From the third term the electric quadrupole moment of the spin 5/2 DFP particle is found to be

$$Q_{\frac{5}{2}}=+\left[\frac{28}{15}+\frac{1}{15}\frac{1}{A+\frac{1}{2}}\right]\left(\frac{\hbar}{mc}\right)^2. \qquad (3.17)$$

As a function of A it is seen that $Q_{\frac{5}{2}}$ is singular at the forbidden value $A=-1/2$ but approaches the value $+(28/15)(\hbar/mc)^2$ quite rapidly as one moves away from that value.

CONCLUSIONS

It has been shown that the magnetic moment of a DFP particle of half-integral spin $s>1/2$ is uniquely determined to be $(1/s)(e\hbar/2mc)$. In general, the values of higher moments must be expected to depend on the parameter A. The spin 3/2 quadrupole moment has however the unique value $+(5/3)(\hbar/mc)^2$. The spin 5/2 quadrupole moment depends on the choice of A.

PHYSICAL REVIEW VOLUME 102, NUMBER 1 APRIL 1, 1956

N-V Potential in the Lee Model

STEVEN WEINBERG
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received November 14, 1955)

An investigation is made of states in the Lee model representing a "physical" V-particle bound to an N-particle. It is found that the energy eigenvalue problem can be reduced to a single transcendental equation involving the parity, energy, renormalized coupling, and N-V separation. Only states of odd parity are considered. For such states this equation leads to a single-valued real potential if the renormalized coupling constant is less than the critical value for the appearance of the so-called "ghost" states of the free physical V-particle. For stronger couplings the potential becomes many-valued with half its real branches corresponding to the presence of "normal" V's, and half to "ghost" V's. For still stronger coupling no real energies exist. It is shown that complex energies appear, at least in the case where the virtual θ particles are assumed nonrelativistic and the N-V separation is sufficiently small. A possible reason for the appearance of these difficulties is suggested.

I. INTRODUCTION

In a recent paper, Lee[1] has presented an extremely interesting example of a quantum field theory in which mass and coupling-constant renormalization can be carried out exactly, i.e., without the use of perturbation theory. In the Lee model there are three fictitious, chargeless, spinless particles: N and V are heavy fermions and θ is a light boson. The Hamiltonian[2] for the model is

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}'+\delta\mathcal{H},$$

$$\mathcal{H}_0=m\int d^3\mathbf{p}\psi_V^*(\mathbf{p})\psi_V(\mathbf{p})+m\int d^3\mathbf{p}\psi_N^*(\mathbf{p})\psi_N(\mathbf{p})+\int d^3\mathbf{k}\omega a^*(\mathbf{k})a(\mathbf{k}),$$

$$\delta\mathcal{H}=-\delta m\int d^3\mathbf{p}\psi_V^*(\mathbf{p})\psi_V(\mathbf{p}),$$

$$\mathcal{H}'=-g_0(2\pi)^{-\frac{3}{2}}\int d^3\mathbf{p}d^3\mathbf{p}'d^3\mathbf{k}\delta(\mathbf{p}'+\mathbf{k}-\mathbf{p})\times f(k)(2\omega)^{-\frac{1}{2}}[\psi_V^*(\mathbf{p})\psi_N(\mathbf{p}')a(\mathbf{k})+\psi_V(\mathbf{p})\psi_N^*(\mathbf{p}')a^*(\mathbf{k})], \qquad (1)$$

where g_0 is the unrenormalized coupling-constant, $f(k)$ a high-momentum cut-off function, $\omega=\omega(k)$ the energy of a free θ of momentum $\mathbf{k}$. The field operators ψ_V,ψ_N,a obey the usual commutation (anticommutation) relations for boson (fermion) unrenormalized field operators in a Schrödinger representation. The mass of the N and of the "physical" V is m; $m-\delta m$ is the mass of the "bare" V; and μ is the mass of θ. There is no distinction between "bare" and "physical" N's, θ's, or vacuum. We use units with $\hbar=c=\mu=1$.

