Interpretation of the $Be^9(p,d)$ Reaction at Energies of 5 to 30 Mev*

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The angular distribution of the $Be^9(p,d)$ reaction is substantially constant between 5 and 30 Mev. The Butler theory, used with a finite radius as cutoff on the radial integral, does not give a satisfactory interpretation of this fact, but the "transparent-nucleus" Born approximation pick-up theory does. For incident energies between 16 and 31 Mev this theory gives a momentum distribution for the picked-up neutron which is consistently defined, and which agrees quantitatively with that obtained from the 95-Mev data; at lower energies, 5 to 8 Mev, the theory does not appear to work quite as well. The results of the analysis give support to the general validity of interpreting results of the high-energy pick-up process in terms of a momentum distribution and a single-particle model.

THE Butler theory of the pick-up process usually gives satisfactory results in the low-energy region. However, it has been recently noted that this theory does not give agreement with the observations of the $Be^9(p,d)$ reaction. Reynolds and Standing[1] have found that the angular distributions at several low energies cannot be fitted to Butler curves for the same value of r_0. Finke[2] was unable to explain his data at 31.5 Mev on the Butler theory with any value of this parameter. He was, however, able to obtain agreement (at angles less than 75°) with a Born approximation applied with a transparent nucleus model (i.e., with no restriction of the region of integration), a type of calculation which Daitch and French some time ago suggested might be more correct than the original Butler theory.[3]

The failure of the Butler theory may be attributed to the low binding energy of the "outer" neutron of Be^9 and the consequent diffuseness of its wave function, in contrast to the relatively small size of the core of the nucleus. That this neutron does spend so much time outside the region of strong nuclear interaction permits one to believe that the transparent Born approximation might give a satisfactory treatment of this problem. This type of calculation was first suggested for the deuteron pick-up process at high energies by Chew and Goldberger,[4] and from recent work it seems likely that this approximation is reasonably valid for light nuclei at high energies, of the order of 100 Mev.[5] It is of some interest to see whether such a calculation can give agreement with experimental data for Be^9 over a range of low energies, in order both to understand the data and to test the validity of the model at such energies.

Observations of the $Be^9(p,d)$ angular distributions (leading to the ground state of Be^8) have recently been accomplished at several energies between five and thirty Mev.[1,2,6,7] The results indicate that the angular distribution is substantially constant within this energy range. Since the Butler theory involves a sharp nuclear boundary of radius r_0, it will predict a behavior of the cross section which is characteristically diffraction-like, showing minima at angles which depend on kr_0, and so on the energy. The general nature of the effect is that the angular distribution becomes narrower at higher energies. This behavior is generally observed to occur; this can be seen from the result that the nuclear radius for a given isotope determined from Butler-type analysis of stripping curves at different energies is roughly constant.[8] For the $Be^9(p,d)$ reaction, however, the fact that the angular distribution is substantially independent of energy cannot be explained on the Butler-type theory unless an energy dependence is attributed to the nuclear radius. Such a suggestion has been made by Reynolds and Standing to explain the behavior of the $Be^9(p,d)$ angular distribution.

However, there is reason to believe that the assumptions involved in the Butler theory are not justified for the treatment of the present problem because of the low binding energy of the picked-up particle. The Butler theory of this process presumes that outside the nucleus the exact wave function represents incident free protons and outgoing free deuterons. However, Be^9 may be pictured as behaving not as if it had a well-defined radius outside of which the above approximation for the wave function is valid, but rather as though it had a core of 8 nucleons of radius $1.4A^{\frac{1}{3}}\times10^{-13}$ cm with a loosely bound "outer" neutron which spends much of its time outside of the core (calculation with a trial potential indicates a 50% probability for this neutron to be outside the nuclear core). Thus the required form of the exact wave function cannot be obtained for any reasonable value of r_0.

