one-dimensional analysis. However, if the wave has been only slightly slowed down, then the longitudinal E-field component will be small compared to the transverse E-field component, almost all of the wave energy will be outside the dielectric, and interaction of an em wave in the fundamental mode with the electron beam will strongly resemble the idealized situation which has been analyzed. Matching of fields at the moving edge of the beam will, to be sure, require utilization of higher modes of propagation, both in reflection and transmission, but under the stated conditions there will be negligible energy in these higher modes. The slower the em wave, the greater will be the departure of the laboratory experiment from the idealized calculation.

In so far as a laboratory experiment is to be related to the theoretical results obtained in Sec. II, there is a further consideration, namely that the foregoing calculations refer only to steady-state solutions. Actually, there is likely to be a considerable transient following incidence of the wave on the electron beam because of the high dispersion of the electron-gas medium. A transmitted wave inside the gas must, so to speak, "hunt" for its proper, steady-state frequency.

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Thermomolecular Pressure Ratios for He^3 and He^4†

THOMAS R. ROBERTS AND STEPHEN G. SYDORIAK
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

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Thermomolecular pressure ratios of He^3 have been measured for a tube extending from room temperature to a bulb at 2°K. The results agree within experimental accuracy with He^4 ratios calculated from the Weber-Schmidt equation. Detailed calculations from this equation are given. Limitations of the equation due to large deviations from ideal viscosity behavior in the liquid helium range are discussed.

INTRODUCTION

MEASUREMENTS of the vapor pressure of liquid He^3 below 1°K have required a knowledge of the thermomolecular pressure ratio, p_{cold} to p_{warm}. The theory of this ratio has been developed in detail by Weber[1] and applied experimentally to He^4 by him and his collaborators.[2,3] Their research showed that this pressure ratio can be expressed as a function only of the ratios of the tubing radius, R, to λ_c and λ_w, the mean free paths at the cold and warm ends of the tube. Thus in the classical region where the mean free paths of He^4 and He^3 are equal, the thermomolecular pressure ratios also must be equal.

The quantum-mechanical calculations of de Boer and Cohen,[4] confirmed by the experiments of Becker, Misenta, and Schmeissner,[5] have shown that below 5°K the viscosities, and hence the mean free paths, of He^3 and He^4 are very unclassical in behavior. In fact the He^3 viscosity, η_3, is nearly twice as great as the He^4 viscosity, η_4, instead of being equal to $(\frac{3}{4})^{\frac{1}{2}}\eta_4$. Nevertheless, the present measurements show that within the experimental accuracy there is no difference between measured ratios for He^3 and calculated ratios for He^4.

Since He^3 is scarce, vapor pressure thermometry using He^3 may be forced to rely on considerable thermomolecular pressure ratio corrections below 1°K. Therefore we have calculated the ratios in more detail than heretofore available for convenience in making the corrections graphically. The results of the experiments and calculations are given below.

CALCULATED p_c/p_w RATIOS FOR HELIUM FOUR

Weber and Schmidt[3] found that the following equation best fittted their He^4 data. The subscripts c and w designate cold and warm quantities, respectively.

$$\log\frac{p_c}{p_w}=\frac{1}{2}\log\frac{T_c}{T_w}+0.18131\log\frac{y_c+0.1878}{y_w+0.1878}+0.41284\log\frac{y_c+1.8311}{y_w+1.8311}-0.15823\log\frac{y_c+4.9930}{y_w+4.9930}, \tag{1}$$

where

$$y=\frac{R}{\lambda}=\left(\frac{273.15\text{°K}}{T}\right)^{1.147}\frac{Rp}{13.42}. \tag{2}$$

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† Work done under the auspices of the U. S. Atomic Energy Commission.

[1] S. Weber, Leiden Comm. No. **264b** (1936); **264d** (1936), and Suppl. **71b** (1932).

[2] Weber, Keesom and Schmidt, Leiden Comm. No. **246a** (1936).

[3] S. Weber and G. Schmidt, Leiden. Comm. No. **246c** (1936).

[4] J. de Boer and E. G. D. Cohen, Physica **17**, 993 (1951).

[5] Becker, Misenta, and Schmeissner, Z. Physik **137**, 126 (1954).

