# Spin of the $\Lambda^0$ Particle

MALVIN RUDERMAN AND ROBERT KARPLUS
*Physics Department, University of California, Berkeley, California*
(Received December 7, 1955)

An analysis of the mesonic and nonmesonic decay of hyperfragments shows that the spin of the $\Lambda^0$ particle is 1/2 or 3/2. If the spin is 3/2, the parity is that of the proton.

## INTRODUCTION

AMONG the early suggestions to explain the long lifetime of the new heavy mesons and hyperons was the hypothesis that they possess a high spin.[1] The high centrifugal barrier which the decay products had to penetrate could be very effective in reducing the transition probability. However, in correlating and predicting strange particle decays and production, this kind of model has not proved nearly as fruitful as those based upon isotopic spin.[2] The relationship of the ordinary to the isotopic spin depends upon the detailed structure of such models. In particular, the usual "strangeness" classification leaves entirely open the assignment of ordinary spins and parities among the strange particles.

For the $\tau$ meson, which has a three-pion decay, it seems possible to infer the spin and parity from the angular and energy distribution of the decay mesons.[3] No such inference is possible for those strange particles which are defined by a two-particle decay such as $\Lambda^0 \rightarrow p+\pi^-$. Here measurement of angular correlation between the $\Lambda^0$ decay and its line of flight or its plane of production have been suggested as possible ways to obtain spin and parity information,[4,5] but experimental data are far from definitive. In eight examples of $\pi^- + p \rightarrow \Lambda^0 + \theta^0$ observed in the Brookhaven pion beam, the $\Lambda^0$ decay plane is always within 45° of the production plane.[6] On the other hand, no such correlation has been found for $\Lambda^0$ particles produced by cosmic-ray particles in heavier nuclei.[7] If such a high degree of correlation is borne out, it would imply a high spin $>9/2$.

The existence of bound $\Lambda^0$ hyperons[8] (hyperfragments) suggests a simple way to decide the $\Lambda^0$ parity (relative to a proton) and to estimate its spin. Such hyperfragments decay either by complete conversion of the $\Lambda^0$—nucleon mass difference into nucleon kinetic energy or through pion emission. The former mechanism dominates for all nuclei with $Z>2$. It has been pointed out by Cheston and Primakoff[9] that this is expected because of the high probability for internal conversion of the pion by neighboring nucleons. This stimulated process is much more effective than self-absorption in contributing to the nonmesonic decay. However, the ratio of mesonic to nonmesonic decay turns out to be such a very sensitive function of the orbital momentum of the pion-proton in the decay $\Lambda^0 \rightarrow p+\pi^-$ that the orbital momentum can be inferred from present data with only an approximate knowledge of the interaction of the pion field with nuclear matter.

This possibility is quite analogous to the use of the internal conversion coefficient to determine the multipolarity of a nuclear electromagnetic transition,[10] except that the energy is transferred to nucleons via a mesonic field instead of being transferred to electrons via the electromagnetic field. For a radiator with frequency $\omega$, the radial dependence of the radiated wave at large distances is $e^{i\omega t}e^{ikr}/r$, where $k=(\omega^2-\mu^2)^{\frac{1}{2}}$ and $kr\gg 1$. Close to the source, the wave amplitude becomes large like $1\cdot 1\cdot 3\cdots(2l-1)/r(kr)^l$, where $l$ is the multipolarity of the radiation. Because of the high Fourier components of such a rapidly varying field, the results for internal conversion of the $\Lambda^0$ are relatively insensitive to any assumed correlations between the $\Lambda^0$ and any of the nucleons in the fragment. It is the very large nucleon mass relative to the energy transferred (~180 Mev) that makes this hyperfragment internal conversion so very sensitive to multipolarity.

The contributions of self-absorption to nonmesonic decays are estimated from data on photomeson production. The partial lifetime for the mesonic decay of a bound $\Lambda^0$ is also lengthened by the restrictions of the exclusion principle on the possible states for the residual proton. A precise estimate of these effects is not essential to the conclusion that $l=1$ or 0 in $\Lambda^0 \rightarrow p+\pi^-$.

---

[1] E. Fermi and R. P. Feynman (unpublished); see M. Gell-Mann and A. Pais, *Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, London and New York, 1955), p. 342; and R. Gatto, Nuovo cimento **1**, 378 (1955).

[2] M. Gell-Mann and A. Pais, reference 1; M. Goldhaber, Phys. Rev. **92**, 1279 (1953) and Phys. Rev. **101**, 433 (1956); A. Salam and J. C. Polkinghorne, Nuovo cimento **2**, 685 (1955).