It is again as a consequence of the low binding energy of the "outer" neutron that one would expect the Born approximation to give satisfactory results for the Be^9

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[1] J. B. Reynolds and K. G. Standing, Phys. Rev. **101**, 158 (1956).
[2] R. Finke, University of California Radiation Laboratory Report UCRL-2789, Berkeley, 1954 (unpublished).
[3] P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952).
[4] G. F. Chew and M. L. Goldberger, Phys. Rev. **77**, 470 (1950).
[5] W. Selove, Phys. Rev. **101**, 231 (1956).
[6] B. L. Cohen *et al.*, Phys. Rev. **90**, 323 (1953).
[7] J. A. Harvey, Massachusetts Institute of Technology LNSE Progress Report, January 1, 1951 (unpublished).
[8] See e.g., R. Huby, Progr. Nuc. Phys. **3**, 177 (1953).

problem. In this method the exact wave function for the system is replaced by the initial wave function of a bound neutron plus incident plane wave protons. One thus ignores the modification of the proton waves in the neighborhood of the nuclear potential, as well as the scattered deuterons. Since the wave function of the picked-up neutron remains large outside the region of high nuclear density, the dominant contribution to the Born approximation integral can arise from the outside region, and the distortion of the proton wave function may not manifest itself strongly in the calculation of the Born approximation result.

Hence the large size of the wave function of the picked-up particle compared with the size of the nuclear core explains both the applicability of the Born approximation and the inapplicability of the Butler theory (or what is roughly equivalent, the Born approximation with a restricted range of integration[9]) to this problem. More generally, a sufficient (but perhaps not necessary) condition for the Born approximation to be valid would be that the exponential decay length α^{-1} of the wave function of the picked-up particle outside the nucleus be at least as large as the nuclear radius. In terms of the binding energy in Mev, B, and assuming a core radius of $1.4(A-1)^{\frac{1}{3}}\times 10^{-13}$ cm, we have approximately $B<10A^{-\frac{2}{3}}$. One sees that this condition is satisfied for deuterium and beryllium, but not for other nuclei. Thus, even though we may expect this theory to give acceptable results for beryllium, as indeed it appears to do (see below), we can unfortunately not extend this expectation directly to the general case.

The result of applying the Born approximation to the pick-up problem is well known. If we assume a Hulthén wave function for the deuteron, it is straightforward to derive, for the center-of-mass differential cross section leading to a particular final state, the formula[4,5]

$$\frac{d\sigma}{d\omega}=\frac{24\pi^2\alpha}{1+(\alpha/\beta)-4\alpha/(\alpha+\beta)}\,\frac{A(A-1)}{(A+1)^2}\times\frac{K}{k}|F|^2N(n)\left(\frac{\beta^2-\alpha^2}{\beta^2+q^2}\right)^2,$$

where $N(n)=(4\pi)^{-1}\int d\Omega|u(\mathbf{n})|^2$, $u(\mathbf{n})$ is the momentum space wave function of the neutron, $\mathbf{n}=\mathbf{K}-[(A-1)/A]\mathbf{k}$ is $\hbar^{-1}$ times the momentum of the picked up neutron, $\mathbf{q}=\mathbf{k}-\frac{1}{2}\mathbf{K}$ similarly corresponds to the internal momentum of the deuteron, $\mathbf{k}$ and $\mathbf{K}$ to the momenta of the incident proton and the formed deuteron respectively, and F is the fractional parentage coefficient[10] and takes into account competing processes in which the residual nucleus is left in an excited state. α and β are the Hulthén wave function parameters. It is seen that the momentum density of the neutron of

[9] E. Gerjuoy, Phys. Rev. **91**, 645 (1953).

[10] A. M. Lane and D. H. Wilkinson, Phys. Rev. **97**, 1199 (1955).

