Mass Degeneracy of the Heavy Mesons*

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(Received December 29, 1955)

Consequences of the assumption that the θ-τ mass degeneracy is not accidental, but follows from some symmetry principle, are discussed. This assumption implies, among other things, the existence of doublets of opposite parity for all particles of odd strangeness number, such as Λ^0 and $\Sigma^\pm$, Σ^0.

IN recent months, as experimental information accumulates, there is growing evidence[1] of the approximate identity of masses, excitation functions, and apparent lifetimes for the $K_{\pi 2}{}^+ \equiv \theta^+$ and $K_{\pi 3}{}^+ \equiv \tau^+$ particles. On the other hand, there is also growing evidence of the nonidentity of spin-parity properties of the two particles.[1] In regard to the approximate identity of the apparent lifetimes and excitation functions, a suggestion has been made in a recent publication[2] that a genetic relationship between the two particles may lead to such phenomena. We wish to address ourselves here to the problem of the identity of the masses.

It is of course always possible that this mass degeneracy may be entirely accidental.[3] One notices, however, that in all other cases where two different elementary particles have the same or about the same masses, the mass degeneracy is always a consequence of an exact or approximately exact invariance law. Thus, for example, charge conjugation invariance implies the identical mass values of the electron and the positron, and isotopic spin invariance is directly related to the smallness of the neutron-proton mass difference.

We therefore shall make the assumption that the θ^+-τ^+ mass degeneracy is not accidental, but results from certain invariance laws. Such invariance laws would imply, in contrast to the usual invariance laws, symmetries between states of different space transformation properties. The necessity of such laws, however, seems to be strongly suggested by the experimental mass degeneracy of particles of different spin parity.

PARITY CONJUGATION AND PARITY DOUBLETS

In the following, θ^+ and τ^+ are assumed to have the same spin but opposite parity (such as, e.g., $0+$ and $0-$).

The basic point is that if an invariance law is responsible for the mass degeneracy of θ^+ and τ^+, the *minimum symmetry* must contain invariance with respect to the interchange of these two particles.[4] We shall call this interchange "parity conjugation" and shall denote it by C_P. It interchanges the particles τ^+ and θ^+, but leaves the ordinary particles (neutrons, protons, and pions) unchanged.[5] The invariance law states that the part of the Hamiltonian including all strong interactions, called H_s, commutes with the operation C_P:

$$C_P H_s - H_s C_P = 0. \tag{1}$$

The weak interactions give rise to the other part of the Hamiltonian, and do not commute with C_P, producing a small mass difference between τ^+ and θ^+. We shall return to this point later.

Now Eq. (1) implies that for every strong reaction (i.e., fast reaction) there exists a parity-conjugated reaction of equal strength. In particular, for the reaction

$$\pi^+ + n \rightarrow \Lambda_1{}^0 + \theta^+, \tag{2}$$

one obtains a reaction of equal amplitude by taking the parity conjugation of all the particles:

$$\pi^+ + n \rightarrow \Lambda_2{}^0 + \tau^+. \tag{3}$$

Here $\Lambda_2{}^0$ is the parity-conjugated state of $\Lambda_1{}^0$. Corresponding particles in the two reactions have the same spin and orbital states. Therefore $\Lambda_2{}^0$ must have the opposite intrinsic parity to that of $\Lambda_1{}^0$, and consequently must be a different particle.

Extending the foregoing line of reasoning to Σ, one would conclude that there are two types of Σ with opposite parity. In fact, it is evident that all particles

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* Research supported in part by the joint program of the U. S. Atomic Energy Commission and the Office of Naval Research.

[1] Private communications from Berkeley, Columbia, and the Massachusetts Institute of Technology.

[2] T. D. Lee and J. Orear, Phys. Rev. **100**, 932 (1955).

[3] The spatial extension of these particles is small ($\sim 10^{-13}$ cm), so that any internal interaction will be violent ($\sim$ several hundred Mev). A small mass difference of a few Mev would be highly accidental unless there is an invariance law.

[4] The argument behind this statement is as follows: The symmetry must be represented by a group of 2×2 unitary matrices that is irreducible. Furthermore, this group must contain the parity operator which is in this case $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. It then follows that the group must contain an element of the form $\begin{pmatrix}0 & x\\ y & 0\end{pmatrix}$ which will be taken as C_P.

