Compton Scattering and Bremsstrahlung of Spin-3/2 Particles

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The Fierz-Pauli theory of particles of spin-$\frac{3}{2}$ in interaction with the electromagnetic field is used to calculate the cross section in Born approximation for Compton scattering by such particles. The bremsstrahlung cross section is estimated by the method of virtual quanta. The magnitude of these cross sections rules out the possibility that the mu meson is such a particle.

I. INTRODUCTION

A THEORY of particles of arbitrary spin has been developed by Dirac[1] and by Fierz.[2] Fierz and Pauli[3] have extended the theory to cover the interaction of such particles with the electromagnetic field, and Gupta[4] has pointed out that the theory for spin-$\frac{3}{2}$ particles can be put in a form which strongly resembles the familiar Dirac theory for electrons, and that calculations can be made by the same techniques as are available for problems involving photons and electrons. In particular, Feynman's[5] rules for calculating matrix elements in quantum electrodynamics still apply, with a few obvious modifications.

In Part II of this paper, the Fierz-Pauli-Gupta theory is briefly reviewed. In Part III the analog of the Klein-Nishina formula for Compton scattering is calculated. Owing to the complexity of the problem, the cross section is only calculated to highest order in the energy of the incident photon. We have calculated exact matrix elements, but a great deal of labor would be involved in combining these into a cross section. In Part IV the bremsstrahlung of spin-$\frac{3}{2}$ particles is estimated by the Weizsäcker-Williams method[6] of virtual quanta.

This work was motivated by the possibility that the mu meson might have a spin of $\frac{3}{2}$. In Part V it is pointed out that the calculations of this paper make such a possibility highly unlikely.

II. FIERZ-PAULI-GUPTA THEORY OF SPIN-3/2 PARTICLES

In this paper we shall set $\hbar=c=1$, and also use the Feynman summation convention on repeated indices:

$$a\cdot b\equiv a_\mu b_\mu\equiv a_4b_4-a_1b_1-a_2b_2-a_3b_3.$$

Gupta[4] writes the spinor equations of reference 3 in the form

$$i\alpha_\mu\nabla_\mu\psi=m\psi, \tag{1}$$

where $\nabla_\mu\equiv(-\boldsymbol{\nabla},\,\partial/\partial t)$ and the α's are 16×16 matrices given in reference 4, except that our α_1, α_2, α_3 are i times Gupta's matrices. Interaction with the electromagnetic field is described in the usual way by replacing (1) by

$$\alpha_\mu(i\nabla_\mu-eA_\mu)\psi=m\psi,$$

where A_μ is the electromagnetic potential four-vector:

$$A_\mu\equiv(\mathbf{A},\varphi).$$

Calculations of electromagnetic processes may be made by Feynman's rules, except that the particle propagator in momentum space is now

$$i(\not{p}-m)^{-1},$$

where $\not{p}\equiv p_\mu\alpha_\mu$, p_μ being the energy-momentum four-vector of the virtual particle. By using the relation[4]

$$\sum(\alpha_\mu\alpha_\nu-\delta_{\mu\nu})\alpha_\sigma\alpha_\rho=0, \tag{2}$$

where $\sum$ means a summation over all permutations of the indices μ, ν, σ, ρ, and $\delta_{\mu\nu}$ is the usual Kronecker delta except that $\delta_{11}=\delta_{22}=\delta_{33}=-1$, we see that

$$\not{p}^4=p^2\not{p}^2\quad(p^2=p\cdot p=E^2-\mathbf{p}^2),$$

and from this it follows that

$$i(\not{p}-m)^{-1}=i(\not{p}^2+m^2-p^2)(\not{p}+m)/m^2(p^2-m^2). \tag{3}$$

Such a "rationalized" propagator, with no matrices in the denominator, is of course needed for calculations.

