sorptive reaction to be nonexistent within experimental observation. These two conditions allow the use of a simple Born-approximation single-scattering model. The total differential cross section has two terms: the coherent elastic scattering, and the inelastic scattering.

$$\sigma=\{Z(f_C+\delta f_P)+N\delta f_N\}^2F^2 \text{ (elastic)}$$
$$+\{Z(f_C+f_P)^2+Nf_N^2\}(1-F^2) \text{ (inelastic)},$$

where f_C, f_P, and f_N are the Coulomb, proton, and neutron scattering amplitudes (excluding possible charge exchange) and, in general, depend on θ. F^2 is the nuclear form factor and δ^2 is the fraction of no-spin-flip scatterings. We can evaluate the single-nucleon cross section by counting the K^+-scattering events at large angles when ZF^2 and f_C are small. In our calculations of F^2, we have used a Gaussian density distribution to fit electron scattering experiments.[2] Taking all events with $\theta>60°$, the elastic events are computed to be less than 10% of all scatterings. This does not agree with the data as seen in the plates, where an elastic event was defined as one showing no recoil particles and no measurable change in grain density. However, high-energy electron and proton scattering experiments[3,4] have shown a high probability for slightly inelastic events (e.g., excitation of nuclear levels) comparable, at large momentum transfers, with elastic events. Such small energy losses would not have been observed in the emulsion events.

The cross sections in angle and at two different energies are the same within statistics. This is in agreement with S-wave scattering. A P-wave threshold dependence ($\sigma\propto p^4$ or E^2) would give a ratio of 3.6 to 1 in the cross sections for the two energy intervals selected, 30–70 Mev and 70–120 Mev. Over all energies and angles, if one assumes neutron and proton equality in nuclear emulsions, the differential cross section is

$$f_N^2+f_P^2=(6.4\pm1.6)\times10^{-28} \text{ cm}^2/\text{sterad}.$$

We assume a spherically symmetric distribution to obtain the scattering amplitudes at forward angles. The elastic cross section is relatively insensitive to the ratio of f_N to f_P if the latter are of the same sign. To make a specific calculation we assume $f_N/f_P=+\frac{1}{2}$, corresponding to an interaction in a $T=1$ state; this gives a ratio of scattering to charge exchange of 5 to 1, and the observation is that there are only a few possible charge-exchange events.

In order to cumulate the data, we note that the cross section can be expressed to a good approximation as a function of $4p\beta\sin^2(\frac{1}{2}\theta)$,

$$f_C=e^2/[2p\beta\sin^2(\tfrac{1}{2}\theta)].$$

The form factor is a function of the momentum imparted to the scattering center, $2p\sin(\frac{1}{2}\theta)$.

$$4p^2\sin^2(\tfrac{1}{2}\theta)\simeq2mc[2p\beta\sin^2(\tfrac{1}{2}\theta)].$$

This holds fairly well in our case, where $\beta\simeq0.5$. We then predict the number of scatterings (elastic and inelastic)

TABLE I. Summary of results.[a]

$4p\beta\sin^2(\frac{1}{2}\theta)$ (Mev/c)	Equivalent track length (meters)	Number of events Predicted Nuclear attractive	Nuclear repulsive	Observed
10–30	19.0	7.6	49.5	8
30–100	25.2	11.5	25.0	11

[a] The statistical errors in the predicted values are about ±25%, corresponding to the error in the measurement of the single-nucleon cross section.

to be expected in nuclear emulsions for the track length scanned, having due regard for the small-angle cutoffs that would escape detection. This correction is expressed as a reduction in the effective track length scanned. These data include 3 meters of track length examined by the author for small-angle scatterings, as well as the data of the work cited.[1] Table I gives the results for $\delta=1$.

Under the foregoing assumptions, the attractive nuclear force is very much favored. The alternative of assuming a large amount of spin flip to agree with a repulsive force would not give as satisfactory agreement with the data, since the predicted ratio of events in the two angular intervals would still be roughly 2 to 1. If we retained an S-wave interaction, any spin flip would require a vector K-meson. This analysis includes all K^+- and τ^+ particles in the Cosmotron K^+ "beam"; since there is no evidence that these include only one species of particle, these results must be considered as an average effect over all types found.

* This work was supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] Chupp, Goldhaber, Goldhaber, Iloff, Lannutti, Pevsner, and Ritson, University of California Radiation Laboratory Report UCRL-3021 (unpublished). We wish to thank the authors of this article for making their data available before publication.
[2] Hofstadter, Fechter, and McIntyre, Phys. Rev. **91**, 422 (1953); Phys. Rev. **92**, 978 (1953).
[3] J. H. Fregeau and R. Hofstadter, Phys. Rev. **99**, 1503 (1955).
[4] K. Strauch and W. F. Titus, Phys. Rev. **95**, 854 (1954).

