A detailed description of the technique applied in this investigation will be given later.

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Elementary Theory of the Excitations in Liquid Helium: New Model for Rotons*

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(Received April 6, 1956)

CONSIDERABLE light is thrown on the properties of liquid He^3 by a pair-rotator model in which it is assumed that N atoms form $N/2$ pairs.[1] If these pairs have one lowest energy level, three excited levels at about 0.8 calorie per pair-mole, and eight at about 5.4 calories per pair-mole, it is possible to account roughly for the thermodynamic properties at low temperatures.[2] These energy levels are about correct for a plane rotator moving in a hindering potential of about 12 calories per pair-mole due to the neighboring atoms.[3] The level at 0.8 is threefold because it is a spin triplet. The next level should also be a spin triplet, but presumably has a greater multiplicity because of random pairing[3] of a given atom with any of its neighbors.

The spin-triplet levels in He^3 do not exist in He^4 because of the lack of spin and the different statistics. The first excited (roton) level in He^4 would have an excitation of about 11 calories per pair-mole if it had the same hindering potential.[4] Actually the observed specific heat requires that it have an excitation energy of about 20, but the hindering potential would be expected to be greater than in He^3 on account of closer approach of the atoms. It is difficult to estimate how much increase in the zero-point energy of the atoms in the pair making the rotation, in the direction crosswise to the direction of rotation, contributes to the effective hindering potential. If, however, the difference between He^4 and He^3 arises from greater penetration of the rotating He^4 pairs into the steep part of the potential-energy curve, which begins at about 2.9 A, we may make an estimate of the difference in the hindering potential. If, for example the distance of approach of each of the atoms of a rotating pair to its neighbor, at the height of the hindering potential, were 2.8 A in He^3, it would, if proportional to the cube root of the molecular density, be about 2.55 A in He^4. From the potential-energy curve,[5] this could result in a fivefold increase in the hindering potential, giving an excitation energy of about 22 calories for a rotating pair in He^4.

The multiplicity of the excited level of He^4 appears to be about 18, reckoned on the pair basis. It has already been pointed out that this might result from the involvement of more than two atoms in a roton.[3] We now wish to propose a definite model for He^4, in which two pairs rotate cooperatively, somewhat as interlocking gears. With this model, the reduced mass μ would be doubled and the parameter[4] q would be doubled for a given hindering potential. The value of $16q+4\alpha$ would, however, increase only about 50%, and when allowance is made for the changed μ, the actual excitation energy would go down from 22 to about 16 calories. However, cooperative motion of this sort would require an increase in the zero-point energy, by the uncertainty principle. (This is the reason the levels of He^3 which involve less excitation are free of this effect.) On the other hand, the cooperative action itself might result in some lowering of the hindering potential.

The expected number of pairs of pairs, assuming four nearest neighbors in the liquid, would be $3^2 \times 4N/2$ (the number of nearest neighbor pairs is $4N/2$; each of the atoms in such a pair can connect with any one of the three remaining neighbors, all spatially different, to form a pair of pairs). Dividing by $N/2$, the number of pairs in the cell theory, we obtain an effective multiplicity of 36 per pair. This is twice the observed multiplicity, but it need not be expected that this degeneracy would be fully developed. On the whole, this elementary and pictorial theory gives a surprisingly good account of the experimental results for both He^3 and He^4, both as regards energy and multiplicity of the elementary excitations.

The question now arises, how does this picture fit in with that recently discussed by Feynman and others,[6] in which the roton is described as a sort of "smoke ring" of helium atoms? A cooperating pair of atoms certainly does resemble a smoke ring in some respects. There is, however, a difference in the method of counting the energy levels. We have used a cell theory, or what might be called a space configuration statistics. On account of the large zero-point energy this obviously involves an approximation. With heavier atoms than helium the approximation would be better, but we might expect that even with helium we would at least get correct orders of magnitude. In the theory of Feynman and the earlier theory of Landau, on the other hand, the multiplicity of levels is taken care of by considering a gas of excitations, and the momentum of these excitations is involved.

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