In performing the renormalization, one obtains the relations

$$|\gamma/\gamma_0|=1-(\gamma/\gamma_c), \qquad (2)$$

$$\delta m=-\mathcal{B}\gamma/1-(\gamma/\gamma_c), \qquad (3)$$

where

$$\gamma_0\equiv(g_0/2\pi)^2 \quad \gamma\equiv(g/2\pi)^2$$

(g being the renormalized coupling constant), and

$$\gamma_c^{-1}=\int_0^\infty\frac{k^2f^2(k)dk}{\omega^3},\quad \mathcal{B}=\int_0^\infty\frac{k^2f^2(k)dk}{\omega^2}. \qquad (4)$$

Now (2) leads to a contradiction if $\gamma>\gamma_c$. The introduction of an indefinite metric with metric operator

$$\eta=\begin{cases}\exp\left(i\pi\int d^3\mathbf{p}\psi_V^*(\mathbf{p})\psi_V(\mathbf{p})\right) & (\gamma\geq\gamma_c)\\ 1 & (\gamma<\gamma_c)\end{cases}$$

[1] T. D. Lee, Phys. Rev. 95, 1329 (1954).
[2] G. Källén and W. Pauli, Kgl. Danske Videnskab Selskab Mat.-fys. Medd. 30, No. 7 (1955). (The notation of this paper will be adhered to as closely as possible.)

removes this difficulty, replacing (2) by

$$\gamma/\gamma_0=1-(\gamma/\gamma_c). \qquad (2')$$

If $\gamma>\gamma_c$, γ_0 must be negative and $\mathcal{H}'$ non-Hermitian. (Though, of course, $\eta\mathcal{H}'$ is Hermitian.) Also, in addition to the normal physical state of the V-particle, a "ghost" state appears with negative norm and with energy $m+\omega_1$, where $\omega_1(\gamma)<0$ is the root of

$$H(\omega_1)+1/\gamma=0, \qquad (5)$$

$$H(\omega')\equiv\omega'\int_0^\infty \frac{k^2f^2(k)dk}{\omega^3(\omega-\omega')}. \qquad (6)$$

Despite these considerations, Lee asserted that the theory would retain its plausibility even for $\gamma>\gamma_c$, i.e., the energy spectrum would remain real, and the S-matrix would be unitary. Later, it was shown[2] that in the scattering of θ's on V's, transitions of the V's to their "ghost" states would make the S-matrix nonunitary.

We shall show that in the bound states of the N-V system, complex energies appear for large γ; the "ghost" states will again play an important role.

II. SOLUTION OF THE EIGENVALUE EQUATION

If N_V, N_N, N_θ are the operators representing the number of V, N, and θ particles, respectively, then the quantities N_V+N_N and N_N-N_θ are conserved. Therefore let us look for solutions of

$$\mathcal{H}|\omega_0,\mathbf{p}_0\rangle=(2m+\omega_0)|\omega_0,\mathbf{p}_0\rangle, \qquad (7)$$

with the general form

$$|\omega_0,\mathbf{p}_0\rangle=N\left\{\int d^3\mathbf{p}\phi_1(\mathbf{p})|\mathbf{p}_0-\mathbf{p};\mathbf{p};0\rangle + \int\int d^3\mathbf{p}_1d^3\mathbf{p}_2\phi_2(\mathbf{p}_1,\mathbf{p}_2)|0;\mathbf{p}_1,\mathbf{p}_2;\mathbf{k}_{12}\rangle\right\}, \qquad (8)$$

where

$$|\mathbf{p}';\mathbf{p};0\rangle\equiv\psi_V^*(\mathbf{p}')\psi_N^*(\mathbf{p})|0\rangle,$$
$$|0;\mathbf{p}_1,\mathbf{p}_2;\mathbf{k}\rangle\equiv2^{-\frac{1}{2}}\psi_N^*(\mathbf{p}_2)\psi_N^*(\mathbf{p}_1)a^*(\mathbf{k})|0\rangle = -|0;\mathbf{p}_2,\mathbf{p}_1;\mathbf{k}\rangle,$$
$$\mathbf{k}_{12}\equiv\mathbf{p}_0-\mathbf{p}_1-\mathbf{p}_2,$$

and ϕ_1, ϕ_2 are as yet arbitrary, except that ϕ_2 is chosen antisymmetric under exchange of its arguments. The factor $N=g/g_0$ in (8) is for convenience in calculation of the norm of $|\omega_0,\mathbf{p}_0\rangle$. Now substitution of (8) into (7) yields, after a straightforward calculation,

$$0=(\delta m+\omega_0)\phi_1(\mathbf{p}_1)+g_0(2\pi)^{-\frac{3}{2}}\int d^3\mathbf{p}_2\frac{f(k_{12})\phi_2(\mathbf{p}_1,\mathbf{p}_2)}{\sqrt{\omega_{12}}}, \qquad (9)$$

$$(\omega_{12}-\omega_0)\phi_2(\mathbf{p}_1,\mathbf{p}_2)=g_0(2\pi)^{-\frac{3}{2}}[\phi_1(\mathbf{p}_1)-\phi_1(\mathbf{p}_2)]\frac{f(k_{12})}{2\sqrt{\omega_{12}}}. \qquad (10)$$