Variational Principle in the Hydrodynamical Formulation of the Dirac Field

T. TAKABAYASI
Physical Institute, Nagoya University, Chikusa-ku, Nagoya, Japan
(Received November 21, 1955)

THE Dirac field, which is usually described by a Dirac spinor ψ, can be represented[1] equivalently (at least in c-number theory) by the following set of real tensor quantities[2]:

$$\begin{array}{ll} \text{scalar} & \rho, \\ \text{pseudoscalar} & \theta, \\ \text{vector} & v_\mu, \\ \text{pseudovector} & w_\mu, \\ \text{vector} & k_\mu, \end{array} \qquad (1)$$

which are restricted by the set of subsidiary conditions

$$v_\mu^2=-1,\quad w_\mu^2=1,\quad v_\mu w_\mu=0, \tag{2}$$

and

$$\partial_{[\mu}k_{\nu]}=-(i/2\kappa)\epsilon_{\alpha\beta\gamma\delta}v_\alpha w_\beta(\partial_\mu v_\gamma\partial_\nu v_\delta - \partial_\mu w_\gamma\partial_\nu w_\delta)-(e/mc^2)F_{\mu\nu}, \tag{3}$$

where $F_{\mu\nu}$ is the electromagnetic field strength acting on the Dirac field, $\epsilon_{\alpha\beta\gamma\delta}$ is Levi-Civita's antisymmetric unit pseudotensor, and $\kappa=mc/\hbar$, $\partial_\mu=\partial/\partial x_\mu$, $x_4=ict$.

The Dirac equation can then be replaced[1] equivalently by the following simultaneous equations of motion for the quantities defined by (1):

$$\partial_\mu(\rho v_\mu)=0, \tag{4_1}$$

$$\partial_\mu(\rho w_\mu)=-2\kappa\rho\sin\theta, \tag{4_2}$$

$$w_\mu\partial_\mu\theta+2\kappa v_\mu k_\mu+i\epsilon_{\alpha\beta\gamma\delta}v_\alpha w_\beta\partial_\gamma v_\delta=-2\kappa\cos\theta, \tag{4_3}$$

$$v_\mu\partial_\mu\theta+2\kappa w_\mu k_\mu+i\epsilon_{\alpha\beta\gamma\delta}v_\alpha w_\beta\partial_\gamma w_\delta=0, \tag{4_4}$$

$$v_\nu\partial_\nu(\rho v_\mu)-w_\nu\partial_\nu(\rho w_\mu) =-\partial_\mu\rho-\rho v_{[\mu}w_{\nu]}w_\rho\partial_\nu v_\rho+2i\kappa\rho\epsilon_{\mu\nu\kappa\lambda}v_\nu w_\kappa k_\lambda, \tag{4_5}$$

$$v_\nu\partial_\nu w_\mu=w_\nu\partial_{[\nu}v_{\mu]}+i\epsilon_{\mu\nu\kappa\lambda}v_\nu w_\kappa\partial_\lambda\theta. \tag{4_6}$$

The variables (1) are interpreted[1] as defining a spinning hydrodynamical field. That is to say, v_μ means the 4-velocity, ρ the particle density in the rest frame, w_μ the spin (i.e., intrinsic angular momentum) distribution,[3] k_μ the particle momentum distribution (which is not proportional to v_μ in our case), and θ an extra intrinsic degree of freedom other than spin, representing the possibility of negative mass density. (In the nonrelativistic approximation, θ is interpreted as an internal magnetic pole, which is indeed a pseudoscalar quantity.) The equations of motion (4) then describe the relativistic hydrodynamics for that spinning fluid.

For the interacting Dirac and electromagnetic fields, the foregoing equations are to be combined with the Maxwell equations:

$$\epsilon_{\kappa\lambda\mu\nu}\partial_\lambda F_{\mu\nu}=0, \tag{5_1}$$

$$\partial_\nu F_{\mu\nu}=e\rho v_\mu, \tag{5_2}$$

but then (4_1) becomes a consequence of (5_2), while (5_1) is found to be a consequence of (3). We thus have our new formulation of spinor electrodynamics which consists of the system of equations:

$$[(2),\ (3),\ (4_2)\text{–}(4_6),\ (5_2)], \tag{6}$$

for the basic field variables

$$[\rho,\ \theta,\ v_\mu,\ w_\mu,\ k_\mu,\ F_{\mu\nu}]. \tag{7}$$

The construction above outlined presents a completely gauge-independent and self-contained tensor formulation for the Dirac field, where the ψ spinor and γ matrices are completely suppressed. It describes the motion of a quantum Dirac particle with classical conceptions only.

