Spin and Parity Analysis of Bevatron τ Mesons*

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The data of 71 τ^+ decays found in emulsions exposed at the Bevatron are presented and analyzed. These data are free from selection bias favoring short-ranged pions. Of these 71 events, 13 have negative pions under 10 Mev and one event has a π^+ of (0.38 ± 0.03) Mev. Assuming the distribution functions proposed by Dalitz, the relative probabilities that the 71 events turn out the way they did are 1, $10^{-7.9}$, $10^{-33.5}$, $10^{-8.8}$, $10^{-10.5}$, and $<10^{-2}$ for the respective (0−), (1+), (1−), (2+), (3−), and (3+) spin and parity configurations. The (0−) distribution function of Dalitz is statistically a good fit to the data. In addition, 55 Massachusetts Institute of Technology and 100 Berkeley τ decays are shown to behave essentially the same way. No indication of polarization of the Bevatron τ beam was found. Conclusions are drawn from the data that the τ and θ mesons have different spin-parity configurations and that the only reasonable possibilities for the τ are (0−) and (2−). The data are also used to give an upper limit for the s-wave pion-pion interaction.

I. INTRODUCTION

SEVERAL years ago Dalitz[1] suggested that a study of the energy distribution of the 3 pions in τ decays would give information about the spin and parity of the τ meson. By the time of the Rochester Conference in February, 1955 he had collected data on 53 τ mesons from the cosmic rays in which the pion identities were known. Although such data were subject to selection biases which could not be ascertained, he was able to conclude that the τ meson should have a spin and parity such that decay into two pions was forbidden.[2] Such a conclusion is of vital importance to the study of fundamental particles. It leaves us with at least two different particles which happen to have almost the same mass, lifetime, production cross sections, and scattering cross sections. A possible explanation has recently been given by Lee and Yang.[3] They propose a new conservation law of physics which would require all particles of odd strangeness to have parity doublets.

Thus it is important to collect data which are free from selection bias in order to strengthen Dalitz' conclusion, and perhaps narrow down the spin-parity possibilities. In nuclear emulsion the decay pions may have ranges up to 3.5 cm. For this reason many of the cosmic-ray experiments had a selection bias favoring τ mesons with a low-energy π^-. Now that monoenergetic τ beams are available at the Bevatron[4] and Cosmotron[5] it is possible to place endings at a position in a large emulsion stack which has at least 3.5 cm of emulsion in all directions. Then all selection bias would be eliminated. Thus far three groups have completed experiments approximating these conditions. These are the MIT group, who have circulated data on 55 τ decays,[6] the Berkeley group with 100 τ decays,[7] and the Columbia group with 71 τ decays, which are presented in this paper.

The 71 Columbia τ mesons were located by systematic scanning in an experiment designed to measure the τ lifetime.[8] In 68 of our cases the pion identities are completely known. In the other three cases the π^- is known to be one of two tracks which happen to have close to the same energy. So, for the purpose of spin-parity analysis, our sample of 71 τ mesons to the best of our knowledge is free from selection bias.

The nonrelativistic energy distribution functions of the 3 decay pions which have been obtained by Dalitz[1] and others[9,10] are listed in Table I. Because of energy and momentum conservation, the energies and directions of the 3 pions in the decay plane are determined by only two independent variables. The variables used here are ϵ and x, where $x=(\epsilon_1-\epsilon_2)/\sqrt{3}$. ϵ is the π^- kinetic energy divided by its maximum possible value ($\frac{2}{3}$ of the Q-value). ϵ_1 and ϵ_2 are the corresponding quantities for the π^+ energies, where $\epsilon_1>\epsilon_2$. The relative probabilities that our data turn out the way they did, based on these distribution functions, are displayed in column (c) of Table I.

The meaning of these relative probabilities is discussed in Sec. III of this paper. The experimental data are displayed in Table II and discussed in Sec. II. In Sec. IV the data are shown to pass randomness tests when the (0−) distribution function is assumed to be correct. The validity of the relative probabilities in Table I is discussed in Sec. VI, with the conclusion that (0−) and (2−) are the only reasonable possibilities below spin 4. (3+) is shown to be improbable but not completely ruled out. Also arguments are given

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TABLE I. Column (c) contains the likelihood ratios of different spin and parity possibilities for the τ meson calculated using Eq. (1) and the nonrelativistic distribution functions in column (d).[a] Only the 71 Columbia events were used.

	(a) J	(b) P	(c) $P_{J,P}/P_{0-}$	(d) $\frac{1}{8}\pi f_{J,P}(\epsilon,x)$
2-pion decay forbidden	0	$-$	1	1
	1	$+$	$10^{-7.4}$, $(10^{-7.9})^{\text{b}}$	2ϵ
	2	$-$	could be ~ 1	$\frac{16}{5(A^2+B^2)}[A^2\epsilon^2+B^2(1-\epsilon)^2\pm AB\epsilon(1-\epsilon)(3\cos^2\theta-1)]$
	3	$+$	less than $10^{-2.0}$	$\frac{32}{7(A^2+3B^2)}\{A^2\epsilon^3+7B^2\epsilon(1-\epsilon)^2+(3\cos^2\theta-1)[B^2\epsilon(1-\epsilon)^2\pm 3AB\epsilon^2(1-\epsilon)]\}$
2-pion decay permitted	1	$-$	$10^{-33.5}$	$192\epsilon^2(1-\epsilon)^2\sin^2\theta\cos^2\theta$
	2	$+$	$10^{-8.8}$	$16\epsilon(1-\epsilon)^2\sin^2\theta$
	3	$-$	$10^{-10.5}$	$(48/7)\epsilon^2(1-\epsilon)^2\sin^2\theta(5+3\cos^2\theta)$

[a] $\cos\theta=\frac{x}{x_{\max}}=\frac{(1/\sqrt{3})(\epsilon_1-\epsilon_2)}{[\epsilon(1-\epsilon)]^{\frac{1}{2}}}$, $\int_0^1 d\epsilon\int_0^{x_{\max}} dx f_{J,P}=1$.
[b] Calculated relativistically.

against higher spin and the possibility that the τ could decay into two pions. It is pointed out that the pion-pion s-wave cross section should be less than $4\pi R^2$, where R is the pion-pion interaction distance. Otherwise the effects of this interaction should show up in the data.