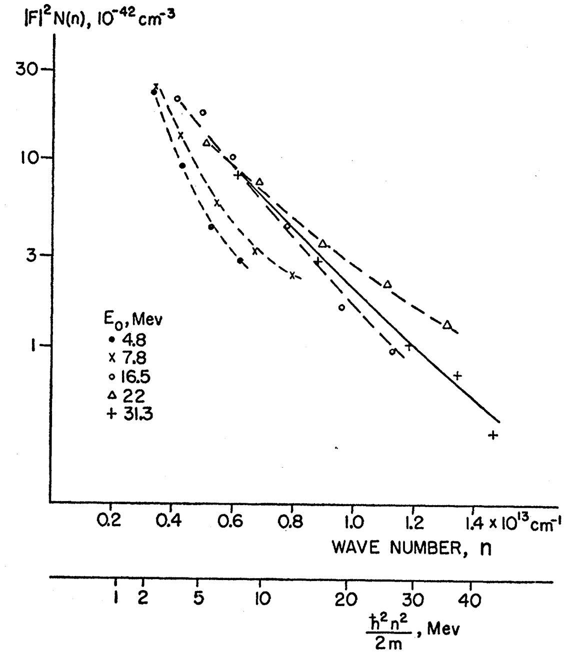

FIG. 1. Momentum density (times fractional parentage coefficient), $|F|^2N(n)$, calculated from the experimental pick-up cross sections. See text for discussion of probable errors. Revised values of the absolute cross sections for the 5- and 8-Mev data have been given by Harvey[11] since this figure was prepared—the ordinates for these data should be increased 20%.

argument n, is determined by the differential cross section at angle θ and at incident energy E_0. Deuterons emerging at different angles will correspond to different values of the momentum, n, of the picked-up neutron. Thus, at each value of the incident proton energy, the differential cross section will determine $N(n)$ for a range of values of n. The result of this determination is shown in Fig. 1 for experiments performed at 5, 8, 16 22, and 31 Mev. Data from large scattering angles have been omitted since for such angles there is a possible preponderance of compound nucleus formation (the dividing point has been arbitrarily chosen as 75°).

The individual statistical errors in the experimental data at a given energy are of the order of a few percent. The absolute-value normalization is somewhat poorer, of the order of 20% at each energy, except for the 5- and 8-Mev data. The 5- and 8-Mev data are internally precise to a few percent, but carry together an uncertainty of about 40% in absolute-value normalization.[11] The 95-Mev data[5] on this reaction give values of $|F|^2N(n)$ which are in agreement within experimental uncertainty with the 16- to 31-Mev results. Thus for incident energies between 16 and 95 Mev the transparent Born model calculation for $Be^9(p,d)$ gives an $N(n)$ which is consistently defined; at somewhat lower energies this model does not appear to work quite as

[11] J. A. Harvey, private communication.

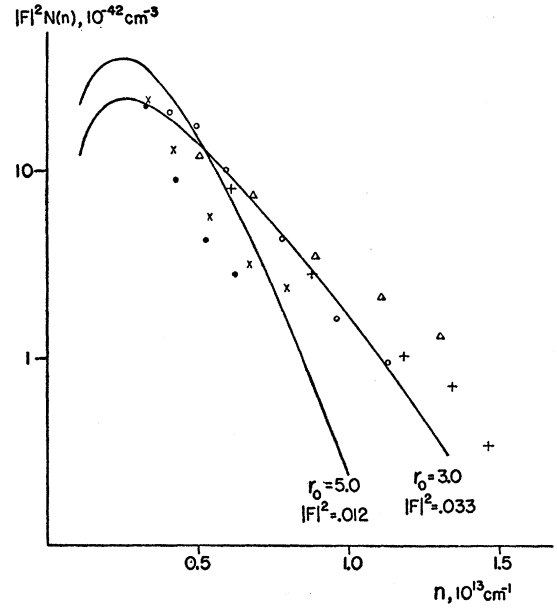

FIG. 2. Theoretical momentum densities calculated for a tapered square-well model of radius $r_0=3.0$ and 5.0×10^{-13} cm.[13] The points shown are the experimentally determined points, reproduced from Fig. 1. The values of $|F|^2$ used on the theoretical curves have been chosen for best fit. The remark on the Fig. 1 caption concerning the 5- and 8-Mev data also applies here.

well, although the data is still almost consistent within experimental uncertainty.[12]

Numerical solutions to the Schrödinger equation of appropriate binding energy (1.67 Mev) and appropriate angular momentum ($l=1$) have been computed for two reasonable tapered square-well potentials.[13] The momentum densities given by Fourier inversion of these solutions are shown in Fig. 2 along with the momentum densities calculated from the experimental data. The values of $|F|^2$ have been chosen to give the best fit. It is seen that for those data which give consistent values of $N(n)$—here, the 16- to 31-Mev data—best agreement is obtained for a core radius r_0 of 3×10^{-13} cm. At the higher values of n, the experimental points indicate a higher momentum density than is predicted by the strict single-particle model; this type of deviation has been observed in the results of experiments done at higher energies and its interpretation has been discussed in reference 5.