The units of R are cm and of p are microns of mercury. Equation (1) must be solved by successive approximations since the pressures cannot be separated out easily. The equation has been solved by use of the IBM 701 calculator for various values of T_w, T_c, and Rp_w, with the results shown in Tables I, II, and III. The limiting ratio as $Rp_w \rightarrow 0$ is $(T_c/T_w)^{\frac{1}{2}}$. Curves of p_c/p_w for each pair of T_w, T_c values can be drawn on semilogarithmic paper as a function of $\log(Rp_w)$. Using commercially available 25-cm/cycle paper, one can read p_c/p_w to ± 0.001. Linear interpolation is adequate at either T_w or T_c.

Table I is for use between room temperature and liquid helium temperatures with a constant radius pressure sensing tube. Tables II and III may be used for a stepped pressure sensing tube having a change of radius at liquid air temperatures. Table III also may be used to calculate approximate pressure drops in tubes at liquid helium temperatures according to Eq. (1). For example, from values for $Rp_w = 0.01$ and $T_w = 85$°K, the ratio p_c/p_w for $T_w = 4$°K, $T_c = 1$°K, and $rp_w = 0.002213$ is $0.1142/0.2213 = 0.516$. Such calculations in the liquid helium range should not be very reliable because of the previously mentioned deviations of the viscosity from ideality. The limitations of Eq. (1) for He^3 and He^4 below 4°K will be discussed in a later section.

TABLE I. Calculated p_c/p_w for He^4, room temperature to liquid He temperature.

Rp_w cmμ of Hg	T_c °K	p_c/p_w $T_w = 0$°C	20°C	30°C
$\rightarrow 0$	4	0.1210	0.1168	0.1149
	2.5	0.0957	0.0924	0.0908
	1	0.0605	0.0584	0.0574
	0.5	0.0428	0.0413	0.0406
1	4	0.2330		
	2.5	0.2190	0.2086	0.2038
	1		0.1961	0.1913
	0.5		0.1936	0.1887
1.6	4	0.2895		
	2.5	0.2810	0.2679	0.2618
	1.0		0.2618	0.2557
	0.5		0.2608	0.2547
2.5	4	0.3624		
	2.5	0.3582	0.3424	0.3352
	1		0.3399	0.3326
	0.5		0.3396	0.3322
4.0	4	0.4570		
	2.5	0.4550	0.4375	0.4290
	1		0.4365	0.4281
	0.5		0.4364	0.4280
6.2	4	0.5555		
	2.5	0.5548	0.5360	0.5271
	1−0.5		0.5356	0.5268
10	4	0.6656		
	2.5	0.6654	0.6469	0.6381
	1−0.5		0.6468	0.6380
16	4−0.5	0.7663	0.7500	0.7419
25		0.8468	0.8336	0.8268
40		0.9102	0.9008	0.8962
62		0.9498	0.9435	0.9406
100		0.9754	0.9720	0.9703
160		0.9888	0.9869	0.9862
250		0.9948	0.9940	0.9936
400		0.9978	0.9975	0.9972
160		0.9990	0.9989	0.9989

TABLE II. Calculated p_c/p_w for He^4. Room temperature to liquid air temperature.

Rp_w cmμ	T_c °K	p_c/p_w $T_w = 20$°C	30°C
$\rightarrow 0$	85	0.5385	0.5296
	75	0.5058	0.4974
0.1	85	0.5436	0.5347
	75	0.5114	0.5029
0.16	85	0.5466	0.5376
	75	0.5146	0.5060
0.25	85	0.5508	0.5415
	75	0.5192	0.5104
0.4	85	0.5575	0.5485
	75	0.5265	0.5178
0.62	85	0.5666	0.5577
	75	0.5361	0.5274
1.0	85	0.5806	0.5714
	75	0.5513	0.5423
1.6	85	0.5995	0.5896
	75	0.5717	0.5626
2.5	85	0.6232	0.6140
	75	0.5976	0.5884
4	85	0.6558	0.6462
	75	0.6328	0.6235
6.2	85	0.6944	0.6852
	75	0.6748	0.6658
10	85	0.7464	0.7377
	75	0.7318	0.7231
16	85	0.8054	0.7976
	75	0.7959	0.7881
25	85	0.8624	0.8560
	75	0.8568	0.8504
40	85	0.9140	0.9092
	75	0.9113	0.9065
62	85	0.9497	0.9466
	75	0.9484	0.9453
100	85	0.9745	0.9728
	75	0.9739	0.9722
160	85	0.9879	0.9871
	75	0.9877	0.9868
250	85–75	0.9944	0.9940
400		0.9975	0.9975
620		0.9989	0.9989
1000		0.9996	0.9995