In Sec. V an independent, but weak, test for the spin of the τ is given. This is the possibility that the Bevatron τ beam be polarized. In general, the production interaction for τ mesons is spin dependent and thus the amplitudes into final states of different m-value (z-component of τ spin) are different. Thus in general the Bevatron τ beam should have some polarization unless the τ has spin zero. In an attempt to detect a possible polarization, the Euler angles of the τ-decay system with respect to the laboratory system were measured for each event. The distribution of these angles is shown to pass randomness tests when the events are assumed to be unpolarized.

II. MEASUREMENT PROCEDURE

In addition to determining ϵ and x for the τ-decay analysis, we were interested in checking all our τ mesons for the possibility of anomalous Q-values and departures from coplanarity. Within accuracies of measurement, all 71 τ-meson decays were coplanar and consistent with a Q-value of 75 Mev. Since the sines of the decay angles are proportional to the pion momenta, the Q-value could always be checked by normalizing these 3 momenta to the energy of one of the pions which had been traced through the stack to its ending. The relativistic relation between momentum and energy was always used.

In order to establish which is the unlike or negative pion, first one secondary must be traced through the stack until it ends. If it gives a π-μ-e decay, a second track must be traced through to its ending. Actually in 10 of our events all 3 tracks were traced through. Although tracing of the third track was not necessary for the purpose of τ-decay analysis, it was useful as an additional check and provided increased accuracy. In 16 of the events only one secondary was traced to the end. Except for events 47, 48, and 51, all the single secondaries were negative pions. Fortunately, in these 3 events where the precise identity of the unlike pion is unknown, the other two pions (one of which must be the unlike pion) had close to the same momentum as determined from the decay angles. Coplanarity was checked from the decay-angle measurements.

The pion energies used in calculating the likelihood ratios were obtained as follows. In the case where only one pion range was known, ϵ and x were determined from the decay angles and normalized to a Q-value of 75 Mev. In the cases where two pion ranges were known, the third energy was obtained by subtraction from the assumed Q-value of 75 Mev. Then ϵ and x were calculated from these 3 energies. In the cases where all 3 ranges were known, the 3 energies were normalized to give a Q-value of 75 Mev. Except in the case of visible scatterings, the distance from the τ decay to the pion ending was used as the range. The density of the emulsion had been measured to an accuracy better than 1% when the stack was assembled. We estimate that our accuracy in energy determinations is mainly limited by straggling and would amount to $\sim 3\%$ error in the energy. The decay angle determinations are estimated to have an accuracy better than $\pm 2°$. These space angles and the 3 Euler angles which specify the orientation of the event in the laboratory system were determined graphically from the measured azimuth and dip angles by means of a stereographic projection. The accuracy of this graphical method is better than 1°. The complete coordinates of each event are listed in Table II.

III. THE RELATIVE PROBABILITIES

For a given spin and parity, the 3-pion final state can be expanded in spherical waves of the unlike pion (angular momentum l) and the two-particle system of the like pions (angular momentum L). Dalitz,[1] Fabri,[9] and Feld[10] have discussed the validity of taking only the lowest order term (smallest possible value of $l+L$) of this expansion for the distribution function $f_{JP}(\epsilon,x)$.

J is the spin of the τ, and P the parity. The function $f_{JP}(\epsilon,x)$ is a probability density; *viz.*, the probability of finding an event in the region $d\epsilon dx$ is $d^2P=f_{JP}d\epsilon dx$. Dalitz, Fabri, and Feld point out that if the interaction operator is of short range ($\sim$Compton wavelength of the τ) and if it is not strongly a function of the pion momenta, the first term of the final state expansion will have a coefficient much larger than all the others. The nonrelativistic τ-decay distribution functions obtained in this way are listed in Table I.

Suppose $f_{0-}(\epsilon,x)$ were the true distribution function. Then the probability of finding an event with coordinates (ϵ_i,x_i) is proportional to $f_{0-}(\epsilon_i,x_i)$. In comparing random sets of 71 events, the probability of a given set is proportional to

$$\prod_{i=1}^{71} f_{0-}(\epsilon_i,x_i).$$

If *a priori* there were no preference between (0−) or (1+) as the spin and parity of the τ meson, we would say that the "relative probability" of (1+) to (0−) is

$$\frac{P_{1+}}{P_{0-}}=\frac{\prod_{i=1}^{71} f_{1+}(\epsilon_i,x_i)}{\prod_{i=1}^{71} f_{0-}(\epsilon_i,x_i)}. \tag{1}$$

Statisticians have called this quantity the likelihood ratio, the ratio of the inverse probabilities, or the figure of merit. Precisely it is the probability that our experiment turn out the way it did assuming f_{1+} is the true distribution, divided by the probability that our experiment turn out the way it did assuming f_{0-} is the true distribution. In this sense it is the "odds" of (1+) *vs* (0−) based on our experiment alone. The likelihood ratio based on all Bevatron τ mesons is obtained by multiplying together the separate values from each laboratory. We have calculated that this ratio is $10^{-7.9}$ for our 71 events, $10^{-7.0}$ for the 100 Berkeley events,

TABLE II. Pion energies and angles of 71 τ decays. The first two columns are ϵ and x, the parameters used in calculating the likelihood ratios. The next 3 columns are the measured pion energies. α_{32} is the measured space angle between track 3 (the π^-) and track 2 (the lower energy π^+). Θ, Φ, and Ψ are the Euler angles which specify the orientation of the decay with respect to the laboratory system as defined in Sec. V.