A wave function describing free particle motion of momentum p in the z-direction is

$$\psi=u\exp[i(pz-Et)]. \tag{4}$$

Since it follows from (1) and (2) that ψ must satisfy the Klein-Gordon equation

$$\nabla_\mu\nabla_\mu\psi=-m^2\psi,$$

p and E in (4) must satisfy the familiar relation $E^2=p^2+m^2$. We shall only be interested in positive energy solutions, with

$$E=+(p^2+m^2)^{\frac{1}{2}}.$$

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) **A155**, 447 (1936).
[2] M. Fierz, Helv. Phys. Acta **12**, 3 (1939).
[3] W. Pauli and M. Fierz, Helv. Phys. Acta **12**, 297 (1939); M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939).
[4] S. N. Gupta, Phys. Rev. **95**, 1334 (1954).
[5] R. P. Feynman, Phys. Rev. **76**, 749 (1949); Phys. Rev. **76**, 769 (1949).
[6] E. J. Williams, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **13**, No. 4 (1935); C. F. V. Weizsäcker, Z. Physik **88**, 612 (1934).

Substituting (4) into (1), we see that the constant column vector u must satisfy the equation

$$(E\alpha_4 - p\alpha_3 - m)u = 0.$$

Four linearly independent solutions are

u_a: $u_{a1}=x$, $u_{a9}=-y$, all other $u_{ai}=0$,
u_b: $u_b=y$, $u_{b14}=-x$, all other $u_{bi}=0$,
u_c: $u_{c3}=y^3$, $u_{c5}=x\sqrt{2}$, $u_{c11}=-x^3$, $u_{c13}=-y\sqrt{2}$, all other $u_{ci}=0$,
u_d: $u_{d2}=-y\sqrt{2}$, $u_{d4}=-x^3$, $u_{d10}=x\sqrt{2}$, $u_{d12}=y^3$, all other $u_{di}=0$,

where u_{a1}, u_{a2}, $\cdots$, u_{a16} denote the elements of the column vector u_a reading from top to bottom in Gupta's representation, and x and y are defined by

$$x = +(E-p)^{\frac{1}{2}}/m^{\frac{1}{2}}, \quad y = +(E+p)^{\frac{1}{2}}/m^{\frac{1}{2}}.$$

(In the nonrelativistic limit, these four states have the spin projections $S_z = -\frac{3}{2}, +\frac{3}{2}, +\frac{1}{2}, -\frac{1}{2}$, respectively.) The normalization is

$$\rho = \tilde{u}\alpha_4 u = u^*\eta\alpha_4 u = 2E/m \text{ for } u_a \text{ and } u_b$$
$$= 6E/m \text{ for } u_c \text{ and } u_d.$$

ρ is the fourth component of the current four-vector, i.e., the particle density. u^* denotes the Hermitian conjugate of u, and $\tilde{u}$ is the "adjoint" of u, defined by

$$\tilde{u} = u^*\eta.$$

The matrix η is given in reference 2.

III. CALCULATION OF CROSS SECTION FOR COMPTON SCATTERING

We shall assume our particle to be initially at rest, and choose the z-axis along the direction of particle recoil after collision. (See Fig. 1.) The four-momenta of the incident photon, scattered photon, initial particle and recoil particle are given by

$$\mathbf{q}_1 = \omega_1\alpha_4 - \omega_1 \sin\theta_1\alpha_1 + \omega_1 \cos\theta_1\alpha_3,$$
$$\mathbf{q}_2 = \omega_2\alpha_4 - \omega_2 \sin\theta_2\alpha_1 - \omega_2 \cos\theta_2\alpha_3,$$
$$\mathbf{k}_1 = m\alpha_4,$$
$$\mathbf{k}_2 = E\alpha_4 - p\alpha_3,$$

respectively. Conservation of energy and momentum implies that

$$\mathbf{k}_1 + \mathbf{q}_1 = \mathbf{k}_2 + \mathbf{q}_2.$$

If we denote incident and final photon polarizations by $\mathbf{e}_1$ and $\mathbf{e}_2$, there are four possibilities:

$$\mathbf{e}_1 = \begin{cases} \alpha_2 \\ \alpha_1 \cos\theta_1 + \alpha_3 \sin\theta_1, \end{cases} \quad \mathbf{e}_2 = \begin{cases} \alpha_2 \\ \alpha_1 \cos\theta_2 - \alpha_3 \sin\theta_2. \end{cases}$$