MEASURED p_c/p_w FOR HELIUM THREE

Helium-three thermomolecular pressure ratios were measured by the single-tube method of Weber and Keesom.[6] A vacuum-jacketed Inconel tube of radius 0.211 cm. was connected to an Inconel bulb of 25 cc volume. This volume was held at a constant temperature below the λ point by using a Sommers[7] electronic helium bath regulator. Metered amounts of gas were successively admitted to the system and the equilibrium

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[6] S. Weber and W. H. Keesom, Leiden. Comm. No. **223b** (1932).

[7] H. S. Sommers, Jr., Rev. Sci. Instr. **25**, 793 (1954).

TABLE III. Calculated p_c/p_w for He^4, liquid air temperature to liquid He temperature.

Rp_w cmμ	T_c °K	p_c/p_w	
		$T_w=75$°K	85°K
$\rightarrow 0$	4	0.2309	0.2169
	2	0.1633	0.1534
	1	0.1155	0.1085
	0.5	0.0816	0.0767
0.01[a]	4	0.2358	0.2213
	2	0.1690	0.1585
	1	0.1220	0.1142
	0.5	0.0889	0.0832
0.016[a]	4	0.2386	0.2239
	2	0.1723	0.1614
	1	0.1257	0.1176
	0.5	0.0930	0.0868
0.025[a]	4	0.2428	0.2276
	2	0.1771	0.1657
	1	0.1310	0.1223
	0.5	0.0989	0.0921
0.040[a]	4	0.2494	0.2336
	2	0.1845	0.1725
	1	0.1395	0.1299
	0.5	0.1082	0.1004
0.062[a]	4	0.2586	0.2419
	2	0.1951	0.1820
	1	0.1512	0.1405
	0.5	0.1213	0.1121
0.1[a]	4	0.2732	0.2552
	2	0.2119	0.1972
	1	0.1703	0.1577
	0.5	0.1433	0.1318
0.16[a]	4	0.2942	0.2745
	2	0.2365	0.2196
	1	0.1990	0.1836
	0 5	0.1770	0.1622
0.25[a]	4	0.3226	0.3008
	2	0.2706	0.2508
	1	0.2398	0.2208
	0.5	0.2246	0.2055
0.40[a]	4	0.3651	0.3403
	2	0.3228	0.2990
	1	0.3017	0.2779
	0.5	0.2935	0.2695
0.62[a]	4	0.4202	0.3922
	2	0.3900	0.3621
	1	0.3779	0.3497
	0.5	0.3741	0.3457
1	4	0.5000	0.4687
	2	0.4830	0.4513
	1	0.4775	0.4456
	0.5	0.4761	0.4441
1.6	4	0.5952	0.5620
	2	0.5871	0.5537
	1	0.5850	0.5515
	0.5	0.5845	0.5510
2.5	4	0.6914	0.6592
	2	0.6879	0.6556
	1	0.6871	0.6547
	0.5	0.6869	0.6545
4.0	4	0.7868	0.7585
	2	0.7854	0.7571
	1−0.5	0.7851	0.7568
6.2	4	0.8610	0.8384
	2	0.8605	0.8379
	1−0.5	0.8603	0.8377
10	4	0.9206	0.9050
	2−0.5	0.9204	0.9048
16	4	0.9582	0.9487
	2−0.5	0.9581	0.9486
25	4−0.5	0.9788	0.9734
40	4−0.5	0.9903	0.9876
62	4−0.5	0.9956	0.9942
100	4−0.5	0.9982	0.9976

[a] Equation (1) is not reliable for these entries.

warm pressures measured by a specially designed McLeod gauge with a range of 0.1 to 3400 microns of mercury. The gauge has a 50 cm long capillary. It was calibrated in the range 80 μ to 3000 μ by a mass-balance method. The calibration was checked at 3000 μ against an oil manometer. The corresponding p_c values were calculated and the results of two runs are given in Table IV. About 1.3% of the gas was calculated[8] as being in the Inconel tube, assuming the temperature distribution corresponding to a thermal conductivity proportional to T and a pressure of $(p_w+p_c)/2$.

