_{2}E_{+}) = \frac{y_{1}y_{2}y_{+}}{[1 - \exp(-y_{1})][1 - \exp(-y_{2})][\exp(y_{+}) - 1]} \\ \times \int \frac{d\Omega_{1}d\Omega_{2}d\Omega_{+}}{(4\pi)^{3}} \sum_{\substack{\text{all} \\ \text{spins}}} \{|X_{12}^{p}|^{2} - 2X_{12}^{p*}X_{12}^{e} \\ -X_{12}^{p*}X_{21}^{e} + |X_{12}^{e}|^{2} \\ + (\text{same but with 1 and 2 interchanged})$$

-(same but with 1 and 2 interchanged)
$$-2[X_{12}{}^{p*}X_{21}{}^{p}+X_{12}{}^{e*}X_{21}{}^{e}]\}, \quad (11)$$

where the factor in front of the integral represents the nonrelativistic Sommerfeld factors for the Coulomb

424

wave functions of the electrons, with

$$y = \frac{2\pi Z}{137} \frac{x}{(x^2 - 1)^{\frac{1}{2}}}, \quad x = \frac{E}{mc^2}.$$
 (12)

The X's in the integrand of (11) are the same as those given by Arley and Møller.¹ Their explicit evaluation requires performing some spin traces by standard methods.

For pure beta decay, the probability per second of observing an electron with energy E in interval dE is

$$W(E)dE = \frac{mc^{2}}{\hbar} \frac{\eta^{2}}{(2\pi)^{3}} \left(\frac{e^{2}}{\hbar c}\right)^{-2} \langle K \rangle^{2} \\ \times \frac{(E_{0} - E)^{2} p E dE}{(mc^{2})^{4} (mc)} \frac{y}{[1 - \exp(-y)]}.$$
 (13)

The electrons observed in the experiment will be due overwhelmingly to those from pure beta decay, since pair creation occurs only once in about 10^9 beta transitions. Hence

$$N^{-} = \frac{2\pi Z}{137} \frac{\langle K \rangle^2}{(2\pi)^3} \frac{mc^2}{\hbar} \left(\frac{\eta}{e^2/\hbar c}\right)^2 H(x_0), \qquad (14)$$

where

$$H(x_0) = \int_1^{x_0} dx \frac{x^2(x_0 - x)^2}{[1 - \exp(-y)]}, \quad x_0 = \frac{E_0}{mc^2}.$$
 (15)

with y defined by (12). From (11) we have

$$N^{+} = \frac{2\eta^{2}}{\pi^{5}} \left(\frac{mc^{2}}{\hbar}\right) \langle K \rangle^{2} \left(\frac{2\pi Z}{137}\right)^{3} G(x_{0}), \qquad (16)$$

where

$$G(x_0) = \int_1^{x_0-2} dx_+ \frac{x_+^2}{\exp(y_+) - 1} F(x_+)$$
(17)

and

$$F(x_{+}) = \int_{1}^{x_{0}-x_{+}-1} dx_{2} \int_{1}^{x_{0}-x_{2}-x_{+}} dx_{1}$$

$$\times \frac{(x_{0}-x_{+}-x_{2}-x_{1})^{2}(x_{1}x_{2})^{2}}{[1-\exp(-y_{1})][1-\exp(-y_{2})]} f(x_{1}x_{2}x_{+}) \quad (18)$$

is the positron spectrum. Hence

$$\frac{N^{+}}{N^{-}} = \frac{16Z^{2}}{(137)^{4}} \frac{G(x_{0})}{H(x_{0})}.$$
(19)

Arley and Møller did not take into account the influence of the Coulomb field on the electron wave functions, and they approximated $f(E_1E_2E_+)$ by a constant 9/128 the limit approached when all particles are emitted at rest. In our calculation, the integrals (15) and (17), aside from containing the Coulomb effect via the nonrelativistic Sommerfeld factor, will be treated more accurately by numerical integration. The only approximation is incurred in the treatment of the angular dependences. A typical angular factor in $f(E_1E_2E_+)$ is of the form

 $[(E_1E_++m^2)-p_1p_+\cos\theta]^{-1}, \quad \theta=$ angle between $\mathbf{p}_1, \mathbf{p}_+,$ which we replace by

$$(E_1E_++m^2)^{-1}\left[1+\frac{p_1p_+\cos\theta}{E_1E_++m^2}\right]$$

The error is estimated to be less than 20%, for the range of energies considered.