[3] R. H. Dalitz, Phil. Mag. **44**, 1068 (1953); R. H. Dalitz, Phys. Rev. **94**, 1046 (1954); E. Fabri, Nuovo cimento **11**, 479 (1954).

[4] R. Adair, Phys. Rev. **100**, 1540 (1955).

[5] Treiman, Reynolds, and Hodson, Phys. Rev. **97**, 244 (1955); S. P. Treiman, Phys. Rev. **101**, 1217 (1956).

[6] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **98**, 121 (1955); W. D. Walker (private communication).

[7] J. D. Sorrels, *Proceedings of the Fifth Annual Rochester Confereuce* (Interscience Publishers, Inc., New York, 1955).

[8] M. Danysz and J. Pniewski, Phil. Mag. **44**, 348 (1953); Fry, Schneps, and Swami, Phys. Rev. **99**, 1561 (1955); and see bibliography in R. Gatto, reference 1.

[9] H. Primakoff and W. B. Cheston, Phys. Rev. **92**, 1537 (1953). This effect has also been considered by Pais and Serber and by Goldhaber, Brueckner, and Goldberger.

[10] J. M. Blatt and V. G. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 614.

## CALCULATION

Since only the ratio of the rate of internal conversion to the rate of ordinary decay is needed, the absolute value of the interaction between the hyperon and the nucleon and meson fields is immaterial. The principal feature of the interaction that we shall employ is its invariance under spatial rotation. The results are independent of the specific description used here but depend only upon the angular momenta. Since the heavy particles are nonrelativistic, we can describe the proton of mass $M$ by a two-component operator $\psi_\nu^{(p)}(\mathbf{r})$ $(\nu=\frac{1}{2}, -\frac{1}{2})$ and the $\Lambda^0$ particle with spin $j$ and mass $M'$ by a $(2j+1)$-component operator $\psi_\mu^{(\Lambda)}(\mathbf{r})$ $(-j\leq\mu\leq j)$. We shall write the interaction operator, with units such that $\hbar=c=\text{mass}_\pi=1$,[11]

$$H_i=\int d\mathbf{r}\,\psi_\nu^{(p)*}(\mathbf{r})d^3\mathbf{r}'T_{\nu\mu}(\mathbf{r}-\mathbf{r}')\phi(\mathbf{r}')\psi_\mu^{(\Lambda)}(\mathbf{r}), \quad (1)$$

where $\phi(\mathbf{r})$ is the pseudoscalar negative meson field operator and $T_{\nu\mu}$ is a matrix which projects the pion-proton system on the angular momentum state that will insure the conservation of parity and angular momentum. It is therefore possible for the meson to be emitted with angular momentum $j+\frac{1}{2}$ or $j-\frac{1}{2}$, depending on the relative parity $\alpha$ of the $\Lambda^0$ particle and a nucleon.

The two possible forms for $T_{\nu\mu}$ are

$$T_{\nu\mu}(\mathbf{r}) = \begin{cases} \dfrac{\lambda_-}{(2\pi)^3}\displaystyle\int d\mathbf{q}\,e^{i\mathbf{q}\cdot\mathbf{r}}q^{j-\frac{1}{2}}Y_{j-\frac{1}{2}}{}^{\mu-\nu}(\theta,\varphi); \quad \alpha=(-1)^{j+\frac{1}{2}} \\ \dfrac{\lambda_+}{(2\pi)^3}\displaystyle\int d\mathbf{q}\,e^{i\mathbf{q}\cdot\mathbf{r}}q^{j-\frac{1}{2}}\left[\left(\frac{j-\mu}{2j}\right)^{\frac{1}{2}}Y_{j-\frac{1}{2}}{}^{\mu+\frac{1}{2}}\boldsymbol{\sigma}_{\nu,-\frac{1}{2}}\cdot\mathbf{q} + \left(\frac{j+\mu}{2j}\right)^{\frac{1}{2}}Y_{j-\frac{1}{2}}{}^{\mu-\frac{1}{2}}\boldsymbol{\sigma}_{\nu,\frac{1}{2}}\cdot\mathbf{q}\right] \end{cases} \quad (2)$$

$$=\frac{\lambda_+}{(2\pi)^3}\int d\mathbf{q}\,e^{i\mathbf{q}\cdot\mathbf{r}}q^{j+\frac{1}{2}}\left[-\left(\frac{j+\mu+1}{2j+2}\right)^{\frac{1}{2}}Y_{j+\frac{1}{2}}{}^{\mu+\frac{1}{2}}\delta_{\nu,-\frac{1}{2}} + \left(\frac{j-\mu+1}{2j+2}\right)^{\frac{1}{2}}Y_{j+\frac{1}{2}}{}^{\mu-\frac{1}{2}}\delta_{\nu,\frac{1}{2}}\right]; \quad \alpha=(-1)^{j-\frac{1}{2}}.$$

