Compound Nucleus Effects in Deuteron Reactions: $C^{13}(d,\alpha)B^{11}$ and $C^{13}(d,t)C^{12}$†

JERRY B. MARION* AND GUSTAV WEBER‡
Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California
(Received February 27, 1956)

Excitation functions for the $C^{13}(d,\alpha)B^{11}$ and $C^{13}(d,t)C^{12}$ reactions have been measured at several laboratory angles in the range of deuteron energy from 1 to 3 Mev. Two prominent resonances were observed in the (d,α) reaction at 1.80 Mev ($\Gamma=55\pm10$ kev) and at 2.20 Mev ($\Gamma=22\pm4$ kev). The latter resonance was observed at 14 angles of observation from 25° to 140°. The 1.80-Mev resonance was observed at 9 angles in the (d,α) reaction and at 3 angles in the (d,t) reaction. The 2.20-Mev resonance was not found in the (d,t) reaction, but the α-particle group leaving B^{11} in the first excited state was observed to be resonant at this energy. Both of the narrow resonances exhibit striking interference effects with the nonresonant background. An analysis of the interference effects at the 2.20-Mev resonance indicates that deuterons with 3 or 4 units of angular momentum are responsible for the formation of the corresponding N^{15} level. The differential cross section for the (d,α) reaction was measured at 17 angles from 11.2° to 144.6° (c.m.) at $E_d=2.28$ Mev. Terms at least as high as $\cos^4\theta$ are necessary to fit the forward peak. This forward peaking in the (d,α) reaction persists over the entire energy range studied in spite of the large number of resonances in this region. This behavior suggests that some of the approximations usually made in the theory of highly excited states of nuclei are not of general validity.

INTRODUCTION

WHEN a nucleus is bombarded with deuterons the intermediate nucleus is formed in a highly excited state. For the light nuclei, these excitation energies are usually 15 Mev or greater. The virtual levels that are formed can almost always emit highly energetic protons, neutrons, and α particles. Consequently the energy states in these regions are generally not well defined and are difficult to study. The stripping theory has had great success in obtaining the spins and parities of the states of the residual nuclei formed in (d,n), (d,p), and (d,t) reactions, and much effort has recently been placed on this aspect of deuteron reactions. For these reasons the compound nucleus effects in these reactions have not been emphasized nor well studied except for reactions produced in the deuteron bombardment of the tightly bound nuclei, C^{12} and O^{16}. In these cases, the excitation energy in the compound nucleus is sufficiently low that many discrete resonances (with widths ≈ 0.1 Mev) have been observed.

It has been found that, in the light nuclei, the angular distributions obtained for (d,n) and (d,p) reactions are usually well described by the Butler[1] theory when the bombarding energy is about 4 Mev or greater, i.e., when the deuteron energy exceeds the Coulomb barrier. At energies of less than about 1 Mev, the stripping contribution to the cross section is usually small and the cross sections increase with bombarding energy following the penetration function, indicating that a compound nucleus is formed. It is therefore to be expected that the range of deuteron energies from about 1 Mev to about 4 Mev will produce competition between compound nucleus formation and the rising contribution from stripping.

These effects have been observed and studied in the $C^{12}+d$ reactions up to bombarding energies of 6.1 Mev.[2–4] At the lower energies, the reaction appears to proceed almost entirely by compound nucleus formation, while the stripping process appears to dominate at the higher energies. Even up to 6 Mev, however, compound nucleus effects are still quite apparent, giving rise to a much larger cross section for angles in the backward hemisphere than is predicted on the basis of the stripping theory.

Pronounced resonances also have been observed[5] in the deuteron bombardment of O^{16}. In this case compound nucleus effects in the (d,p) reaction appear to be more pronounced for the ground-state proton group than for the group leaving O^{17} in the first excited state. The $O^{16}(d,n)F^{17}$ reaction has also been found to exhibit weak, broad resonances, some of which correspond to those observed in the (d,p) reaction.[6]

Such detailed information as exists for the $C^{12}+d$ and $O^{16}+d$ reactions has not yet been obtained for the rest of the light nuclei, although a few angular distributions and excitation functions for at least one angle of observation have been measured for the (d,p)

† Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission. A report of this work was given at the Houston meeting of The American Physical Society, February, 1956 [Bull. Am. Phys. Soc. Ser. II, 1, 94 (1956)].
* National Science Foundation Postdoctoral Fellow.
‡ International Corporation Administration Research Fellow, on leave of absence from Max Planck Institut für Chemie, Mainz, Germany.