However, it is well known (and can be demonstrated either directly or by gauge-invariance arguments) that we may replace $\mathbf{e}_i$ by $\mathbf{e}_i + a\mathbf{q}_i$, where a is an arbitrary

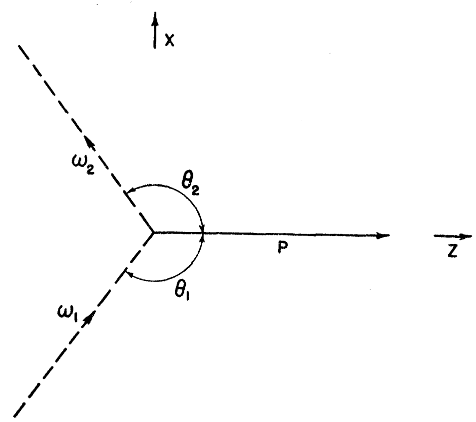

FIG. 1. Coordinate system and angles used in calculations.

constant, without changing our results. If we use this fact and employ the polarizations

$$\mathbf{e}_1 = \begin{cases} \alpha_2 \\ \mathbf{e}_1' \end{cases} \quad \text{and} \quad \mathbf{e}_2 = \begin{cases} \alpha_2 \\ \mathbf{e}_2' \end{cases},$$

where

$$\mathbf{e}_1' = \alpha_3\omega_1 + \alpha_4\omega_1 \cos\theta_1 \quad \text{and} \quad \mathbf{e}_2' = -\alpha_3\omega_2 + \alpha_4\omega_2 \cos\theta_2,$$

a considerable amount of algebra is avoided, a linear combination of α_3 and α_4 being easier to handle than a linear combination of α_3 and α_1.[7]

The cross section for Compton scattering in the Born approximation is

$$d\sigma = 2\pi \times \tfrac{1}{8} \sum_{e_1 u_i} \sum_{e_2 u_f} |(\tilde{u}_f M u_i)|^2 N^{-1} \rho(E).$$

N is the product of the normalizations used in the wave functions, and it must also include a factor $(-e_1' \cdot e_1') = \omega_1^2 \sin^2\theta_1$ if $e_1 = e_1'$, and a factor $(-e_2' \cdot e_2') = \omega_2^2 \sin^2\theta_2$ if $e_2 = e_2'$. $\rho(E)$ is the usual density of states for Compton scattering. The matrix M is the sum of two contributions, one from each diagram for the process. (See Fig. 2.) Applying the usual Feynman rules, we have

$$M = -4\pi i e^2 (M_{\mathrm{I}} + M_{\mathrm{II}}),$$

where

$$M_{\mathrm{I}} = \mathbf{e}_2 (\mathbf{k}_1 + \mathbf{q}_1 - m)^{-1} \mathbf{e}_1,$$

and

$$M_{\mathrm{II}} = \mathbf{e}_1 (\mathbf{k}_1 - \mathbf{q}_2 - m)^{-1} \mathbf{e}_2.$$

Making use of (3), we write

$$\begin{aligned} 2m^3\omega_1 M_{\mathrm{I}} &= \mathbf{e}_2((\mathbf{k}_1+\mathbf{q}_1)^3 + m(\mathbf{k}_1+\mathbf{q}_1)^2 \\ &\qquad - 2m\omega_1(\mathbf{k}_1+\mathbf{q}_1) - 2m^2\omega_1)\mathbf{e}_1 \\ &= \mathbf{e}_2 N_{\mathrm{I}} \mathbf{e}_1, \\ -2m^3\omega_2 M_{\mathrm{II}} &= \mathbf{e}_1((\mathbf{k}_1-\mathbf{q}_2)^3 + m(\mathbf{k}_1-\mathbf{q}_2)^2 \\ &\qquad + 2m\omega_2(\mathbf{k}_1-\mathbf{q}_2) + 2m^2\omega_2)\mathbf{e}_2 \\ &= \mathbf{e}_1 N_{\mathrm{II}} \mathbf{e}_2, \end{aligned}$$

[7] I am indebted to R. P. Feynman for making this suggestion.