	ϵ	x	T_3 (Mev)	T_1 (Mev)	T_2 (Mev)	α_{32}	α_{31}	Θ	Φ	Ψ
1	0.024	0.063	1.18			65.5°	123.0°	67.0°	146.0°	20.5°
2	0.037	0.058	1.68			75.0	116.5	73.0	50.0	−107.0
3	0.038	0.029	1.91	37.8		97.7	100.5	20.5	−86.5	166.5
4	0.050	0.048	2.33			88.5	105.5	24.0	20.0	−163.0
5	0.051	0.139	2.57		30.2	35.5	152.0	111.5	105.0	31.5
6	0.089	0.179	4.46	43.0		66.0	130.0	117.5	−173.0	34.5
7	0.093	0.187	4.64		27.1	55.0	142.0	117.5	−18.0	50.5
8	0.115	0.187	5.75		26.5	11.69	172.75	89.5	−60.0	−76.0
9	0.142	0.249	7.1		23.2	51.0	147.5	75.0	95.0	49.5
10	0.154	0.021	7.7		32.7	96.0	107.0	26.5	−65.0	−33.0
11	0.169	0.142	8.8			78.0	127.5	111.0	−55.0	−91.0
12	0.184	0.155	9.2	39.6		74.5	133.5	157.5	130.0	124.0
13	0.195	0.284	9.7			47.5	151.5	62.5	−85.5	21.5
14	0.224	0.342		46.7	17.1	49.0	155.0	113.5	−93.5	36.5
15	0.228	0.173	11.4		24.3	88.0	128.0	88.5	−126.5	126.5
16	0.228	0.026	11.5			103.0	110.5	113.0	144.0	140.5
17	0.249	0.198	12.3	39.3	22.4	88.0	132.0	129.0	−141.0	93.5
18	0.260	0.134	12.2			89.5	125.5	100.0	−71.0	9.0
19	0.271	0.039	13.6			103.5	114.0	18.5	108.0	161.0
20	0.282	0.216	14.1		21.1	76.0	135.0	135.0	−92.0	−138.0
21	0.301	0.124	14.6			94.0	125.0	100.5	−126.0	11.5
22	0.357	0.395	17.8		11.5	29.0	166.0	96.5	49.0	−96.0
23	0.376	0.009	18.8		27.7	115.5	117.0	75.0	−69.0	129.0
24	0.380	0.333	19.0		13.6	65.0	150.0	137.0	−175.5	−111.0
25	0.382	0.232	19.1		17.9	85.0	140.5	92.0	−134.0	−125.0
26	0.396	0.390	19.8		10.7	56.0	154.0	75.0	88.5	130.0
27	0.396	0.201	19.4			90.5	135.5	112.5	40.0	64.0
28	0.447	0.280	22.7	39.0	14.4	85.5	143.0	80.5	47.0	−47.0
29	0.495	0.194	24.7		16.7	100.5	136.5	61.5	−103.0	−173.5
30	0.502	0.300	25.5	38.6	12.1	88.0	149.5	140.5	−2.5	164.0
31	0.518	0.339	26.1	39.5	10.0	79.5	153.5	53.5	−149.0	33.5
32	0.520	0.065	26.9			115.0	127.0	55.0	111.5	148.0
33	0.524	0.308		37.7	11.1	88.5	150.5	44.5	−142.0	−16.0
34	0.526	0.285		36.7	12.0	86.0	148.5	109.0	12.0	69.0
35	0.552	0.261	28.3	35.9	12.7	96.5	149.5	116.5	177.0	138.0
36	0.554	0.020	28.2	25.0	23.2	118.5	127.5	118.0	−15.0	116.0
37	0.555	0.042	28.1			119.5	126.5	56.5	116.5	−147.5
38	0.572	0.192	28.6		14.9	99.5	142.5	110.5	15.0	166.0
39	0.578	0.018	28.9	23.8		127.5	126.0	118.0	134.5	154.0
40	0.585	0.085	30.0	27.2	19.7	111.5	137.5	122.0	167.5	62.5
41	0.590	0.290	29.5		10.2	90.5	149.5	79.5	−72.5	71.5

TABLE II.—*Continued.*

	ϵ	x	T_3 (Mev)	T_1 (Mev)	T_2 (Mev)	α_{32}	α_{31}	Θ	Φ	Ψ
42	0.591	0.252	29.5			102.0	145.0	73.5	59.5	−49.0
43	0.626	0.017		22.6	21.1	115.0	132.0	95.0	103.0	−58.0
44	0.638	0.006		21.8	21.3	125.5	130.5	110.0	−109.0	−46.0
45	0.642	0.255	32.1		10.4	102.0	148.0	124.5	44.0	150.0
46	0.642	0.060		24.1	18.8	123.0	140.5	158.0	39.0	−46.0
47[a]	0.649	0.259			8.2	109.0	148.5	82.5	−63.5	−77.5
						102.5		97.5	116.5	110.5
48[a]	0.664	0.283			10.2	116.0	152.5	92.0	−109.0	31.0
						91.5		88.0	71.0	−4.0
49	0.684	0.021	33.8	21.0	19.3	130.0	131.0	49.5	44.5	−178.0
50	0.700	0.185	35.0		12.0	117.5	145.5	49.0	177.0	−98.5
51[a]	0.708	0.360			4.80	131.0	164.0	67.0	14.5	−85.5
						65.0		113.0	−165.5	101.5
52	0.714	0.120	34.1			125.0	142.0	28.0	−106.0	−18.0
53	0.717	0.282		31.8	7.35	113.0	156.5	77.0	−61.0	121.0
54	0.735	0.312		32.6	5.63	111.5	157.0	65.5	42.0	88.5
55	0.742	0.153		25.6	12.3	125.0	148.5	112.5	−43.5	−57.5
56	0.744	0.203		27.7	10.1	125.0	153.5	47.5	−177.5	83.5
57	0.760	0.419	38.0		0.38	96.5	175.0	13.5	−143.5	−18.0
58	0.774	0.018		18.9	17.4	138.5	139.0	134.0	158.0	1.0
59	0.794	0.255		28.7	6.6	138.0	163.0	66.0	164.5	−159.5
60	0.810	0.047		19.3	15.2	142.5	149.5	119.0	62.0	−137.5
61	0.816	0.345		32.0	2.16	130.5	171.5	40.0	163.0	−128.0
62	0.820	0.143		23.2	10.8	138.5	156.5	169.5	−30.5	−34.0
63	0.828	0.279		28.9	4.71	129.0	161.5	20.0	142.0	133.5
64	0.828	0.146		23.1	10.5	139.5	153.5	138.0	29.0	75.5
65	0.840	0.060		19.1	13.9	143.5	150.0	39.0	−89.0	56.5
66	0.846	0.131		22.0	10.7	141.0	155.5	55.5	99.0	98.5
67	0.849	0.087	44.0	20.7	13.0	148.0	156.5	164.0	−145.0	175.0
68	0.858	0.177		23.7	8.40	143.0	159.0	112.0	−29.5	−58.0
69	0.874	0.124		21.0	10.3	144.0	156.5	129.0	−81.5	−28.5
70	0.915	0.121		19.9	9.4	157.5	163.5	109.5	−111.5	−90.0
71	0.966	0.054	46.5	15.1	10.6	170.0	172.0	24.5	−137.0	−170.5