Here $\lambda_-$ and $\lambda_+$ are the relevant coupling constants; $q$, $\theta$, and $\varphi$ are the spherical coordinates of $\mathbf{q}$. The total rate for decay of the $\Lambda^0$ particle into a free negative meson which has momentum $q=0.73$, is

$$R^{(-)}{}_{\text{mesonic}}=\begin{cases}(\lambda_-^2/2\pi)q^{2j}, & \alpha=(-1)^{j+\frac{1}{2}}; \\ (\lambda_+^2/2\pi)q^{2j+2}, & \alpha=(-1)^{j-\frac{1}{2}}.\end{cases} \quad (3)$$

To calculate the internal conversion, we take the pion-nucleon coupling

$$H_i'=g\int d^3r\psi^*(\mathbf{r})\tau_i\boldsymbol{\sigma}\cdot\nabla\varphi_i(\mathbf{r})\psi(\mathbf{r}). \quad (4)$$

For low momentum mesons, the interaction (4) is known to give a quantitatively accurate description of the matrix element for the absorption of a meson by a nucleon when both initial and final nucleon states are approximately free.[12] Analyses of meson-nucleon scattering and of photomeson production give[12] $g^2/4\pi\sim0.08$. With the interaction (4), the process is described by the Feynman diagram in Fig. 1, where the $\Lambda°$ and the initial proton are assumed at rest and uncorrelated and the final-state nucleons are assumed to be free. They have equal and opposite momenta of magnitude $k\sim3$, so that exchange effects with other nucleons in the nucleus have been neglected. At the vertex $A$, of course, the appropriate Fourier transform of $T_{\nu\mu}$ must be inserted. The resulting matrix element is

$$M(\mathbf{k})=g(2\rho_p)^{\frac{1}{2}}\frac{\left(\bar{u}^{(n)}\boldsymbol{\sigma}\cdot\mathbf{k}u^{(p)}\right)\left(\bar{u}_\nu{}^{(p)}\int d^3\mathbf{r}T_{\nu\mu}(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}u_\mu{}^{(\Lambda)}\right)}{k^2+1-[(k^2+M^2)^{\frac{1}{2}}-M]^2},$$

where the $u^{(i)}$ are the normalized spinors describing the baryons and $\rho_p$ is the actual proton density ($\rho_p\sim3/8\pi$ in all but the lightest nuclei). The total rate for the nonmesonic decay subject to these assumptions is

$$R^{(-)}{}_{\text{nonmesonic}}=\begin{cases}\pi^{-1}g^2\lambda_-^2k^2M[M(M'-M)+1]^{-2}k^{2j}\rho_p, & \alpha=(-1)^{j+\frac{1}{2}}; \\ \pi^{-1}g^2\lambda_+^2k^2M[M(M'-M)+1]^{-2}k^{2j+2}\rho_p, & \alpha=(-1)^{j-\frac{1}{2}}.\end{cases} \quad (5)$$

The ratio of the two rates, (3) and (5), finally is

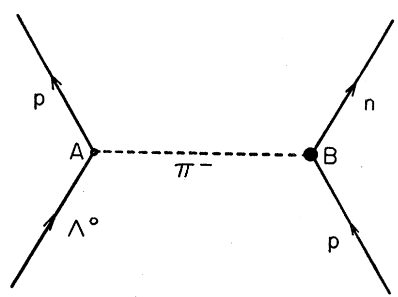


FIG. 1. Feynman diagram for internal conversion of $\pi$ meson emitted by $\Lambda^0$.

[11] This interaction represents the decay of a $\Lambda^0$ particle into a proton and a pion. We shall not inquire into the mechanism of this reaction except to assume that it is localized in a region smaller than the wavelength of the neutron emitted as a result of internal conversion. If the range is of the same magnitude as the $K$-particle Compton wavelength, for instance, this requirement is satisfied.

[12] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954); G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570, 1579 (1956).

$$Q^{(-)}=\frac{R^{(-)}{}_{\text{nonmesonic}}}{R^{(-)}{}_{\text{mesonic}}}$$
$$=2g^2k^2M[M(M'-M)+1]^{-2}\rho_p\left(\frac{k}{q}\right)^{2l+1}\cong 5\rho_p(17)^l, \quad (6)$$

where $l$ is the angular momentum of the meson actually emitted. The approximation that the nucleon density is constant is adequate as long as it does not change appreciably over one wavelength of the final nucleons. This is certainly true if the interaction between the $\Lambda^0$ and any single nucleon of the nucleus is not very strong.