Since we are concerned with "bound" states, we only look for solutions such that $\omega_0\neq\omega$ for any k; ω_0 may be anywhere on the complex plane except along the real axis from one to infinity. Therefore we can divide (10) by $\omega_{12}-\omega_0$ and, using (2′), (3), and (6), eliminate ϕ_2, obtaining:

$$|\omega_0,\mathbf{p}_0\rangle=\int d^3\mathbf{p}_1\phi_1(\mathbf{p}_1)\left\{N|\mathbf{p}_0-\mathbf{p}_1;\mathbf{p}_1;0\rangle + g(2\pi)^{-\frac{3}{2}}\int d^3\mathbf{p}_2\frac{f(k_{12})|0;\mathbf{p}_1,\mathbf{p}_2;\mathbf{k}_{12}\rangle}{(\omega_{12}-\omega_0)\sqrt{\omega_{12}}}\right\} \qquad (11)$$

$$[H(\omega_0)+\gamma^{-1}]\phi_1(\mathbf{p}_1)=(1/4\pi)\int d^3\mathbf{p}_2\frac{f^2(k_{12})\phi_1(\mathbf{p}_2)}{\omega_0\omega_{12}(\omega_{12}-\omega_0)}. \qquad (12)$$

The latter integral equation can easily be solved. Transforming from momentum to position coordinates, we let

$$\phi(\mathbf{r})=(2\pi)^{-\frac{3}{2}}\int d^3\mathbf{p}\phi_1(\mathbf{p})\exp[i\mathbf{r}\cdot(\mathbf{p}-\tfrac{1}{2}\mathbf{p}_0)], \qquad (13)$$

and obtain

$$[H(\omega_0)+\gamma^{-1}]\phi(\mathbf{r})=\lambda(r,\omega_0)\phi(-\mathbf{r}). \qquad (14)$$

$$\lambda(r,\omega_0)\equiv(r\omega_0)^{-1}\int_0^\infty\frac{kf^2(k)\sin kr\,dk}{\omega(\omega-\omega_0)}. \qquad (15)$$

Thus, for a given ω_0 and γ, $\phi(\mathbf{r})=0$ except on those spherical shells of radius r, where

$$H(\omega_0)+\gamma^{-1}=\pm\lambda(r,\omega_0) \qquad (16\pm)$$

the parity of $\phi(\mathbf{r})$ on the shell being even or odd according to whether (16+) or (16−) holds. Except for a factor of two, r can be interpreted as the separation of the N- and V-particles in this state. The delta-function behavior of $\phi^2(\mathbf{r})$ arises from the neglect of recoil for the N- and V-particles, and allows us to speak of ω_0 as a potential energy.

Because of the rather complicated nature of the λ function, it will be necessary in analyzing (16±) to use some properties of λ which do not necessarily hold for all reasonable $\omega(k)$ and $f(k)$. The special cases that will be referred to are as follows:

$$f(k)=1,\quad \omega(k)=(1+k^2)^{\frac{1}{2}}, \qquad (\mathrm{R})$$

$$f(k)=1,\quad \omega(k)=1+\tfrac{1}{2}k^2. \qquad (\mathrm{NR})$$

Case (NR) is roughly equivalent to (R) modified by a cutoff at about $k=1$, but avoids the introduction of a cutoff parameter. In case (R), $\gamma_c=0$; while in case (NR), $\gamma_c\neq0$. In both cases the integrals representing the λ and H functions converge. It is to be expected that any other reasonable choice of $\omega(k)$ and $f(k)$ would lead to final results qualitatively similar to those obtained below for cases (R) and (NR).