Adsorption corrections were made based on the data of Keesom and Schweers.[9] For purposes of extrapolation a straight line was drawn through a plot of X *versus* $\log(p/p_0)$ with the equation $X=[39.5+3.1\times\log(p/p_0)]\times10^{-10}$ mole/cm^2. The adsorption correction amounts at most to about 1.5% and leads to a slight improvement in the fit of the He^3 data to Eq. (1).

The two runs in Table IV were taken nearly a year apart with two different McLeod gauge capillaries. The last column gives the percentage difference between the observed p_c/p_w as corrected for adsorption and the He^4 ratio read from a graph of the calculated data of Table I. This percentage error is defined as Δ in Table V and is compared as to algebraic and absolute magnitude averages with measurements on He^4 by other workers. The present work compares favorably in scatter with the measurements of Weber, Keesom, and Schmidt,[2] on which Eq. (1) is based. The other two sets of data show larger systematic deviations.

Only the Weber and Keesom[6] data were obtained by the single-tube method. The other measurements were made by the differential method in which the pressure difference is measured between the warm ends of the capillary tube and a considerably wider tube. The wider tube pressure is corrected by Eq. (1) to find p_c.

Using this differential method, van Itterbeek and

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[8] S. G. Sydoriak and H. S. Sommers, Jr., Rev. Sci. Instr. 22, 915 (1951).

[9] W. H. Keesom and J. Schweers, Physica 8, 1007, 1020 (1941).

de Grande[10] found a consistent difference in thermomolecular differences, $p_w - p_c$, between H_2 and D_2 at T_c values of 20.4° and 14.15°K. However, the maximum observed deviation in p_c/p_w between the two gases was 1.5%. This they attributed to the fact that $\lambda(H_2)/\lambda(D_2)$ is larger than 1, being 1.134 at 20°K and 1.194 at 14°K.

The ratio of mean free paths for He^3 to He^4 is calculated from the data of Becker and his co-workers[5,11] according to the relation $\lambda_3/\lambda_4 = (M_4/M_3)^{\frac{1}{2}}\eta_3/\eta^4$, where M is the atomic weight of the corresponding helium atom. Although the mean free path does not have a clear meaning in the helium temperature range where the quantum statistics are dominant, the ratio calculated by the above classical relation serves as a measure of departure from ideal gas behavior. The results are shown in Table VI. Apparently the large deviations from ideality in the helium temperature range do not measurably change the thermomolecular ratios for He^3, at least for Rp_w values greater than 5 cm micron of Hg. An explanation for this fact is discussed in the next section.

TABLE IV. Experimental helium-3 thermomolecular pressure ratios.

Rp_w cmμ(Hg)	(1) (p_c/p_w) calc	(2) $(p_c/p_w)_{obs}$ Uncorrected for adsorption	(3) $(p_c/p_w)_{obs}$ Corrected for adsorption	$\left(\frac{(3)-(1)}{(1)}\right)$ ×100
		Run I: $T_c = 2.15°K$; $T_w = 301.2°K$		
4.87	0.4730	0.4783	0.4719	−0.2%
8.12	0.5917	0.5939	0.5857	−1.0
10.94	0.6608	0.6683	0.6619	+0.2
13.67	0.7096	0.7155	0.7101	+0.1
18.97	0.7781	0.7834	0.7794	+0.2
24.2	0.8225	0.8206	0.8174	−0.6
29.0	0.8527	0.8573	0.8546	+0.2
33.9	0.8756	0.8804	0.8781	+0.3
43.8	0.9074	0.9079	0.9061	−0.1
53.1	0.9272	0.9347	0.9332	+0.6
63.0	0.9419	0.9448	0.9435	+0.2
89.2	0.9648	0.9654	0.9645	0.0
115.3	0.9761	0.9758	0.9752	−0.1
167.2	0.9870	0.9832	0.9828	−0.4
291	0.9950	0.9944	0.9943	−0.1
409	0.9973	(0.9973)	(0.9973)	
		Run II: $T_c = 1.90°K$; $T_w = 305.7°K$		
8.56	0.6000	0.6004	0.5955	−0.8%
14.8	0.7235	0.7142	0.7095	−2.0
20.25	0.7897	0.7967	0.7930	+0.4
24.7	0.8227	0.8370	0.8338	+1.3
29.0	0.8492	0.8646	0.8620	+1.5
36.4	0.8820	0.8953	0.8932	+1.3
46.8	0.9122	0.9196	0.9179	+0.6
55.5	0.9303	0.9414	0.9400	+1.0
66.4	0.9457	0.9482	0.9470	+0.1
95.6	0.9682	0.9734	0.9726	+0.5
155	0.9846	0.9842	0.9836	−0.1
244	0.9930	0.9908	0.9903	−0.3
390	0.9971	0.9934	0.9933	−0.4
646	0.9990	(0.9990)	(0.9990)	