We can apply exactly the same calculation to the other possible meson decay mode of the $\Lambda^0$,

$$\Lambda^0\rightarrow\pi^0+n. \quad (7)$$

The neutral pion can be absorbed by either protons or neutrons, but the coupling constant is only $g/\sqrt{2}$. The ratio of conversion to emission is proportional to the total density $\rho$ of nucleons. For the mode (7), Eq. (6) becomes

$$Q^{(0)}=2.5\rho(17)^l\sim Q^{(-)}. \quad (8)$$

## APPLICATIONS

We shall make use of the observations of Fry, Schneps, and Swami[13] and of Blau,[13] according to whom the heavier hyperfragments ($Z>2$) decay almost exclusively by nonmesonic decay while mesonic and nonmesonic decay compete in helium and hydrogen. Moreover, for $Z\leq 2$ the $\Lambda^0$ binding energy is so small that the $\Lambda^0$ spends most of its time outside of the nucleus. For larger $Z$, the binding energy increases sufficiently to confine the $\Lambda^0$ pretty well within the nuclear volume. The application of Eqs. (3), (5), and (6) for $Z>2$ and $Z\leq 2$ leads to a result which is consistent with both sets of data.

For $Z\leq 2$, corrections for self-absorption of the emitted $\pi^-$ and for the effects of the exclusion principle on the residual proton are almost certainly negligible if the $\Lambda^0$ is so weakly bound that its wave function is spread over a volume much larger than $4\times(4\pi/3)\times(\hbar/\mu c)^3$. Moreover, the relative wave function of the $\Lambda^0$ and the nucleus is completely determined by the binding energy. The density of protons around the $\Lambda^0$ varies slowly enough so that Eq. (6) may be used with confidence. The average density of protons near a loosely bound $\Lambda^0$ is the product of the proton density in the nucleus times the probability of the $\Lambda^0$ being in the nucleus, approximately $(3Z/4\pi R^3)R(2M'E_b)^{\frac{1}{2}}$, where $E_b$ is the small binding energy and $R$ is the nuclear radius. Using $R=A^{\frac{1}{3}}$, $Z=2$, $A=3$, and Eq. (6), we obtain the ratio of nonmesonic to mesonic decay given in Table I. For helium, the observed ratio based on nine events is 1.25. The reported binding energies for $\Lambda^0$ in hydrogen and helium isotopes are of the order of one Mev or less. Thus the helium data alone strongly

TABLE I. Ratio $Q^{(-)}$ of nonmesonic to mesonic decay for helium hyperfragments as a function of the pion angular momentum $l$ and the $\Lambda^0$ binding energy $E_b$ in Mev.

| $l$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $Q^{(-)}$ | $0.4(E_b)^{\frac{1}{2}}$ | $6.8(E_b)^{\frac{1}{2}}$ | $120(E_b)^{\frac{1}{2}}$ | $2000(E_b)^{\frac{1}{2}}$ |

favors $l=0$ or 1. An orbital momentum $l=2$ is not reasonable unless $E_b\lesssim 1$ kev.

Since $\Lambda^0$ is an isotopic singlet, hydrogen and helium hyperfragments of the same mass should have almost identical structures and the ratio $Q^{(-)}$ for ${}_{\Lambda}$H should be just one-half that for ${}_{\Lambda}$He. Of five hydrogen hyperfragments which have been definitely observed, one has had a nonmesonic decay,[14] while two were expected.

The decays of the heavier fragments give additional support to the assignment. In order to evaluate the data, however, an estimate of self-absorption and the much more important correction for exclusion effects must be made.

The 6.5-Mev binding energy which has been reported for a $\Lambda^0$ bound in $Be^9$ makes it a good approximation to consider the $\Lambda^0$ confined to the nuclear volume. Then the density $\rho_p$ of Eq. (5) is approximately $3/8\pi$.