III. REAL ENERGIES

The following properties of λ and H will be used. (See Figs. 1 and 2.) Assuming $\omega_0<1$, we have

$$\omega_0 H(\omega_0)>0, \tag{17}$$

$$H(0)=0, \tag{18}$$

$$\lim_{\omega_0\to-\infty} H(\omega_0)=-\gamma_c^{-1}, \tag{19}$$

$$\frac{d}{d\omega_0}H(\omega_0)>0, \tag{20}$$

$$\omega_0\lambda(r,\omega_0)>0, \tag{21}$$

$$\lambda(r,\omega_0)\to\pm\infty \text{ for } \omega_0\to 0\pm,\ r \text{ fixed}, \tag{22}$$

$$\lambda(r,\omega_0)\to 0 \text{ for } \omega_0\to-\infty,\ r \text{ fixed}, \tag{23}$$

$$\lambda(r,\omega_0)\to 0 \text{ for } r\to\infty,\ \omega_0 \text{ fixed}, \tag{24}$$

$$(\partial/\partial r)\lambda(r,\omega_0)>0 \text{ for } \omega_0<0. \tag{25}$$

For any fixed r, the function $\lambda+H$ attains a maximum value $-\gamma_m^{-1}(r)$ in the range $\omega_0<0$, and

$$\gamma_m(r)>\gamma_c. \tag{26}$$

Equations (17), (18), (19), (20), (23), (24) hold in general, i.e., for any reasonable choice of the functions $\omega(k)$ and $f(k)$, and follow directly from (15) and (6). Equations (21) and (25) hold in cases (R) and (NR). (See Appendix and Sec. IV.) Equation (22) holds whenever (21) holds. It is easy to see that (26) will follow from (17), (18), (19), (21), (22), and (23), if we can show that $\lambda+H>-\gamma_c^{-1}$ for some finite $\omega_0<0$. For $\gamma_c=0$ this is trivial. If $\gamma_c\neq 0$ and $\mathcal{B}$ is finite [as in case (NR)], then, as $\omega_0\to-\infty$,

$$\lambda\to-(r\omega_0^2)^{-1}\int_0^\infty \frac{kf^2(k)\sin krdk}{\omega},$$

while

$$H\to-\gamma_c^{-1}-\mathcal{B}\omega_0^{-1}.$$

Thus

$$\lambda+H+\gamma_c^{-1}\to-\mathcal{B}\omega_0^{-1}>0,$$

and Eq. (26) is therefore proven.

Now, using (5), (20), (21), and (17), we see that (16+) cannot be satisfied for real ω_0 unless $\omega_0<\omega_1$ or $\omega_0>0$, and that (16−) cannot be satisfied for real ω_0 unless $\omega_1<\omega_0<0$. Thus, for a given real ω_0 and γ, $\phi(\mathbf{r})$ will have the same parity on all shells where it does not vanish; in other words, the parity operator not only commutes with $\mathcal{H}$, but is a function of it. Only states of odd parity ($\omega_1<\omega_0<0$) will be considered. For such states, there can only be one shell on which $\phi(\mathbf{r})$ does not vanish. [See (25).] The norm of such a state can be shown (see Appendix) to be

$$-\partial/\partial\omega_0(H+\lambda), \tag{27}$$

except for positive factors. Now for fixed r,

$$\lambda+H+\gamma^{-1}\to\begin{cases}\gamma^{-1}-\gamma^{-1} & (\omega_0\to-\infty)\\ -\infty & (\omega_0\to 0-).\end{cases} \tag{28}$$

For $\gamma<\gamma_c$, we do not use the indefinite metric, so there can only be states of positive-definite norm. But it is clear from (27) and (28) that if for any fixed r and fixed $\gamma<\gamma_c$ there is more than one root of (16−), then some of the roots will correspond to states of negative or zero norm. Thus for $\gamma<\gamma_c$ we have a single-valued potential $\omega_0(r,\gamma)$.

For $\gamma>\gamma_c$ the situation is not so simple. Discarding for the moment the possibility of states with zero norm, we see from (27) and (28) that there must be an even

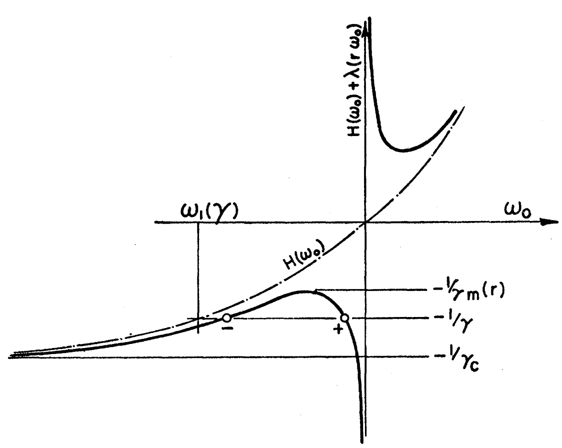

FIG. 1. Schematic representation of the eigenvalue problem for odd parity and fixed r. A typical γ with $\gamma_c<\gamma<\gamma_m(r)$ is shown; the circled intersection points correspond to states with norm ± 1 and real ω_0. If $\gamma<\gamma_c$ there will be one real ω_0 with positive norm, and if $\gamma>\gamma_m(r)$ there will be no real ω_0.