[a] In these 3 events the π^- is one of two possibilities, both of which are listed. In these 3 cases ϵ and x are the averages of the 2 possibilities.

and $10^{1.1}$ for the 55 MIT events. Thus the combined ratio favors (0−) over (1+) by a factor of $10^{13.8}$. It will be shown in the next section that the MIT result is not as much out of line as one might think. MIT happened to get very few events with low-energy negative pions compared to Columbia and Berkeley. The strength of such low-energy events in rejecting (1+) can be seen by going to the limiting case of a single event with a zero-energy π^-. Such an event by itself would be conclusive proof that the τ must have even spin, and no matter what other data existed, Eq. (1) would always give $P_{1+}/P_{0-}=0$.

The likelihood ratios for all spin values less than 4 based on the Columbia data alone are listed in Table I. In all cases where f_{JP} was a rapidly varying function of the coordinates (this is usually the case near the boundaries of the Dalitz plot), the coordinates of the event were displaced by an amount equal to the estimated measurement error in the direction which would give the most conservative likelihood ratio.

The nonrelativistic f_{JP} functions listed in Table I are given in terms of ϵ and $\cos\theta$, where

$$\cos\theta=x/x_{\max} \quad\text{and}\quad \left\{\begin{array}{l} x=(\epsilon_1-\epsilon_2)/\sqrt{3} \\ x_{\max}=[\epsilon(1-\epsilon)]^{\frac{1}{2}}. \end{array}\right. \tag{2}$$

Nonrelativistically, this quantity is the cosine of the angle between the π^- direction and the relative velocity of the two like pions. As can be seen from the dashed boundary in Fig. 2, for most values of ϵ the relativistic $x_{\max}$ is significantly less than the nonrelativistic expression $\{[\epsilon(1-\epsilon)]^{\frac{1}{2}}$, the solid semicircle$\}$. Since we always used Eq. (2) for $\cos\theta$ with the measured or relativistic energies, for most values of ϵ our $\cos\theta$ would have an upper limit significantly less than one. This procedure has a large effect in underestimating the likelihood ratio when the factor $\sin^2\theta$ is contained in the distribution function, as is the case for all those configurations which permit two-pion decay. For this reason a completely relativistic procedure would have made the (1−), (2+), and (3−) cases even more unfavorable. Only in the case of the (1+) calculation was the extra effort made to use a completely relativistic procedure as outlined by Fabri.[9] In this case the distribution function is independent of $\cos\theta$ and not much change should be expected. The nonrelativistic result was $P_{1+}/P_{0-}=10^{-7.4}$ and the relativistic result was $10^{-7.9}$.

It should be remembered that the likelihood ratios obtained in Table I depend on the assumed shape of f_{JP} which in turn depends on the assumptions mentioned earlier. Independent of these assumptions there are certain general features of the (1+), (1−), (2+), and (3−) distribution functions which would make them unfavorable compared to (0−) or (2−). These general features and our conclusions are discussed in Sec. VI.

IV. STATISTICAL TESTS FOR THE (0−) DISTRIBUTION

In addition to fulfilling the requirement of having the highest relative probability, the true distribution should possess two other properties which can be tested. First, the observed data should be distributed randomly with respect to this distribution; and secondly, the likelihood ratios obtained for the other distributions should be within a standard deviation or so of the calculated mean of these quantities. For example if f_{0-} is assumed to be the true distribution, the mean value of $\log(P_{1+}/P_{0-})$ is

$$\left\langle \log\frac{P_{1+}}{P_{0-}}\right\rangle_{\mathrm{Av}} = N\int\int\left(\log\frac{f_{1+}}{f_{0-}}\right)f_{0-}d\epsilon dx, \tag{3}$$

with a standard deviation of

$$N^{\frac{1}{2}}\left[\left\langle \log^2\frac{f_{1+}}{f_{0-}}\right\rangle_{\mathrm{Av}} - \left\langle \log\frac{f_{1+}}{f_{0-}}\right\rangle_{\mathrm{Av}}^2\right]^{\frac{1}{2}}, \tag{4}$$

where

$$\left\langle \log^2\frac{f_{1+}}{f_{0-}}\right\rangle_{\mathrm{Av}} = \int\int\left(\log\frac{f_{1+}}{f_{0-}}\right)^2 f_{0-}d\epsilon dx. \tag{5}$$

Numerical integrations give

$$\langle\log(P_{1+}/P_{0-})\rangle_{\mathrm{Av}} = -0.085N, \quad \text{with standard deviation} = 0.32\sqrt{N}, \tag{6}$$