We quote the approximate form of $f(x_1x_2x_+)$ used for the numerical computation:

$$f(x_{1}x_{2}x_{+}) = \frac{x_{1}+x_{2}+x_{+}}{4q_{1}^{2}Q^{2}} \left[\left(\frac{2}{x_{+}} - \frac{1}{x_{2}} + x_{2}+2x_{+} \right) + \left(2 - \frac{1}{x_{2}x_{+}} \right) \frac{1}{x_{1}} + x_{1} \right] \\ + \frac{1}{4(x_{1}+x_{+})^{2}q_{1}^{2}} \left(1 - \frac{1}{x_{+}x_{1}} \right) - \frac{x_{2}}{2(x_{1}+x_{+})q_{1}^{2}Q} \left(1 - \frac{1}{x_{+}x_{1}} \right) \\ + \frac{1}{4(x_{1}+x_{+})q_{1}q_{2}Q} \left[\left(x_{2} + \frac{2}{x_{+}} \right) - \left(\frac{x_{2}}{x_{+}} + \frac{2x_{2}+x_{+}}{x_{2}} \right) \frac{1}{x_{1}} + \frac{x_{1}}{x_{2}x_{+}} \right] \\ - \frac{1}{2q_{1}^{2}Q} \left(1 + \frac{1}{2x_{+}x_{1}} \right) + \frac{x_{1}+x_{2}+x_{+}}{4q_{2}^{2}Q^{2}} \left[\left(\frac{2(x_{2}+x_{+})}{x_{2}x_{+}} + x_{2}+2x_{1} \right) - \frac{1+x_{2}x_{+}}{x_{1}x_{2}x_{+}} + x_{1} \right] \\ + \frac{(1-x_{2}x_{+})x_{1}}{2x_{2}x_{+}(x_{2}+x_{+})q_{2}^{2}Q} + \frac{1}{4(x_{2}+x_{+})q_{1}q_{2}Q} \left[\frac{(x_{2}-x_{+})(x_{2}+x_{+})}{x_{1}x_{2}x_{+}} + \frac{2(x_{2}-x_{+})}{x_{2}x_{+}} + \left(1 - \frac{1}{x_{2}x_{+}} \right) x_{1} \right] \\ + \frac{x_{2}x_{+}-1}{4x_{2}x_{+}(x_{2}+x_{+})^{2}q_{2}^{2}} - \frac{2x_{2}x_{+}+1}{4x_{2}x_{+}q_{2}Q} + \frac{1}{2} \frac{2^{-}x_{+}^{2}}{q_{1}q_{2}Q^{2}} \left[\frac{2^{-}x_{+}^{2}}{x_{+}} + \frac{x_{+}^{2}-1}{x_{1}x_{2}x_{+}} \right] \\ - \frac{1}{2q_{1}q_{2}Q} \left(1 - \frac{1}{x_{1}x_{2}} \right) - \frac{1}{8(x_{1}+x_{+})(x_{2}+x_{+})q_{1}q_{2}} \left[1 - \frac{1}{x_{2}x_{+}} + \left(\frac{1}{x_{2}} - \frac{1}{x_{+}} \right) \frac{1}{x_{1}} \right], \quad (20)$$
where

 $x_1 = E_1/m, \quad x_2 = E_2/m, \quad x_+ = E_+/m, \quad q_1 = 1 + x_1x_+, \quad q_2 = 1 + x_2x_+, \quad Q = 1 + x_1x_2 + x_1x_+ + x_2x_+.$