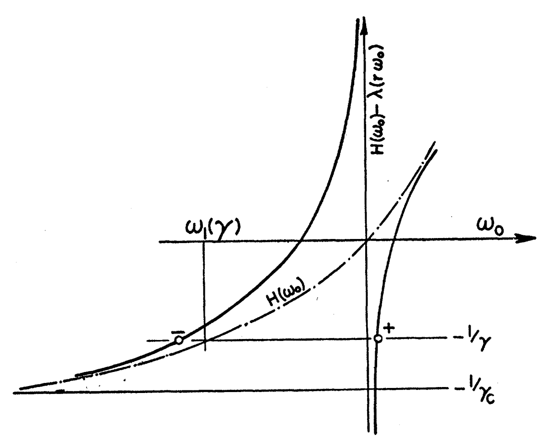

FIG. 2. Schematic representation of the eigenvalue problem for even parity and fixed r. A typical γ with $\gamma_c<\gamma<\gamma_m(r)$ is shown; the circled intersection points correspond to states with norm ± 1 and real ω_0. It is apparent from Figs. 1 and 2 that no mixing of even and odd parities can occur in a stationary state of real energy.

number $2n$ of real roots of (16−); if labeled $\omega_0^{(s)}$ with $\omega_0^{(1)}<\omega_0^{(2)}<\cdots<\omega_0^{(2n)}$, then $\omega_0^{(s)}$ will correspond to a state of norm $(-1)^s$. If γ is small enough, then $n\geq 1$. [See (26).] The physical significance of the negative-norm states can be seen by letting r go to infinity; then for γ sufficiently small we will have just two states, one with $\omega_0=0$ and positive norm, the other with $\omega_0=\omega_1(\gamma)$ and negative norm. Thus the states of negative norm represent a "ghost" V bound to an N.

Now, if γ is slightly smaller than $\gamma_m(r)$ there must exist just two states, one with positive and one with negative norm. As γ is increased, the two states will approach each other in energy, merging when $\gamma=\gamma_m(r)$ into one state of energy $\omega_0(r)$ and zero norm. If $\gamma>\gamma_m(r)$, there can be no states with that r and real ω_0.

IV. COMPLEX ENERGIES

The situation described in the last paragraph leads to the supposition that there must exist complex roots of (16−) when $\gamma>\gamma_m(r)$. In addition to the disappearance of real roots for such γ, note that the norm of any state of complex energy must be zero, since

$$\langle\omega_0|\eta\mathcal{H}|\omega_0\rangle=(\omega_0+2m)\langle\omega_0|\eta|\omega_0\rangle$$

must be real. Thus the existence of complex ω_0 would ensure continuity of the norm at $\gamma=\gamma_m(r)$.

To simplify the actual demonstration of these complex energies as much as possible, we consider the case (NR), with r assumed to be very small. In this case the integrations in (4), (6), (15) are easy to perform, and we obtain

$$a\mathcal{B}=\tfrac{1}{2},\quad a\gamma_c^{-1}=\tfrac{1}{8},$$
$$a\lambda(r/\sqrt{2},\omega_0)=(r\omega_0^2)^{-1}[e^{-rZ}-e^{-r}],$$
$$a[H(\omega_0)+\gamma^{-1}]=(1-Z)\omega_0^{-2}-\tfrac{1}{2}\omega_0^{-1}+\Delta,$$
$$\Delta\equiv a(\gamma^{-1}-\gamma_c^{-1}),$$
$$a\equiv(\pi\sqrt{2})^{-1},$$
$$Z\equiv(1-\omega_0)^{\frac{1}{2}}.$$

These results hold for all ω_0 and r, except of course for $\omega_0\geq 1$. When ω_0 is complex, Z must be taken as having positive real part. (It is easy to see that (21) and (25) now hold.)