Now, it is important that our formalism can be derived from a variational principle. For the Lagrangian density, we adopt the following form:

$$\mathfrak{L}=-mc^2\rho\{(v_\mu k_\mu+\cos\theta) +(1/2\kappa)(w_\mu\partial_\mu\theta+i\epsilon_{\alpha\beta\gamma\delta}v_\alpha w_\beta\partial_\gamma v_\delta)\} +\tfrac{1}{2}(mc^2/e)F_{\mu\nu}\{\partial_{[\mu}k_{\nu]}+(i/2\kappa)\epsilon_{\alpha\beta\gamma\delta}v_\alpha w_\beta \times(\partial_\mu v_\gamma\partial_\nu v_\delta-\partial_\mu w_\gamma\partial_\nu w_\delta)\}+\tfrac{1}{4}F_{\mu\nu}^2. \tag{8}$$

Taking variations of (8) with respect to each of the variables (7), we can actually derive (3), (4), and (5). Our Lagrangian (8) implies a new variational principle for spinor electrodynamics, where the "interaction Lagrangian" term involves the coupling constant e as a reciprocal.

Next, we remark that our formulation has, as a natural result of its tensor character, the advantage that it exhibits the covariance properties of the Dirac field with respect to various transformations more simply and manifestly and also in direct physical terms in comparison with the case of the usual ψ formalism. The transformation properties of our field variables for a proper Lorentz transformation and space inversion, given in (1) make the Lagrangian (8) a scalar. Also this specification fits the physical meanings of those variables.

For time reversal, we should stipulate[4]

$$\begin{aligned}&\rho \text{ scalar},\quad \theta \text{ pseudoscalar},\\ &v_\mu,\ w_\mu,\ k_\mu \text{ pseudovector},\\ &F_{\mu\nu} \text{ pseudotensor},\quad (e \text{ invariant});\end{aligned} \tag{9}$$

while charge conjugation is represented by simultaneous transformations[5]:

$$\begin{aligned}&\rho,\ v_\mu \text{ invariant},\\ &eF_{\mu\nu},\ w_\mu,\ k_\mu \text{ change sign},\\ &\theta\rightarrow\theta+\pi.\end{aligned} \tag{10}$$

Finally, our formulation has also the merit that it derives the nonrelativistic approximation to a Dirac particle through a more simple and physically intuitive procedure than that[6] of the usual formalism. Namely, the simple assumption

$$v_k\ll 1$$

in our formulation leads immediately to the hydrodynamical representation of a nonrelativistic spinning particle, which was set up in our previous article.[7]

[1] T. Takabayasi, Progr. Theoret. Phys. (Japan) **13**, 222 (1955); Nuovo cimento (to be published); Soryusiron-Kenkyu **8**, 429 (1955) (in Japanese).

[2] These quantities are related to ψ as follows: $\rho=(\Omega^2+\Omega'^2)^{\frac{1}{2}}$, $\tan\theta=\Omega'/\Omega$, $v_\mu=s_\mu/\rho$, $w_\mu=s_\mu'/\rho$, $k_\mu=(1/2\kappa\rho^2)(\Omega\delta_\mu\Omega+\Omega'\delta_\mu\Omega')-(e/mc^2)A_\mu$, where $\Omega=\bar\psi\psi$, $\Omega'=i\bar\psi\gamma^5\psi$, $s_\mu=i\bar\psi\gamma^\mu\psi$, $s_\mu'=i\bar\psi\gamma^5\gamma^\mu\psi$, $\delta_\mu\Omega=i(\partial_\mu\bar\psi\cdot\psi-\bar\psi\partial_\mu\psi)$, $\delta_\mu\Omega'=-(\partial_\mu\bar\psi\gamma^5\psi-\bar\psi\gamma^5\partial_\mu\psi)$, and A_μ is the electromagnetic potential.

[3] More precisely, the spin is given by the space components w_k multiplied by $i\hbar/(2v_4)$.

[4] This represents the time reversal of Wigner type.

[5] In this case the electromagnetic field is to be considered as an external field.

[6] W. Pauli, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1933), Vol. 24, Part 1, p. 236; L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

[7] T. Takabayasi, Progr. Theoret. Phys. (Japan) **14**, 283 (1955).