[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[2] Bonner, Eisinger, Kraus, and Marion, Phys. Rev. **101**, 209 (1956).

[3] Holmgren, Blair, Simmons, Stratton, and Stuart, Phys. Rev. **95**, 1544 (1954).

[4] McEllistrem, Chiba, Douglas, Herring, and Silverstein, Phys. Rev. **99**, 632(A) (1955); Takemoto, Dazai, Chiba, Ito, Suganamata, and Watanabe, J. Phys. Soc. Japan **9**, 447 (1954); G. C. Phillips, Phys. Rev. **80**, 164 (1950); Bonner, Evans, Harris, and Phillips, Phys. Rev. **75**, 1401 (1949); Bailey, Freier, and Williams, Phys. Rev. **73**, 274 (1948); Bennett, Bonner, Hudspeth, Richards, and Watt, Phys. Rev. **59**, 781 (1941).

[5] Stratton, Blair, Famularo, and Stuart, Phys. Rev. **98**, 629 (1955); N. P. Heydenburg and D. R. Inglis, Phys. Rev. **73**, 230 (1940); J. C. Grosskreutz, Phys. Rev. **101**, 706 (1956).

[6] Marion, Brugger, and Bonner, Phys. Rev. **100**, 46 (1955).

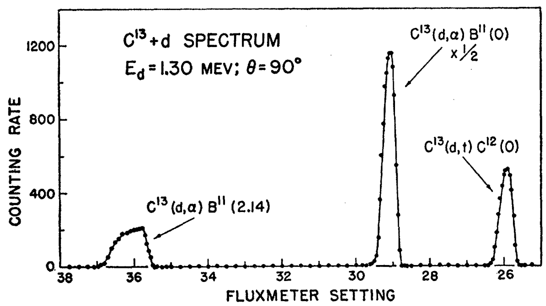

FIG. 1. Momentum profile of the α particles and tritons from the deuteron bombardment of C^{13} at an energy of 1.30 Mev. The observation angle was 90° and the target was a thin C^{13}-enriched foil. The CsI crystal detector was biased to respond to α particles and tritons but not to protons. The fluxmeter setting is proportional to the reciprocal of the momentum.

reactions on Li^6, Be^9, B^{10}, and C^{13}, up to a bombarding energy of 2 or 3 Mev by observing the emitted protons directly; residual activities corresponding to the (d,p) reactions on Li^7, B^{11}, and F^{19} have also been measured as a function of energy.[7] With the exception of the $Li^7(d,p)$ reaction, no pronounced resonances have been observed in these reactions. Much less experimental information concerning (d,α) reactions has been obtained. For bombarding energies above 1 Mev, only the $Li^6(d,\alpha)He^4$ reaction has been studied.[8]

There has not yet been developed an adequate method of making calculations from the theory of the interference between the stripping process and compound nucleus formation[9]; furthermore, the determination of resonance parameters for broad overlapping levels is virtually impossible even when stripping is not important. In spite of the present difficulties in the theoretical analysis of highly excited states in nuclei, it seems desirable to obtain further experimental information concerning this energy region in the hope that more detailed results will hasten the development of the theory.

In the planning of a program designed to investigate compound nucleus effects in deuteron reactions, it was felt desirable to make measurements on at least two different kinds of reactions for each target nucleus, namely, the (d,p) and (d,α) reactions. The (d,p) reaction is expected to exhibit interference effects between stripping and compound nucleus formation, while the (d,α) reaction presumably proceeds entirely through compound states. In addition, for the reactions Be^9+d and $C^{13}+d$, it is possible to obtain information concerning the (d,t) reactions as well.