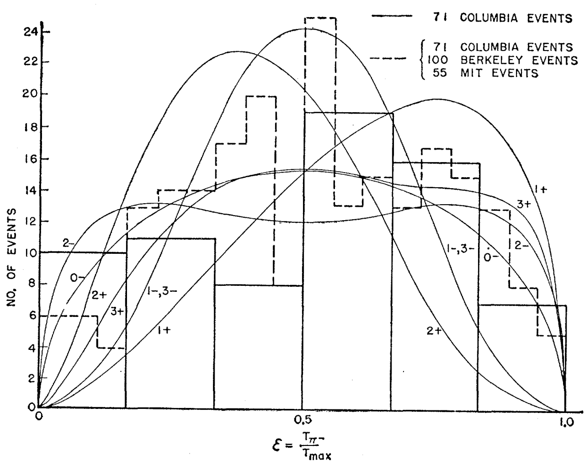

FIG. 1. Histogram of π^- energies in Bevatron τ decays. The 71 Columbia events are shown by the solid lines for 6 equal intervals in the π^- energy. The combined Columbia, Berkeley, and MIT data are given by the dashed lines for 18 equal intervals in ϵ. The curves are the theoretical distribution functions in ϵ alone using the functions listed in Table I. The (2−) curve is plotted for $A/B=1$. The maximum likelihood solution of A/B is used for the (3+) curve.

and

$$\langle\log(P_{1-}/P_{0-})\rangle_{\mathrm{Av}} = -0.39N, \quad \text{with standard deviation} = 0.51\sqrt{N}. \tag{7}$$

Thus our value of $P_{1-}/P_{0-}=10^{-33.5}$ is consistent with $10^{-27.9\pm4.3}$ which is obtained from Eq. (7) by putting $N=71$.

Equation (6) shows why it is possible for as many as 55 events to give a slight preference for f_{1+}, even if the true distribution is f_{0-}. For 55 events the expected ratio is $P_{1+}/P_{0-}=10^{-4.7\pm2.3}$. The value of $10^{1.1}$ obtained by MIT is 2.5 standard deviations off, which is not an unreasonable fluctuation. Equation (6) predicts $10^{-6.1\pm2.7}$ for 71 events. So the Columbia result of $10^{-7.9}$ is about 0.7 standard deviations on the other side. Equation (6) predicts $10^{-19.3\pm4.7}$ for 226 events. The combined Columbia, Berkeley, and MIT data give $10^{-13.8}$ which is about one standard deviation off in spite of the MIT fluctuation.

It is of interest to turn the argument around and ask how large a fluctuation is necessary for the Columbia data to turn out the way it did assuming f_{1+} is the true distribution. Numerical integrations give $P_{1+}/P_{0-}=10^{4.2\pm1.7}$ for 71 events. Our value of $10^{-7.9}$ is 7.1 standard deviations off, which is very unreasonable. For instance, in a Gaussian distribution the relative probability of a value 7.1 standard deviations from the mean is $10^{-10.9}$ of the value at the mean.

Another and more simple way of seeing that there is no serious contradiction between the Columbia and MIT data is to consider only those events with negative pions under 10 Mev. We found 13 and MIT found 2. The (0−) and (1+) distributions predict that we should have found either 10.1 or 2.4 and that MIT should have found 7.8 or 1.8 respectively. Although the 2 events of MIT are closer to the mean of 1.8 predicted by f_{1+}, there is still a chance of one in 80 of getting 2 or less events with a mean of 7.8. The 13 Columbia events are quite consistent with the mean of 10.1 predicted by f_{0-}, but in this case there is only one chance in 1.5×10^6 of getting 13 or more events with the mean of 2.4 predicted by f_{1+}. The ratio of the probabilities of finding 13 events with a mean of 2.4 compared to a mean of 10.1 is $10^{-5.1}$. This type of analysis can be extended to all energies by narrowing the energy interval and forming the combined relative probability. Such a procedure leads to Eq. (1) with the result $P_{1+}/P_{0-}=10^{-7.9}$ for the Columbia data.

Next we test our data to see whether they are random with respect to f_{0-}. Figure 1 shows a histogram of our events *vs* π^- energy. If these 6 points in the histogram were Gaussianly distributed with known standard deviations about the f_{0-} curve, we could perform a χ^2 test. In the limit of large N for each of the intervals this becomes a correct assumption. Since the smallest number of events in any interval is 7, the χ^2 test should be a useful approximation in evaluating our data. In the case where the distribution function is known, the quantity

$$M=\sum_{i=1}^{m}\frac{(N_i-p_iN)^2}{p_iN} \tag{8}$$

is χ^2 distributed with $(m-1)$ degrees of freedom.[11] Here p_i is the area under the curve in the ith interval and m is the total number of intervals. For the 6 experimental values in Fig. 1 the M-value with respect to the f_{0-} curve is 6.0. According to the χ^2 distribution of 5 degrees of freedom, the probability of M being this value or larger is 0.31.

So far only the distribution in ϵ has been tested. In Fig. 2 the 72 events are plotted *vs* both ϵ and x (commonly called a Dalitz plot). Dalitz has pointed out that the nonrelativistic f_{0-} gives a uniform distribution over the entire semicircular area.[1] We have divided this area from the center radially in 6 equal sectors and find 15, 8, 7, 11, 12, and 18 events in the 6 sectors starting from the bottom. In this case the M-value is 7.3, which has a χ^2 probability of 0.20.

V. SEARCH FOR POLARIZATION EFFECTS

Additional information on the τ spin can be obtained by examining the spacial orientations of the τ decays. Polarized τ mesons of nonzero spin would have an anisotropic distribution of the decay planes. However, if the tests for polarization give negative results, the τ spin need not necessarily be zero. On the other hand, if anisotropy is observed, then the spin must be nonzero. This is a weak test for zero spin for two reasons. First, the degree of polarization present may happen to be too small to be detected even though the production interaction be spin-dependent. Secondly, it is not clear to us how much depolarization there would be while the τ is at rest in the emulsion for $\sim 10^{-8}$ sec. This is a problem which bears further study. The depolarization effects while the τ is coming to rest have been studied and are expected to be small.[12]