Thus the angular distributions can be satisfactorily explained by the Born approximation. However, too large a cross section is predicted. In order to make the momentum density deduced from the experimental data agree best in magnitude with that computed by use of the Schrödinger equation, it was necessary to choose $|F|^2=0.033$. Now it appears from the 95-Mev data that removal of the "loose" neutron from Be^9 leaves Be^8 in one of two states, either the ground state or the 2.9-Mev state, with relative intensities I_0 and I_1. $|F|^2$ for the ground state would then be $I_0/(I_0+I_1)$, and this ratio is about 0.3 for both the 30- and 95-Mev data.

The value 0.033 required to fit the "single-particle" theoretical momentum distribution is thus not in good agreement with the value implied by the I_0/I_1 intensity ratio. But since the 95-Mev data shows a long high-momentum tail which is not given by the single-particle calculation in the low-energy region, but which possesses an appropriate high-momentum tail. Such a procedure leads to a value of $|F|^2$ of from 7 to 10%, which compares more favorably with the 30% value indicated by the I_0/I_1 intensity ratio.

The fact that for $n<\sim1.5\times10^{13}$ cm^{-1} the experimental and theoretical angular distributions agree except for an angle-independent factor suggests that any opacity effect operating to decrease the cross section does not appreciably affect the angular distribution. If this is true, it lends important support to the general validity of interpreting results of the high-energy pick-up process in terms of a momentum distribution and a single-particle model. On the conservative side, it should be noted that the explanation of the agreement we have found may not involve such an angle-independent opacity effect; conceivably, if the Born approximation were corrected for the neglect of the proton-nucleus interaction, the theoretical cross sections might be appropriately reduced.[14]

It is important to note that the 16- to 31-Mev data and the 95-Mev data give values of $|F|^2N(n)$ in quantitative agreement with each other. It is perhaps somewhat surprising that any opacity or distorted-wave effects show no appreciable net energy dependence over this range, in view of the fact that the small value of $|F|^2$ obtained suggests that such effects may be present.

Following the conclusion of the major part of this work, the first part of some independent work by Dabrowski and Sawicki[15] on the $Be^9(p,d)$ reaction has appeared. Their approach is similar to ours in examining the validity of the transparent-nucleus type of calculation, but we have thought it worthwhile anyway to publish the present note emphasizing certain aspects of the interpretation. Our calculation using a core radius of 5.0×10^{-13} cm was motivated by their use of this value in their discussion; this value does not fit the data at all as well as the value 3.0, which we had chosen for our first calculation as being closely $1.4A^{\frac{1}{3}}$.

We wish to thank Marshall Baker and Mrs. Mary Hermann for assistance in the calculations.

[12] *Note added in proof.*—We wish to thank M. M. Gordon for valuable comments regarding this report. Gordon has made calculations on the transparent model, but obtained discouraging results since he was attempting to fit the 5-, 8-, and 22-Mev data (at that time the 16- and 31-Mev measurements had not been made).

[13] The potential well that has been used is: $V=-V_0$, $r<r_0-b$; $V=0$, $r>r_0+b$; V linear, $r_0-b<r<r_0+b$. Solutions have been computed for $r_0=3\times10^{-13}$ cm, and 5×10^{-13} cm, both with $b=1\times10^{-13}$ cm.

[14] See J. Horowitz and A. Messiah, Phys. Rev. **92**, 1326 (1953), and W. Tobocman and M. Kalos, Phys. Rev. **97**, 132 (1955).

[15] J. Dabrowski and J. Sawicki, Acta Phys. Polon. **14**, 143 (1955). See also their note in Nuovo cimento **12**, 293 (1954).