TABLE V. Summary of average percentage deviations of p_c/p_w from calculated values.

Investigators	R cm	$\Sigma\Delta/n$	$\Sigma\|\Delta\|/n$
Present work, Run I	0.211	−0.05%	±0.30%
Present work, Run II	0.211	+0.24	±0.08
Weber, Keesom, and Schmidt[a]	0.0521	+0.21	±0.50
Weber, Keesom, and Schmidt[a]	0.1000	+0.02	±0.32
Van Itterbeek and de Grande[b]	0.0618	+0.9	±1.1
Van Itterbeek and de Grande[c]	0.0430	+1.3	±1.5
Weber and Keesom[d]	0.1014	−0.9	±1.8

[a] See reference 2.
[b] See reference 10.
[c] One point at $\Delta = +19.6\%$ has been omitted.
[d] See reference 6.

CALCULATION OF p_c/p_w IN LIQUID HELIUM RANGE

According to Weber,[1] the equation for the high-pressure or hydrodynamic region is obtained by integrating the equation

$$dp/dT = (6k_1/R^2)(\eta_2/\rho T), \tag{3}$$

where k_1 is an empirical constant equal to 1.36 for helium four and ρ is the gas density. Since pressures are less than 1 mm, the ideal gas law is adequate for density calculations. For He^4, the measured[5,12] values of viscosity are approximated by $\eta_4 = 2.7T$ (micropoise) while the calculated[4] and measured[5] He^3 viscosities are roughly given by $\eta_3 = 7.0T^{\frac{1}{2}}$ (micropoise). Integration of Eq. (3) under these assumptions from $T_w = 3°K$ to $T_c = 0.5°K$ yields the following for He^4:

$$1-(p_c/p_w)^2 = 0.0124/(Rp_w)^2, \tag{4}$$

and for He^3:

$$1-(p_c/p_w)^2 = 0.0543/(Rp_w)^2, \tag{5}$$

for Rp_w in units of cmμ. For He^4 from 15°K to 480°K, $\eta_4 = 5.00T^{0.647}$ (μ poise) and integration of this value in Eq. (3) would give

$$1-(p_c/p_w)^2 = 0.0255/(Rp_w)^2. \tag{6}$$

Table VII gives ratios calculated from Eqs. (4), (5), and (6) for Rp_w from 10 to 1 cmμ. These equations cease to be good approximations for lower p_c/p_w values. The table shows that differences between He^4 and He^3 ratios of greater than 1% may be expected for $Rp_w < 2$ cmμ. The high temperature viscosity relationship, used in Eqs. (1), (2), and (6) should give reasonably reliable ratios from room or liquid nitrogen temperatures

TABLE VI. Ratio of viscosities and mean free paths of He^3 to He^4.

T°K	η_3/η_4	λ_3/λ_4
20.33	0.906	1.044
14.15	0.900	1.037
4.15	1.233	1.420
2.64	1.718	1.979
1.30	2.073	2.388

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[10] A. Van Itterbeek and E. de Grande, Physica 13, 289 (1947).
[11] E. W. Becker and R. Misenta, Z. Physik 141, 535 (1955).
[12] Van Itterbeek, Schapink, van den Berg, and van Beek, Physica 19, 1158 (1953).

TABLE VII. p_c/p_w calculated from Eqs. (4), (5), and (6). $T_w=3.0°\text{K}$; $T_c=0.5°\text{K}$.