A natural plane of reference is the plane of production of the K-mesons. The z-axis is taken normal to this plane with up being positive. The K-beam direction is chosen as the y-axis. In the τ-decay system the cross product, $\mathbf{P}_1\times\mathbf{P}_2$, is taken as the z'-axis, where $\mathbf{P}_1$ and $\mathbf{P}_2$ are the momenta of the two positive pions and $P_1>P_2$. The three Euler angles Θ, Φ, and Ψ are used to specify the orientation of the τ system with respect to the production system. Θ and Φ are the usual polar angles of z' in the production system and Ψ is the angle from the line of nodes (i.e., from the positive axis of rotation which generates Θ) to the π^- direction. z' is the axis of rotation which specifies the positive direction of Ψ. Anisotropy of the decay plane will show up in a plot of $\cos\Theta$ *vs* Φ. Whether or not Ψ is a useful angle

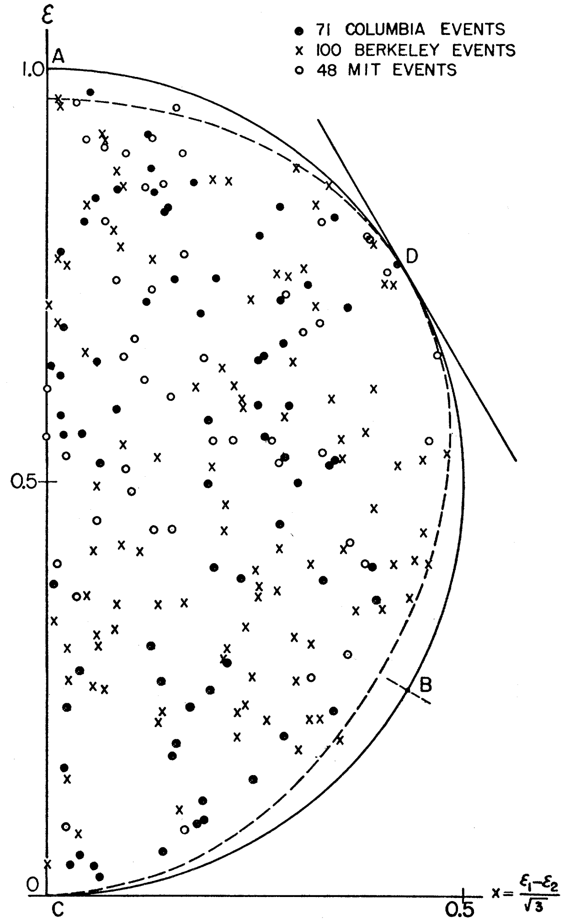

FIG. 2. Dalitz plot of the Columbia, Berkeley, and MIT Bevatron τ decays. Nonrelativistically the 3-pion phase space is proportional to the area. The semicircular boundary ABC is due to momentum conservation using nonrelativistic energies. The data are plotted using their experimental or relativistic energies and within experimental errors should lie inside the relativistic boundary (dashed curve).

to show polarization depends upon the particular spin and parity of the τ. We have not exhausted all the possibilities for a third coordinate. In Table II the five coordinates ϵ, x, Θ, Φ, Ψ are listed for each event. This information is sufficient for any type of correlation study. The 9 coordinates needed to specify the directions and energies of the 3 pions are reduced to 5 independent coordinates by the requirements of energy and momentum conservation.

If there is no polarization the distributions in $\cos\Theta$, Φ, and Ψ would be uniform. It is possible, however, to have strong correlations in a three-dimensional space which do not show up in any of the one-dimensional components of this space. For this reason the two-dimensional distributions of $\cos\Theta$ *vs* Φ, Ψ *vs* Φ, and $\cos\Theta$ *vs* Ψ are shown in Fig. 3. The data have been plotted in such a way that the information as to the separate indentities of the like pions has been given up. This is justified because the final state wave function

[11] H. Jeffreys, *Theory of Probability* (Oxford University Press, Oxford, 1939).
[12] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

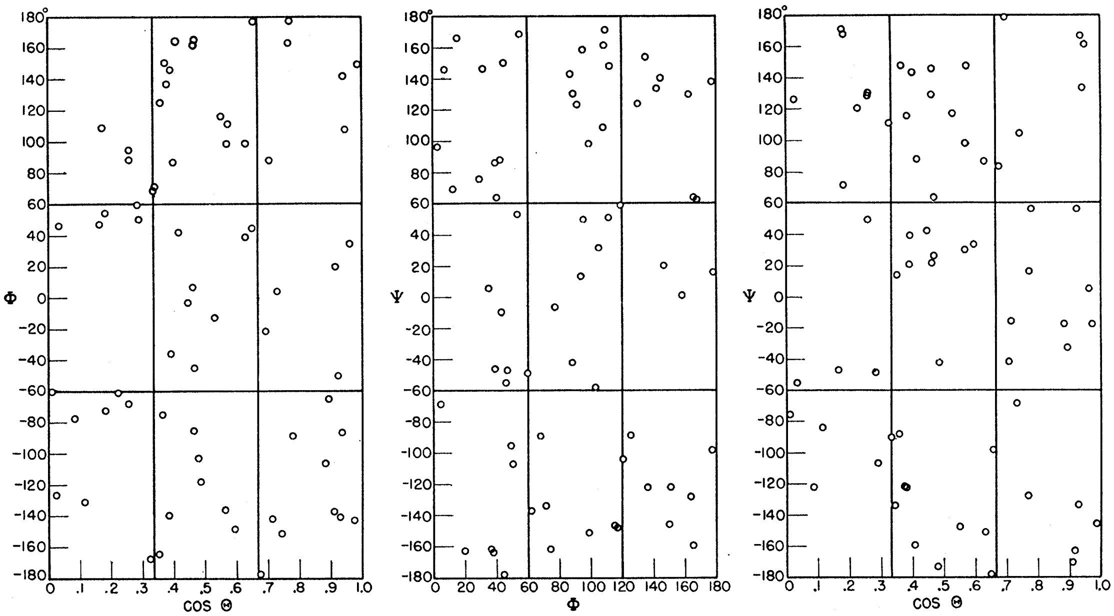

FIG. 3. Orientations of the Columbia τ decays with respect to the laboratory system. Θ, Φ, and Ψ are the Euler angles specifying the τ-decay system. For zero polarization of the τ beam, each of these two dimensional distributions should be uniform. The data has been plotted in such a way that the information as to the separate identities of the like pions has been given up.

must be symmetric with respect to an interchange of the like pions. There appears to be no apparent systematic clumping of events. In a χ^2 test each plot was divided into 9 equal areas and the respective M-values calculated to be 13.0, 4.8, and 7.7. All three of these values are compatible with the assumption of uniform distributions.