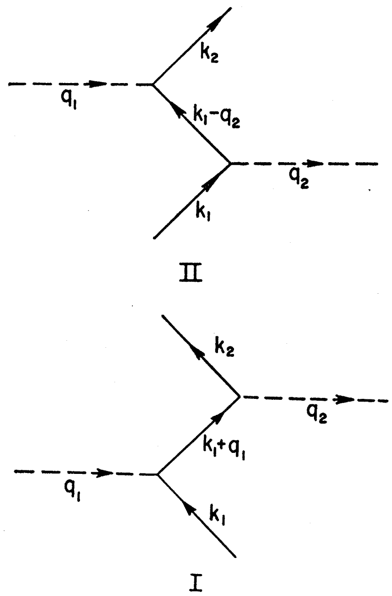

FIG. 2. Feynman diagrams involved in Compton scattering.

where we have used the relations

$$(k_1+q_1)\cdot(k_1+q_1)=m^2+2m\omega_1,$$
$$(k_1-q_2)\cdot(k_1-q_2)=m^2-2m\omega_2.$$

At this point the 64 quantities $(\tilde{u}_f\mathbf{e}_2N_{\mathrm{I}}\mathbf{e}_1u_i)$ and the 64 quantities $(\tilde{u}_f\mathbf{e}_1N_{\mathrm{II}}\mathbf{e}_2u_i)$ were calculated. A great deal of labor would have been involved in combining these directly into a cross section, so the assumption $\omega_1\gg m$ was made, and only the leading terms in each transition amplitude were retained. In order to facilitate comparison with the usual formulas for the Compton effect, angles θ and φ, defined by

$$\theta\equiv\theta_1+\theta_2-\pi,\quad \varphi\equiv\pi-\theta_1$$

were introduced. (See Fig. 3.)

From the well-known relation

$$\omega_2=m\omega_1/[m+\omega_1(1-\cos\theta)],$$

we see that when $\omega_1\gg m$ two cases must be distinguished:

$$\text{(A) } 1-\cos\theta\gg m/\omega_1,$$
$$\text{(B) } 1-\cos\theta\sim m/\omega_1.$$

Cross sections valid in each region were found to be

$$\text{(A) } d\sigma=(1/162)(e^2/m)^2(\omega_1/m)[\csc^6(\theta/2)+9\csc^4(\theta/2)+9\csc^2(\theta/2)]d\Omega,$$
$$\text{(B) } d\sigma=(4/81)(K^2+2K+2)(K+1)^{-5}\times(e^2/m)^2(\omega_1/m)^4d\Omega, \quad (5)$$
$$K\equiv(1-\cos\theta)\omega_1/m.$$

As has been emphasized before, only leading terms in ω_1/m are given. It will be noted that (B), with $K\to\infty$, and (A), with $\theta\to 0$, give the same cross section.

The total cross section computed from (5) is

$$\sigma=(2\pi/27)(e^2/m)^2(\omega_1/m)^3. \quad (6)$$

Corresponding quantities for spin-$\frac{1}{2}$ electrons are obtained from the well-known Klein-Nishina formula:

$$\left.\begin{array}{l}\text{(A) } d\sigma=\frac{1}{4}(e^2/m)^2(m/\omega_1)\csc^2(\theta/2)d\Omega,\\ \text{(B) } d\sigma=\frac{1}{2}(e^2/m)^2(K^2+2K+2)(K+1)^{-3}d\Omega,\end{array}\right. \quad (7)$$

$$\sigma=\pi(e^2/m)^2(m/\omega_1)\ln(2\omega_1/m). \quad (8)$$

The important feature of (5) and (6) is the rapid increase of cross section with increasing photon energy at high energies, especially when compared with the Klein-Nishina results in (7) and (8).