The suppression of the mesonic decay in nuclear matter is qualitatively similar to the changed lifetime of a neutron for $\beta$ decay when it is immersed in nuclear matter. The size of this effect is most simply estimated for an independent-particle model of the nucleus. We first note that, because there is no exclusion principle for $\Lambda^0$ and nucleon, a $\Lambda^0$ in a nucleus has about the same wave function, but not the same energy, as the lowest-energy proton or neutron. Thus the residual proton from $\Lambda^0\rightarrow p+\pi^-$ will be excluded from a wave function similar to that of the $\Lambda^0$. The overlap intergal between initial and final states cannot be 1 as for the free $\Lambda^0$ decay, but rather is

$$I^2\approx\sum_p\left|\int\psi_p{}^*(\mathbf{r})\psi_\Lambda(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}\right|^2. \quad (9)$$

Here $\psi_p$ is any unoccupied proton wave function and $q$ is the $\pi^-$ momentum. Equation (9) is certainly something of an overestimate because the higher energy proton states are accessible only when the pion momentum is very small, and that in turn reduces the density of states and matrix element for pion emission unless $l=0$. If the summation is over a complete set of proton states, $I^2\approx 1$. If the lowest proton state and $\Lambda^0$ state are assumed identical, Eq. (9) becomes

$$I^2=1-\left|\int\psi_\Lambda{}^*(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}\psi_\Lambda(\mathbf{r})d\mathbf{r}\right|^2-\sum_{p'}\left|\int\psi_{p'}(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}\psi_\Lambda(\mathbf{r})d\mathbf{r}\right|^2. \quad (10)$$

[13] Fry, Schneps, and Swami, Phys. Rev. **101**, 1526 (1956); M. Blau, Phys. Rev. (to be published).

[14] It should be noted that nonmesonic decays of ${}_{\Lambda}$H may be difficult to identify unambiguously.

TABLE II. Corrected ratio $Q_c^{(-)}$ of nonmesonic to mesonic decay expected for heavier hyperfragments as a function of the pion angular momentum; the various corrections are discussed in the text.

| $l$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $Q_c^{(-)}$ | 4.8 | 80 | 1400 | 24 000 |

The third term in Eq. (10) is summed over all occupied proton states except the ground state. In light nuclei these are only $p$-state protons; but $\mathbf{q}\cdot\mathbf{r}$ is small, so that we shall neglect this third term compared to the second, again an approximation which overestimates $I^2$. For $A=8$ and $R=2$, an estimate of Eq. (10) then gives $I^2\sim\frac{1}{4}$.

An estimate of the importance of self-absorption can be inferred from photoproduction data. A proton in $C^{12}$, for instance, is only one-half as effective as a free proton for the photoproduction of 42-Mev mesons.[15] According to Lax and Feshbach,[16] however, the effect of the exclusion principle is sufficient to account for the reduction. We shall therefore neglect the effect of self-absorption, which is certainly no greater than a factor of two even in carbon.[17]

A further correction to the observed mesonic decay is necessary for the alternative mode $\Lambda^0\rightarrow n+\pi^0$. If $\Lambda^0\rightarrow$ nucleon+pion is a transition from isotopic spin $T=0$ to $T=\frac{1}{2}$, then the neutral-decay mode is half as prevalent as the $\pi^-$ mode. But since the $\pi^0$ is not seen, such decays are always interpreted as nonmesonic. Including this and the correction to $I^2$, we have the predicted ratio of nonmesonic to mesonic decays in Table II.

The experimental ratio[13] is about 40. The best fit is again $l=0$ or 1.[18]

Thus the conclusions from both the light ($Z\leq 2$) and heavier hyperfragments are as follows:

(a) $l\geq 2$ gives ratios differing from experiment by factors of 100 or more.

(b) The data are not inconsistent with either $l=0$ or $l=1$. If the spin of the $\Lambda^0$ is 3/2, therefore, its parity is the same as that of the proton. Since an angular correlation has been observed in the free decay of the $\Lambda^0$,[6] a spin of 3/2 is favored over the value 1/2. It should be emphasized, however, that the arguments presented here give a limit to the magnitude of $l$ only if the "size" of the $\Lambda^0$ particle is no larger than about $0.5\times 10^{-13}$ cm.

## ACKNOWLEDGMENTS

We are grateful to Professor Gerson Goldhaber, who gave us prepublication information about some of the references listed.

[15] R. F. Mozley, Phys. Rev. **80**, 493 (1950).

[16] H. Feshbach and M. Lax, Phys. Rev. **76**, 134 (1949).

[17] Any internal conversion of the emitted pion by neighboring nucleons of $C^{12}$ can increase the cross section for absorption of photons but does not affect the cross section for making free pions.

[18] The orbital momentum $l=1$ corresponds to that special case which has also been treated by Primakoff and Cheston (see reference 9). With their approximations they predict a $Q_c^{(-)}$ for $l=1$ which is about as large as our result for this case. In general, the ratios in Tables I and II are expected to be an underestimate. For example, a strong correlation in position between the $\Lambda^0$ and neighboring protons, if due to an attraction, will increase the probability for internal conversion.