VI. DISCUSSION AND CONCLUSIONS

Before conclusions can be based on the relative probabilities in Table I, the validity of the distribution functions which were used must be studied. The functions used are based on three assumptions: (1) that the interaction radius is small, (2) pion-pion forces have little effect on the distribution function, and (3) that the interaction operator is essentially momentum independent over the region involved.

Feld[10] has shown that the modifications in the general shape of f_{JP} are small for interaction distances up to 1.4×10^{-13} cm.

Experimentally there is so far no evidence for a strong pion-pion interaction for low-energy pions. Furthermore the effects of such an interaction, if it exists, should show up on the Dalitz plot, Fig. 2. Here ϵ is the vertical axis and $(\epsilon_1-\epsilon_2)/\sqrt{3}$ is the horizontal axis. The advantage of such a plot is that nonrelativistically the area is then directly proportional to the phase space. A strong force between unlike pions should show up as a clustering of events about point B in Fig. 2. Forces between like pions would show up in the region of point A. As can be seen from Fig. 2, there are no such systematic effects. An extremely large s-wave pion-pion cross section would be necessary in order to alter f_{0-} significantly. The effect of a given pion-pion interaction may be calculated by the same method used to calculate the modification of the matrix element due to nuclear forces in the process $p+p\rightarrow p+n+\pi^+$.[13] For simplicity assume a pion-pion square well of depth $(\hbar K)^2/\mu$ and radius R. At $r=0$ the two-pion wave function is

$$\psi_{2\pi}(0)=\frac{k^2+K^2}{k^2+K^2\cos^2[(k^2+K^2)^{\frac{1}{2}}R]}, \tag{9}$$

where $\hbar k=|\mathbf{P}_1-\mathbf{P}_2|$. For a small τ-decay radius one might expect the matrix element to have this dependence on the relative momentum of the two pions. For values of KR less than one, there is almost no change in shape of f_{0-}. For $KR=1.2$ the maximum variation in f_{0-} is about a factor two. $KR=1.2$ corresponds to an s-wave scattering length $a_0=R$, or a zero-energy pion-pion cross section of $4\pi R^2$ which is about 250 millibarns for $R\approx\hbar/\mu c$.

Possible momentum dependence of the interaction operator is discussed by Dalitz.[1] However, even if the interaction were strongly energy dependent, the fact that the centrifugal potential is of long range requires that the distribution of events with low-energy negative pions go as P^n where n is no smaller than $2l$. We are using the notation P for the π^- momentum, l for the π^- angular momentum, and L for the angular

[13] K. A. Brueckner and K. M. Watson, Phys. Rev. 87, 621 (1952).

momentum of the system of the two like pions. The final state can be expanded as a series in these spherical waves with the conditions that L be even, that J be the vector sum of l and L, and that $(-1)^3(-1)^{l+L}=P$ (parity conservation). Table III lists the combinations of l and L meeting the above 3 conditions and having the lowest possible value of $(l+L)$. For even spin and odd parity it is possible to have $l=0$ and $L=J$. For all other spin-parity combinations $l>0$. The low-energy pion events of Fig. 2 appear to be uniformly distributed. A uniform distribution of events in region C of Fig. 2 is possible only in cases where l can be 0; i.e., for even spin and odd parity. The presence of so many low-energy π^- events is a strong argument against odd J or even J with even parity. The lowest-energy negative pion plotted in Fig. 2 is one of (1.18 ± 0.04) Mev found at Columbia. It is relevant to mention that in the Columbia experiment performed with the Brookhaven K^+-beam,[5] out of 21 τ mesons, the lowest energy negative pion had an energy of (0.36 ± 0.02) Mev.

A particle of even spin and odd parity cannot decay into two pions because the parity of the 2-pion system would have to be $(-1)^2(-1)^J$, which is even for even J. Thus the presence of so many τ decays with a low-energy π^- is a strong argument against the τ and θ being the same particle. So by visual inspection of Fig. 2 one sees that the data are unfavorable to all possibilities except (0−), (2−), (4−), etc. There is a further argument against spin and parity combinations which permit 2-pion decay. For all such spin and parity combinations the distribution function must go to zero at the circular boundary at least as fast as $\sin^2\theta$. Again we see thare are many experimental points quite close to the dashed boundary, which are an additional strong argument that the τ and θ must be different particles.

There is one final possibility to be disposed of. Suppose the spin and parity are such that 2-pion decay is permitted. Then the presence of low-energy π^- events might be explained by a strong pion-pion resonance near 75 Mev. Such a resonance might also enhance the distribution near point D on the semicircular boundary. However, Lee has pointed out that the distribution near all the other regions of the semicircle would not be enhanced and the presence of events in these regions rule out this possibility.[14]

Thus, even without knowledge of the precise distribution function, general wave mechanical arguments along with qualitative inspection of Fig. 2 lead to the same results as the quantitative likelihood ratios of Table I. This type of reasoning need not be completely qualitative. For example let us consider an extreme case of how f_{1+} could behave. According to the previous discussion it must start off as proportional to ϵ for small ϵ. We shall consider the extreme type of case where f_{1+} is linear in ϵ only up to 10 Mev and from then on is isotropic. We have calculated for the function

$$\frac{\pi}{8}f_{1+}(\epsilon,x)=\begin{cases}5.3\epsilon & \text{for} \quad \epsilon<0.2\\ 1.06 & \text{for} \quad \epsilon>0.2\end{cases}$$

that $P_{1+}/P_{0-}=10^{-3.0}$, using only the Columbia data.