IV. VIRTUAL-QUANTA ESTIMATE OF BREMSSTRAHLUNG

We wish to estimate the bremsstrahlung cross section for a spin-$\frac{3}{2}$ particle of mass m, charge e, and total energy $E\gg m$, encountering a stationary nucleus of mass M and charge Ze, to produce a photon of energy between ϵE and $(\epsilon+d\epsilon)E$. Since it is the fast, light particle which will produce most of the radiation, we will consider the Lorentz system in which the particle m is at rest while the nucleus M is moving past with energy $E^*=(M/m)E$. The highly flattened Coulomb field of the nucleus will contain, at a radial distance r, a flux of $N(\omega)d\omega$ photons per second per cm^2 of energies between ω and $\omega+d\omega$, where

$$N(\omega)\approx\begin{cases}0 & \text{for } rM>E^*/\omega,\\ (Z^2e^2/\pi^2)(\omega r^2)^{-1} & \text{for } rM<E^*/\omega.\end{cases} \quad (9)$$

In particular, we consider the Compton scattering of one of these photons ω_1 through an angle θ by the particle m. The following relations are easily obtained:

$$\begin{aligned}\omega_2&=(1-\epsilon)\omega_1,\\ 1-\cos\theta&=\epsilon m/\omega_2=(m/\omega_1)(1-\epsilon)^{-2}d\epsilon,\\ d\Omega&=2\pi d(\cos\theta)=2\pi(m/\omega_1)(1-\epsilon)^{-2}d\epsilon.\end{aligned} \quad (10)$$

Then the bremsstrahlung cross section will be

$$d\sigma_b=\int d\omega_1\int 2\pi r dr\left(\frac{d\sigma}{d\Omega}\right)_{\text{Compton}}\times 2\pi\left(\frac{m}{\omega_1}\right)(1-\epsilon)^{-2}d\epsilon\times\left(\frac{Z^2e^2}{\pi^2}\right)\left(\frac{1}{\omega_1 r^2}\right).$$

The limits on the radial integral are easily specified: $r_{\min}=r_n$, the nuclear radius $(\sim m^{-1})$; for $r<r_n$, the assumption of a Coulomb field is certainly not valid, and virtual photons (if any) in this region are ignored. $r_{\max}=E/\omega_1 m$, since for $r>E/\omega_1 m$, $N(\omega_1)=0$ by (9).

The limits on the integration over ω_1 were chosen as follows. If $\omega_1>E/r_n m$, then $N(\omega_1)=0$. Therefore

$(\omega_1)_{\max} \leq E/r_n m$. However, because of the extreme energy dependence of our Compton cross sections, it was decided to ignore virtual photons of energy greater than Am; for high energies ω_1 the cross sections (5) become so large that the Born approximation is certainly no longer valid. To arrive at a reasonable cut-off parameter A,[8] we observe that in the coordinate system in which the total momentum is zero, the angular dependence of the Compton cross section is no longer so sharply peaked in the forward direction, but is fairly smooth. Therefore it appears that only low values of angular momentum (or low multipole orders) contribute, so that the total cross section should be of the order of $\pi\lambda_c^2$, where $\lambda_c \equiv \omega_c^{-1}$ is $(2\pi)^{-1}$ times the wavelength of the radiation in the "center of momentum" system. We shall impose on ω_1 the restriction

$$\sigma(\omega_1) < \pi/\omega_c^2.$$

Using (6) for $\sigma(\omega_1)$ and the relation

$$\omega_c = (m\omega_1/2)^{\frac{1}{2}}$$

for ω_c, we obtain roughly $\omega_1 < 25m$. We shall take $A = 25$; we believe this to be a quite conservative estimate.

From (10), $(\omega_1)_{\min} \geq \frac{1}{2}\epsilon m(1-\epsilon)^{-1}$. If $\epsilon < 0.8$, however, $\frac{1}{2}\epsilon m(1-\epsilon)^{-1} < 2m$. Since our formulas (5) are not expected to retain much validity at such low energies, we choose $(\omega_1)_{\min}$ thus

$$(\omega_1)_{\min} = \begin{cases} \frac{1}{2}\epsilon m(1-\epsilon)^{-1} & \text{if } \epsilon > 0.8, \\ fm & \text{if } \epsilon < 0.8, \end{cases}$$

where $f \approx 2$.

For $(d\sigma/d\Omega)_{\text{Compton}}$ we shall take the second equation of (5), since for $K \gg 1$ the cross section is very small anyway. Furthermore, we approximate this expression by

$$\frac{d\sigma}{d\Omega} \approx \left(\frac{8}{81}\right)(1-\epsilon)^4\left(\frac{e^2}{m}\right)^2\left(\frac{\omega_1}{m}\right)^4.$$

This is exact at $\epsilon = 0$, good to $\sim 5\%$ at $\epsilon = 0.2$, good to $\sim 50\%$ at $\epsilon = 0.6$, and is an underestimate for all $\epsilon \neq 0$.