Rp_w cmμ	Eq. (4) (He^4)	Eq. (5) (He^3)	Eq. (6)
10	0.9999	0.9997	0.9999
3	0.999	0.997	0.999
1	0.994	0.973	0.987

down to Rp_c values of about 1 cmμ. Hence the entries below an Rp_w of 1, indicated by superscript a in Table III, should not be relied on very heavily.

CONCLUSION

Experimental measurements have shown that the thermomolecular ratios for He^3 between room and liquid helium temperatures do not differ within experimental error from values calculated from the Weber-Schmidt He^4 equation, Eq. (1), for Rp_w values down to 5 cmμ. Theoretical considerations based on measured viscosities in the liquid hydrogen and helium regions indicate that deviations from these calculated ratios probably will exceed 1% for lower Rp_w values. When very low pressures are reached, the ratios for both gases should theoretically converge to the molecular-flow-region limiting value of $(T_c/T_w)^{\frac{1}{2}}$.

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The Lindemann and Grüneisen Laws[*]

J. J. GILVARRY
The Rand Corporation, Santa Monica, California

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The Lindemann assumption of direct contact of neighboring atoms at fusion is replaced by the criterion that melting occurs when the root-mean-square amplitude of thermal vibration reaches a critical fraction ρ, presumed the same for all isotropic monatomic solids, of the distance of separation of nearest-neighbor atoms. The Debye-Waller theory of the temperature dependence of the intensity of Bragg reflection of x-rays is used, without further assumptions, to derive a generalized Lindemann law. In contrast to the Lindemann form, all physical quantities involved in this formulation are evaluated at the fusion point, and departure of the average energy of an atomic oscillation from the equipartition value is taken into account by the quantization factor of the Debye-Waller theory. If the Grüneisen constant γ_m of the solid at fusion is evaluated by its definition from the Debye frequency of the solid, use of the generalized Lindemann law and Clapeyron's equation permits one to express γ_m in terms of the bulk modulus of the solid at melting and the latent heat and volume change of fusion. By means of Grüneisen's law applied to the solid at fusion, γ_m can be expressed likewise in terms of the corresponding bulk modulus, thermal expansion, volume, and heat capacity at constant volume; the two evaluations of γ_m connect the Lindemann and Grüneisen laws. These relations permit one to evaluate the slope and curvature of a fusion curve as functions of γ_m, and thus to express in terms of γ_m the conditions that a fusion curve be normal in the sense of Bridgman. Experimental fusion data on 13 cubic metals are used to evaluate the constant of proportionality (inversely proportional to ρ) in the Lindemann frequency; the values are reasonably constant. The corresponding values of ρ for Al and Cu show reasonable agreement with values deduced from x-ray intensity measurements. The average ρ seems significantly below values estimated by Grüneisen. Good agreement with the cubic metals is found for hexagonal close-packed elements, but not for elements with more complex lattices, in general. The two evaluations of the Grüneisen constant γ_m of the solid at fusion are shown to be in good agreement, experimentally, for 14 elements checked, but agreement fails for the elements Ga, Bi, and Sb, which show abnormal fusion curves.

THE order-disorder theory of fusion given by Lennard-Jones and Devonshire[1] yields excellent agreement with experiment.[2] The theory has the important consequence that it provides an approximate theoretical justification[1] for Lindemann's theory of fusion,[3,4] beyond the elementary considerations on which the latter theory is based. Lindemann considers a solid composed of simple harmonic oscillators, as in the Grüneisen theory of solids.[5] He assumes further that the oscillators are arranged in a simple cubic lattice, and that any departure of the average energy of an oscillator from the equipartition value can be represented by the Einstein function appearing in the theory of heat capacity of solids. The basic assumption is made that fusion occurs when the amplitude of thermal vibration of the atoms reaches one-half the distance of separation of neighboring atoms less their diameter; that is, when direct contact of neighboring atoms occurs. A second assumption is made, that the Hooke constant of the thermal oscillation (with the mass of the atom) corresponds to the Einstein frequency of the solid as determined by the heat capacity. These hypotheses determine a characteristic frequency as a function of the melting temperature, which shows good numerical agreement with the characteristic frequencies

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