In addition to the general features of the distribution functions giving information about the spin of the τ, there is a theoretical argument which makes high spin values unreasonable. This is the effect which the centrifugal barrier penetration factor has on slowing down the τ decay. Let $C(l,L)$ be the centrifugal barrier penetration factor. Order of magnitude calculations give $(PR/\hbar)^l(qR/2\hbar)^L$ for the ratio of the final state wave function evaluated at the interaction radius R to the value of the wave function at large distances, where P is the π^- momentum and $q=|\mathbf{P}_1-\mathbf{P}_2|$. Then[15]

$$C(l,L)\sim\left[\left(\frac{\bar{P}R}{\hbar}\right)^l\left(\frac{\bar{q}R}{2\hbar}\right)^L\right]^2$$
$$=10^{-1.55(l+1.1L)} \quad \text{for} \quad R=\hbar/M_\tau c. \qquad (10)$$

In order for the "natural lifetime" of 10^{-22} sec to be slowed down to 10^{-8} sec, $(l+1.1L)$ should be about 9. Hence, spins greater than $\sim$8 become unreasonable due to this effect alone. In addition to this effect, the violation of conservation of strangeness is expected to slow down the τ decay to the realm of the weak beta-decay type interactions, which corresponds to a lifetime of $\sim10^{-8}$ sec already.[16] According to this strong interpretation of the strangeness theory, any spin-parity combination other than (0−), (1+), or possibly (2−) would be ruled out because of the additional lengthening of the τ-lifetime due to the centrifugal barrier penetration factor.

The conclusions we draw are that the only reasonable possibilities for the τ are (0−) and (2−), with (4−) and (3+) as weak possibilities. We also conclude, as previously proposed by Dalitz,[2] that it is very unlikely that the τ and θ have the same spin and parity. The

TABLE III. Lowest order terms in spherical wave expansion of 3-pion final state. l is π^- angular momentum and L is angular momentum of system of the 2 like pions.

2-Pion decay forbidden J	P	l	L	2-Pion decay permitted J	P	l	L
0	−	0	0				
1	+	1	0	1	−	2	2
2	−	0 2	2 0	2	+	1	2
3	+	1 3	2 0	3	−	2	2
4	−	0 4 2	4 0 2	4	+	1 3	4 2

[14] T. D. Lee (private communication).

[15] For the case of (1−), $C(l,L)=10^{-6.5}$ which is not in agreement with the factor $\sim10^{-12}$ mentioned by Fabri in reference 9.

[16] M. Gell-Mann (to be published).

question arises whether there is any means of discriminating between (0−) and (2−). The fact that no polarization was observed is weak, but inconclusive, evidence against nonzero spin. There is no hope of discriminating against (2−) by means of the likelihood ratio because the shape of f_{2-} can be made to resemble f_{0-} very closely for certain values of the arbitrary parameter A/B (see Table I). In Fig. 1 f_{2-} is plotted for the choice $A/B=1$. The small differences between f_{2-} and f_{0-} in this plot could be made to overlap by letting the interaction have a small energy dependence. Thus in principle it is impossible to rule out (2−) by a Dalitz-type analysis. The distribution function f_{2-} also has an arbitrary plus or minus sign which is related to the relative phases of the two spherical waves ($l=0$, $L=2$) and ($l=2$, $L=0$). We assume this relative phase is limited to either 0 or 180 degrees by time reversal arguments.[17] Since the term following this plus or minus sign is so small compared to the other terms, either choice of sign can give $P_{2-}/P_{0-}\sim 1$.

The (3+) distribution function also contains A/B and a plus or minus sign as arbitrary parameters. Because of the common factor ϵ in f_{3+}, it is impossible for P_{3+}/P_{0-} to be ~ 1 for any choice of the arbitrary parameter here. This and higher spin states can be analyzed by finding the maximum-likelihood solution[11] for the arbitrary parameters and using those values to get the relative probability. In the case of f_{3+} we went through this procedure and found $A/B=1.4$ for the maximum-likelihood solution, with the minus sign giving a much better fit. Using just the Columbia data, this gave the relative probability $P_{3+}/P_{0-}=10^{-2}$. The values of $A/B=1.0$ and 1.6 reduced this relative probability a factor 10. We conclude that (3+) is not ruled out for the τ meson, but that it is unlikely.

In summary, we feel that the only reasonable possibilities left for the τ meson are (0−) and (2−) with (3+) and (4−) as weak possibilities. (1+) is strongly ruled out, as are all possibilities which permit two-pion decay. The effects of the centrifugal barrier and conservation of strangeness should rule out all higher spins—at least for spins above ~ 5. The data are statistically quite consistent with f_{0-} and can be made consistent with f_{2-} and f_{4-} when the arbitrary parameters in these distributions are so adjusted. The lack of any indication of pion-pion interaction effects can be used to set an upper limit on the s-wave scattering length for the pion-pion interaction. This upper limit turns out to be $a_0<R$ where R is the range of the pion-pion interaction.

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[17] This is strictly true in the absence of pion-pion forces, but is probably a good approximation in our case. We wish to thank T. D. Lee and V. L. Telegdi for bringing this simplification to our attention.

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Symmetries in Isotopic Spin Space and the Charge Operator

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A general relation is shown to exist between the charge and the operator inducing a symmetry with respect to the 1,2 plane in isotopic spin space. This relation is unique, i.e., it is the same for all types of fields (baryons and mesons).

THE connection between the charge Q and the third component I_3 of isotopic spin is well known. However, the appearance of an additive constant in the relation $Q(I_3)$ and, above all, the fact that this constant must be chosen differently for each type of field have long been a kind of puzzle for some physicists.

The experimental finding of the hyperons and heavy mesons and the discovery of the fact that their main properties are well accounted for by the Gell-Mann model[1] made this question even more acute but at the same time offered some hints to a possible answer. Under the assumption that the strong interaction Hamiltonians are (a) of the Yukawa type, and (b) invariant not only under rotations but also under re-

[1] M. Gell-Mann, Phys. Rev. **92**, 833 (1953); M. Gell-Mann and A. Pais, *Proceedings of the Glasgow Conference on Nuclear and Meson Physics*, 1954 (Pergamon, London, 1955), p. 342.