We then find, for $\epsilon < 0.8$,

$$\frac{d\sigma_b}{d\epsilon} = \left(\frac{32}{243}\right)\bar{\varphi}(1-\epsilon)^2\left(A^3 \ln\left(\frac{E}{Am}\frac{1}{r_n m}\right) + \frac{A^3}{3} - f^3 \ln\left(\frac{E}{fm}\frac{1}{r_n m}\right) - \frac{f^3}{3}\right)$$

$$\approx (32/243)A^3\bar{\varphi}(1-\epsilon)^2[\ln(E/10^{10}\,\text{ev}) + \tfrac{1}{3}] \quad E \geq 10^{10}\,\text{ev},$$

where $\bar{\varphi}$, as in Heitler,[9] is defined by $\bar{\varphi} \equiv Z^2 e^2 (e^2/m)^2$, and we have set $r_n m \approx 4$.

[8] The following estimate of A was suggested by R. F. Christy.

[9] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954).

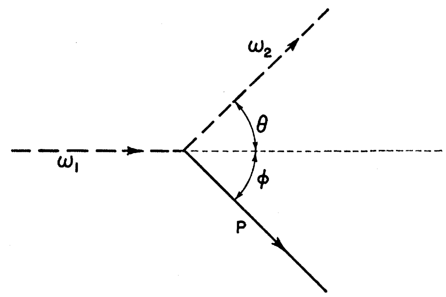

FIG. 3. Compton scattering in lab system.

The cross section for production of bremmstrahlung photons ϵE with $0.2 < \epsilon < 0.8$ is

$$\int_{0.2}^{0.8}\left(\frac{d\sigma_b}{d\epsilon}\right)d\epsilon \approx 3.5 \times 10^2 \times \bar{\varphi}[\ln(E/10^{10}\,\text{ev}) + \tfrac{1}{3}]. \quad (11)$$

For spin-$\frac{1}{2}$ particles at high energy, Christy and Kusaka[10] give

$$\left(\frac{d\sigma_b}{d\epsilon}\right)_{\frac{1}{2}} = \tfrac{2}{3}\bar{\varphi}\epsilon^{-1}(3\epsilon^2 - 4\epsilon + 4)\left[2\ln\left(\frac{2E}{m}\frac{1-\epsilon}{\epsilon}\frac{1}{r_n m}\right) - 1\right],$$

from which we find, approximately,

$$\int_{0.2}^{0.8}\left(\frac{d\sigma_b}{d\epsilon}\right)_{\frac{1}{2}} d\epsilon \approx 5.4\,\bar{\varphi}[\ln(E/10^{10}\,\text{ev}) + 4.3]. \quad (12)$$

For energies $E \approx 10^{10}$ ev, (11) is about 5 times as large as (12); for 10^{11} ev the ratio is about 25.

V. DISCUSSION

The calculations of Christy and Kusaka[11] show that the observed frequency of cosmic-ray burst production at sea level is accounted for quite satisfactorily by bremsstrahlung and knock-on electrons from spin-$\frac{1}{2}$ mesons of energies $\sim 10^9 - 10^{11}$ ev, provided we use the modern value of the mu-meson mass. The bremsstrahlung cross section obtained in Part IV of this paper exceeds the corresponding spin-$\frac{1}{2}$ cross section by a factor of at least 3 throughout most of the photon spectrum at $E = 10^{10}$ ev. At $E = 10^{11}$ ev, the ratio is well over 10 throughout most of the spectrum, and the rise with energy continues rapidly. Since the spin-$\frac{1}{2}$ bremsstrahlung in this energy range accounts for nearly all ($\approx 95\%$) of the bursts, the observations are clearly not in accord with the mu meson's being a spin-$\frac{3}{2}$ particle of the type discussed in this paper. Such mesons, arriving at sea level with the observed mu-meson flux would produce far too many bursts. There-

[10] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 405 (1941).