[5] An implicit assumption here is that C_P commutes with the isotopic spin, the charge, the heavy particle number, and the charge conjugations operator. If C_P does not commute with these, by multiplying C_P with a combination of these operations one may obtain an operator C_P' that does. C_P' should then be used as the parity conjugation operator. In all cases where such an operator C_P' does not exist, invariance with respect to C_P would lead to the existence of charged particles with the same mass as Λ^0. These cases would not be considered in detail.

of odd strangeness S defined by[6]

$$S=2(Q-I_3-\tfrac{1}{2}N)$$

must be "parity doublets," i.e., two particles with opposite parity. Furthermore, for any system with odd strangeness the operation C_P always changes it into a system with the opposite parity, while for all systems with even strangeness, the operation C_P leaves the parity invariant. We therefore have[7]

$$C_P\mathrm{P}-(-1)^S\mathrm{P}C_P=0, \quad (4)$$

where P is the parity operator.

For even strangeness, C_P commutes with P. Hence a particle with even strangeness would have a definite parity and a definite value $=\pm1$ for C_P. Since pions can be produced in fast reactions, they must have $C_P=1$. The C_P of nucleons can be chosen arbitrarily because there is conservation of heavy particles. For convenience we choose it to be $+1$. The other known particle of even strangeness, Ξ^-, may therefore have $C_P=\pm1$. In principle the sign here is measurable, but in practice this is very difficult.

The conservation of parity conjugation leads to a new type of selection rule. For example, particles with $S=0$ and $C_P=-1$ could not disintegrate by fast interactions into nucleons and pions. If γ-ray interactions also satisfy the conservation of parity conjugation (see case B in the next section), one would thus have the possibility of new long-lived particles with $S=0$. The metastability of these particles does not derive from the usual strangeness selection rule, but from the conservation of parity conjugation.

PRODUCTION AND DECAY OF Λ_1^0, Λ_2^0, etc.

Reactions (2) and (3) are in conformity with the known selection rules. So are the reactions

$$\pi^+ + n \rightarrow \Lambda_2^0 + \theta^+, \quad (5)$$

and

$$\pi^+ + n \rightarrow \Lambda_1^0 + \tau^+, \quad (6)$$

which are parity-conjugate reactions and consequently have equal cross sections. The ratio of the cross sections for (5) and (2) is not determined by invariance laws. However, whatever value this ratio takes, equal numbers of θ^+ and τ^+ are always produced, and equal numbers of Λ_1^0 and Λ_2^0. In fact, it is evident that the two members of any parity doublet must be produced with equal abundance and must always have the *same excitation function.*

To discuss the decay schemes, we must differentiate between two possibilities:

Case A.—The electromagnetic interaction does not satisfy the conservation law under parity conjugation.[8] In this case the mass difference between the two members of a parity doublet may be expected to be of the order of magnitude of the mass difference between the π^0 and $\pi^\pm$. It could of course be much smaller.

The heavier of the two components of a parity doublet could undergo electric dipole radiative decay and become the lighter one. For example,

$$\Lambda_1^0 \rightarrow \Lambda_2^0 + \gamma, \quad \Sigma_1^+ \rightarrow \Sigma_2^+ + \gamma, \quad \text{etc.} \quad (7)$$

These processes would have very short lifetimes ($\ll 10^{-12}$ sec) unless the particles have zero spin, or happen to have a very small mass difference of the order of a few kev.

If the genetic relationship explanation[2] of the apparently equal life times of τ^+ and θ^+ is correct, τ^+ and θ^+ must not undergo such rapid γ transitions. Therefore they either have zero spin for which single γ decays are forbidden, or have other spins but have a very small mass difference.

Case B.—The electromagnetic interaction also satisfies the conservation law under parity conjugation. In this case the only interactions not conserving parity conjugation are (presumably) of the strength of the weak interactions responsible for the decays. These are extremely weak. The mass difference between the two members of a parity doublet would therefore be exceedingly small (of the order of 10^{-5} ev). The genetic relationship between τ^+ and θ^+ as an explanation[2] of their equal apparent lifetimes is in this case untenable. One would in fact expect that the lifetimes of τ^+ and θ^+ are not exactly identical.

The two particles Λ_1^0 and Λ_2^0 would in this case both decay into $\pi^- + p$ and/or $\pi^0 + n$ with the same Q value. But the wave functions of the decay products are completely different and the lifetimes would be expected to be nonidentical. This applies also to the Σ's.

[6] M. Gell-Mann (to be published), and K. Nishijima, Progr. Theoret. Phys. Japan **12**, 107 (1954).

[7] This commutation relation is to be contrasted with the commutation relation between the parity P and the charge conjugation C: $C\mathrm{P}-(-1)^N\mathrm{P}C=0$, where N is the total number of fermions.

[8] This would be the case if, for example, Λ_1^0 and Λ_2^0 do not have the same magnetic moments.