The numerical computation for N^+/N^- is carried out for Z = 16, and for three values of $x_0: x_0 = 7, 5, 4.33$. The last value of x_0 corresponds to the experiment of Greenberg and Deutsch, and yields

$$V^+/N^- = 0.55 \times 10^{-9},$$
 (21)

in good agreement with the experimental value (1). The other values of x_0 do not correspond to any physical case, since they are all calculated for Z=16. They are included to show the dependence of N^+/N^- on the energy available. The numerical results may be tabulated as follows:

x_0	$H(x_0)$	$G(x_0)$	N^+/N^-
7	1001.36	2.926	33.9 ×10⊸
5	173.25	0.0461	3.08×10⊸
4.33	80.25	0.00380	0.55×10⊸

Plots of N^+/N^- and the positron spectrum $F(x_+)$ may be found in the paper of Greenberg and Deutsch² and will not be repeated here.

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Decay of the New Nuclide Ne²⁴^{+*}

B. J. DROPESKY AND A. W. SCHARDT Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received January 12, 1956)

The nuclide Ne²⁴ has been produced by bombarding a neon gas target with 1.5-Mev tritons, the reaction being Ne²²(t,p)Ne²⁴. Beta and gamma scintillation spectrometers with coincidence circuits were used to study this nuclide. Ne²⁴ decays with a half-life of 3.38 ± 0.02 min, emitting negative beta groups of 1.98 ± 0.05 and 1.10 ± 0.05 Mev. This decay is also accompanied by gamma rays of 472 ± 5 and 878 ± 9 kev with relative intensities of 100 to 8, respectively. The excited levels in Na²⁴ corresponding to the above transitions are at 0.472 and 1.35 Mev. The 0.472-Mev level is an isomeric state with a half-life of the order of 20 milliseconds. The spins and parities of the Na²⁴ ground state, 0.472-Mev, and 1.35-Mev levels are 4+, 1+, and 1+, respectively. The Ne^{24} - Na^{24} energy difference is 2.449±0.035 Mev, and the Ne^{24} mass is 24.001195 ± 0.000040 atomic mass units.

INTRODUCTION

HE availability of tritons as bombarding particles has made it possible to add two neutrons to stable nuclides by means of the (t,p) reaction.¹ In the region of light elements several of these isotopes have been observed in this laboratory.² The neon activities, Ne²³ and Ne²⁴, resulting from the (t,p) reaction on neon are relatively easy to study because they can be separated from other activities by passing the gas through a cold charcoal trap. Ne²⁴ was of primary interest because it had not been investigated previously, and because its decay scheme was expected to yield information about the levels of the odd-odd nucleus Na²⁴.

Sheline and co-workers,3 considering the nuclides Si³², Mg²⁸, Ne²⁴, and O²⁰, speculated about the properties

of the then unknown Ne²⁴. They estimated the Ne²⁴ $-Na^{24}$ mass difference to be 4.4 MeV by extrapolating the mass differences of Mg²⁸-Al²⁸ and S³⁶-Cl³⁶. Alternately, predictions of mass differences can be made by employing formulas derived from the uniform model of the nucleus.⁴ These predictions have been relatively successful in the region of medium-light nuclei. Values of 3.3 and 2.7 Mev are predicted for the Ne²⁴-Na²⁴ mass difference, depending on whether the parameters are fixed by the $Mg^{28}-Al^{28}$ or $S^{36}-Cl^{36}$ differences. The discrepancy between these values is typical of shell effects, the magic number 20 being involved in the case of S³⁶. With this amount of energy available, it is to be expected that Ne²⁴ would decay to excited levels⁵ of Na²⁴, since the ground-state transition would involve a spin change of 4. Spin and parity assignments⁶ of 1+ or 2+ have been made to the Na²⁴ level at 1.34 Mev and to one of the levels at 0.472 and 0.564 Mev. It

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^{*} A preliminary report of this work appears in Phys. Rev. 100, 954 (A) (1955). ¹ D. N. Kundu and M. L. Pool, Phys. Rev. 73, 22 (1948).

² The only previous publication resulting from this work refers to F²¹ [N. Jarmie, Phys. Rev. **99**, 1043 (1955)].

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