Now if $r=0$, Eq. (16−) becomes

$$Z^3+Z^2+[(2\Delta)^{-1}-1]Z-[3(2\Delta)^{-1}+1]=0.\quad(29)$$

The solution of this equation is now perfectly straightforward, except that care must be taken to reject as spurious all roots with $\mathrm{Re}(Z)\leq 0$. As expected, we obtain for $\gamma\leq\gamma_c$ one real ω_0. For $\gamma_m(0)\geq\gamma\geq\gamma_c$, there are two real ω_0 merging when $\gamma=\gamma_m(0)$ into one real ω_0. For $\gamma>\gamma_m(0)$, there are two complex ω_0, which are not complex conjugates. The quantity $\gamma_m(0)$ is given by

$$\gamma_m(0)=\gamma_c(1+8\Delta_+)^{-1}>\gamma_c,$$

where $\Delta_+\simeq-6.7\times10^{-3}$ is the greater value of Δ for which the discriminant of (29) is zero.

It is now obvious that for any $\gamma>\gamma_m(0)$ we will have complex ω_0 if r is chosen sufficiently small.

V. DISCUSSION

That the Hamiltonian in a theory with indefinite metric is not Hermitian makes it impossible to carry out the standard proofs of the unitarity of the S-matrix[3] and the reality of the energy eigenvalues. These conditions, which are essential for a physically plausible theory, must rather be tested directly for each individual problem. It has been previously shown[2] that in the Lee model the S-matrix does in fact become nonunitary for a renormalized coupling γ larger than a critical value γ_c; we now also see that there exist in the Lee model certain "bound" states of complex energy if the coupling becomes greater than a second critical value $\gamma_m(r)$.

It seems a reasonable conjecture that these difficulties will arise in any theory in which energy eigenstates of negative norm appear which are not degenerate with similar energy eigenstates of positive norm. In the case of quantum electrodynamics this is not the case, for the scalar photons are degenerate with the longitudinal and transverse photons, and so there is room for a subsidiary condition (the Lorentz condition) which describes how these states are to be mixed to form "physical" states of positive norm only. As is well known, the use of the indefinite metric in quantum electrodynamics does not lead to the sort of difficulties described here. In the Lee model, however, no "ghost state" of an N-V bound system of negative parity has the same energy as any similar odd parity state of different r, or as any similar even parity state.

These results serve to emphasize once again the extreme care that must be taken if any departure from the usual formalism of quantum mechanics is to be made.

VI. APPENDIX

A. Calculation of the Norm

Consider two states of the form (11) differing only in their total momenta, and let ω_0 be real. Then the scalar product of these states is

$$M\equiv\langle\omega_0\mathbf{p}_0'|\eta|\omega_0\mathbf{p}_0\rangle$$
$$=\int\int d^3\mathbf{p}_1'd^3\mathbf{p}_1\phi_1^*(\mathbf{p}_1')\phi_1(\mathbf{p}_1)$$
$$\times\Big\{|N|^2\langle\mathbf{p}_0'-\mathbf{p}_1';\mathbf{p}_1';0|\eta|\mathbf{p}_0-\mathbf{p}_1;\mathbf{p}_1;0\rangle$$
$$+g^2(2\pi)^{-3}\int\int d^3\mathbf{p}_2'd^3\mathbf{p}_2$$
$$\times\frac{f(k_{12}')f(k_{12})\langle0;\mathbf{p}_1',\mathbf{p}_2';\mathbf{k}_{12}'|\eta|0;\mathbf{p}_1,\mathbf{p}_2;\mathbf{k}_{12}\rangle}{(\omega_{12}\omega_{12}')^{\frac{1}{2}}(\omega_{12}'-\omega_0)(\omega_{12}-\omega_0)}\Big\}$$

[3] C. Møller, Kgl. Danske Videnskab Selskab Mat.-fys. Medd. **23**, No. 1 (1945); **22**, No. 19 (1946).