[11] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941).

fore the possibility that the mu meson is a spin-$\frac{3}{2}$ particle describable by the Fierz-Pauli-Gupta theory of such particles must be ruled out.

ACKNOWLEDGMENTS

The author is indebted to Professor R. P. Feynman for suggesting this problem and for considerable help with its solution, and to Professor R. F. Christy for many helpful discussions.

APPENDIX

This investigation originated in a conjecture of Professor Feynman that the masses of particles might arise entirely from virtual interactions with other fields. This might easily be the case for the particles with strong interactions, but the muon has no such interactions. Since ordinary spin-$\frac{1}{2}$ theory appears to be unable to account for a large electromagnetic self-mass, it was felt that a spin-$\frac{3}{2}$ theory should be investigated in connection with the muon.

The self-mass is

$$\Delta m=(E/m)(4\pi e^2/i)(\tilde{u}u)^{-1}(2\pi)^{-4} \times\int d^4kc(k)^2k^{-2}(\tilde{u}\alpha_\mu(p-k-m)^{-1}\alpha_\mu u),$$

$$(\tilde{u}u)=2E/m.$$

Since the integral diverges quadratically, we choose the convergence factor $C(k^2)$ to be

$$C(k^2)=[-\lambda^2/(k^2-\lambda^2)]^2.$$

The result, retaining only terms in λ^2, is

$$\Delta m=(5e^2/18\pi)(\lambda/m)^2m.$$

Feynman has suggested[12] that this result, rewritten in the form

$$\Delta(m^2)=(5/9\pi)e^2\lambda^2,$$

remains valid even if $m=0$. The entire muon mass could then be accounted for electromagnetically, if the cutoff λ were given a value equal to several proton masses.

[12] In the case of spin zero, the analogous formula can be shown to remain valid when $m=0$.

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Quantized Field Theory in the Hamilton-Jacobi Formalism*

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A method is developed, and applied to the free Klein-Gordon field, for modifying the author's Hamilton-Jacobi formalism for classical field theory in such a way as to describe field quantization. Although the modified Hamilton-Jacobi equations, as well as the explicit, modified, nonlinear field equations do not contain functionals of the field variables on a space-like surface, but only functions of the field variables at a space-time point, they are implicitly tied to families of space-like surfaces through a normal vector field which cannot be eliminated. Furthermore, the modified Hamilton-Jacobi equations in each case can be found only from the Schrödinger equation for the quantized field system, not directly by a uniform method of generalizing the classical equations. The consistency of the present method with the usual formalism of field quantization is shown by means of a continuity equation, but no explicit solutions, which would exhibit the presence of particle-like quanta, are given. Problems peculiar to fermions are not discussed.

INTRODUCTION

A POSSIBLE description of the effects of field quantization by a classical field theory is presented. This is done by modifying the Hamilton-Jacobi formalism for classical field theory,[1] previously developed by the author, in a manner analogous to that used by Bohm[2] to obtain, from the Schrödinger equation for a particle, a modified Hamilton-Jacobi equation which in principle describes the motion of the particle in the sense of classical mechanics. A preliminary application to field theory was already given by Bohm[3] in the case of the electromagnetic field. Bohm's treatment is not relativistic, but could be made so by rather trivial generalizations. A more serious point is that his Hamilton-Jacobi function S characterizes the entire field system at a time t (relativistically: on a space like surface σ). It is thus not a function of the field variables at a single space-time point, but rather, a functional of the field variables on a space-like surface. Bohm's S-function for fields satisfies functional rather than field equations. It was shown in CFHJ that the basic

* Read at the New York meeting of the American Physical Society, January 30 to Februrary 4, 1956 [Bull. Am. Phys. Soc. Ser. II, 1, 47 (1956)].

[1] H. Freistadt, Phys. **97**, 1158 (1955), hereafter quoted as CFHJ. The notation of CFHJ is used throughout. Equations are quoted as CFHJ (1), CFHJ (2),$\cdots$.

[2] D. Bohm, Phys. Rev. **85**, 166 (1952).

[3] D. Bohm, Phys. Rev. **85**, 180 (1952).