Now, using the properties of η and noting that N is imaginary for $\gamma \geq \gamma_c$ and real for $\gamma \leq \gamma_c$, we have, for any γ,

$$M=\delta(\mathbf{p}_0-\mathbf{p}_0')\left\{\int d^3\mathbf{p}\,|\phi_1(\mathbf{p})|^2 \times\left[N^2+(\gamma/4\pi)\int d^3\mathbf{k}\frac{f^2(k)}{\omega(\omega-\omega_0)^2}\right] -(\gamma/4\pi)\int\int d^3\mathbf{k}d^3\mathbf{p}\phi_1^*(\mathbf{p}_0-\mathbf{p}-\mathbf{k})\phi_1(\mathbf{p})\frac{f^2(k)}{\omega(\omega-\omega_0)^2}\right\}$$

$$=\gamma\delta(\mathbf{p}_0-\mathbf{p}_0')\left\{\int d^3\mathbf{r}\,|\phi(\mathbf{r})|^2 \times\left[\gamma^{-1}-\gamma_c^{-1}+\int_0^\infty \frac{k^2f^2(k)dk}{\omega(\omega-\omega_0)^2}\right] -(1/4\pi)\int d^3\mathbf{r}\phi^*(\mathbf{r})\phi(-\mathbf{r})\int d^3\mathbf{k}\frac{f^2(k)e^{-i\mathbf{r}\cdot\mathbf{k}}}{\omega(\omega-\omega_0)^2}\right\}.$$

Since $\phi(\mathbf{r})=\pm\phi(-\mathbf{r})$, we have

$$M=\gamma\delta(\mathbf{p}_0-\mathbf{p}_0')\int d^3\mathbf{r}\,|\phi(\mathbf{r})|^2\left\{\gamma^{-1}-\gamma_c^{-1} +\int_0^\infty\frac{k^2f^2(k)dk}{\omega(\omega-\omega_0)^2}\mp\frac{1}{r}\int_0^\infty\frac{kf^2(k)\sin krdk}{\omega(\omega-\omega_0)^2}\right\}d^3\mathbf{r}.$$

If we assume that (16±) can only be satisfied for one value of r, the norm must have the same sign as the quantity

$$\gamma^{-1}-\gamma_c^{-1}+\int_0^\infty\frac{k^2f^2(k)dk}{\omega(\omega-\omega_0)^2}\mp\frac{1}{r}\int_0^\infty\frac{kf^2(k)\sin kr}{\omega(\omega-\omega_0)^2}dk,$$

and, using (16±), this becomes

$$\omega_0\frac{\partial}{\partial\omega_0}[H(\omega_0)\mp\lambda(r,\omega_0)],$$

which reduces in the case of odd parity and negative ω_0 to expression (27).

B. Relativistic λ Function

In case (R) we have $f^2(k)=1$, $\omega(k)=(1+k^2)^{\frac{1}{2}}$, so that

$$\lambda(r,\omega_0)=(2ir\omega_0)^{-1}\int_{-\infty}^{\infty}\frac{ke^{ikr}dk}{\omega(\omega-\omega_0)}.$$

Let C be a closed contour enclosing the upper half of the complex plane except for that part of the imaginary axis from $z=i$ to $z=i\infty$. Then for all ω_0 with nonzero real part,

$$\lambda(r,\omega_0)-\lambda_0(r,\omega_0)=(2ir\omega_0)^{-1}\oint_C\frac{ZdZe^{irZ}}{\omega(Z)[\omega(Z)-\omega_0]},$$

$$\lambda_0(r,\omega_0)=-\frac{1}{r}\int_0^\infty\frac{\exp[-r(1+u^2)^{\frac{1}{2}}]du}{\omega_0^2+u^2}.$$

Thus, for real ω_0 we have

$$\lambda(r,\omega_0)=\begin{cases}\lambda_0(r,\omega_0) & (\omega_0<0)\\ \lambda_0(r,\omega_0)+(\pi/\omega_0 r)\times\exp(-r(1-\omega_0^2)^{\frac{1}{2}}) & (0<\omega_0<1).\end{cases}$$

It is now obvious that (25) and (21) hold for $\omega_0<0$. For $1>\omega_0>0$ note that

$$\lambda_0(r,\omega_0)>-\frac{e^{-r}}{r}\int_0^\infty\frac{du}{\omega_0^2+u^2}=\frac{-\pi}{2\omega_0 r}e^{-r},$$

$$\lambda(r,\omega_0)>\frac{-\pi}{2\omega_0 r}e^{-r}+\frac{\pi}{\omega_0 r}\exp[-r(1-\omega_0^2)^{\frac{1}{2}}]>0,$$

and therefore (21) also holds for $1>\omega_0>0$.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Dr. Gunnar Källén for suggesting this problem, and for many valuable discussions. He also wishes to thank Professor Niels Bohr for the hospitality of the Institut for Teoretisk Fysik of Copenhagen, where this work was begun.

This research was performed during the tenure of a National Science Foundation predoctoral fellowship.