In general, both (d,p) and (d,α) reactions have large positive Q-values. It is frequently possible to separate the proton groups by pulse-height analysis in a scintillation detector after stopping the slower deuterons in a thin foil. By using an angular distribution chamber with observation ports at a number of angles, one may readily obtain excitation functions and angular distributions for the (d,p) reactions. The situation becomes more complicated for the (d,α) and (d,t) reactions, since the emitted particles tend to be stopped in foils thick enough to prevent the deuterons from entering the detector, and magnetic or electrostatic analysis of these reaction products is usually necessary.

In this paper we shall describe the investigation of the $C^{13}(d,\alpha)B^{11}$ and $C^{13}(d,t)C^{12}$ reactions, using a magnetic spectrometer to separate the reaction products, and in a later paper[10] we shall describe the investigation of the $C^{13}(d,p)C^{14}$ reaction, using an angular distribution chamber and pulse height analysis in a scintillation detector to separate the desired proton group. Studies of deuteron reactions in Be^9, B^{10}, N^{14}, and F^{19} are in progress.

EXPERIMENTAL PROCEDURE

A. General

The separation of the reaction products in the deuteron bombardment of C^{13} was made with the Kellogg Laboratory's 16-in. double-focusing 180° magnetic spectrometer. The momentum profile obtained at a bombarding energy of 1.30 Mev and at an observation angle of 90° is shown in Fig. 1. The target was a thin C^{13}-enriched foil. The CsI crystal detector was biased to allow pulses from α particles and tritons, but not protons, to be counted. The protons could be readily discriminated against since the thickness of the crystal was approximately 0.0007 in. and protons with an energy greater than about 1.4 Mev pass through the crystal and consequently give only small pulses. The α particles and tritons were completely stopped and gave pulses corresponding to their full energy. The peaks shown in the spectrum are due to tritons leaving C^{12} in the ground state and α particles leaving B^{11} in the ground and first excited (2.14 Mev) states. The large width of the lower energy α-particle group is due to straggling in the foil target.

The differential cross section for a particular reaction may be computed from a knowledge of the spectrometer resolution and solid angle, the number of target atoms per cm², the deuteron flux, and the area under the peak in the momentum profile obtained according to the relation,

$$A=\int\frac{N(I)}{I}dI,$$

where I is the fluxmeter current and $N(I)$ is the number of particles detected per unit deuteron flux at a fluxmeter setting, I.[11] In order to obtain an excitation

[7] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).
[8] W. Whaling and T. W. Bonner, Phys. Rev. **78**, 258 (1950); Heydenburg, Hudson, Inglis, and Whitehead, Phys. Rev. **74**, 405 (1948).
[9] W. Tobocman, "A review of deuteron stripping at low and intermediate energies," 1955 (unpublished); R. G. Thomas, Phys. Rev. **100**, 25 (1955).
[10] J. B. Marion and G. Weber (to be published).
[11] Snyder, Rubin, Fowler, and Lauritsen, Rev. Sci. Instr. **21**, 852 (1950).

function for a reaction, one needs to determine the area under the profile peak at each bombarding energy, since this quantity is directly proportional to the cross section. To facilitate the taking of data, only the maximum counting rate in each profile at a particular energy was determined. Ordinarily, five or six points were necessary to establish the peak. An empirical correction curve, relating the peak counting rate to the cross section, was determined by taking complete profiles at six bombarding energies between 1 and 3 Mev with the foil target. This curve was found to be identical for the ground-state α particles and tritons and is shown in Fig. 2. At low bombarding energies, straggling of the particles in the carbon foil and the greater thickness of the target to the incident deuterons increase the width of the profile and the shape tends to become trapezoidal. This effect is evident for the lower energy α-particle group in Fig. 1. As a result, the correction curve rises steeply as the bombarding energy is decreased below about 1.8 Mev. At higher energies the effects tending to increase the profile width are less important and the peak counting rate is almost directly proportional to the cross section.

The correction curve of Fig. 2 was obtained at an observation angle of 90°. That the same curve was applicable for other angles was checked by taking profiles at 45° and 135°. No significant deviations from the 90° curve were noted. Owing to the greater straggling of the α-particle group leaving B^{11} in the first excited state, a separate correction curve was necessary for this group.

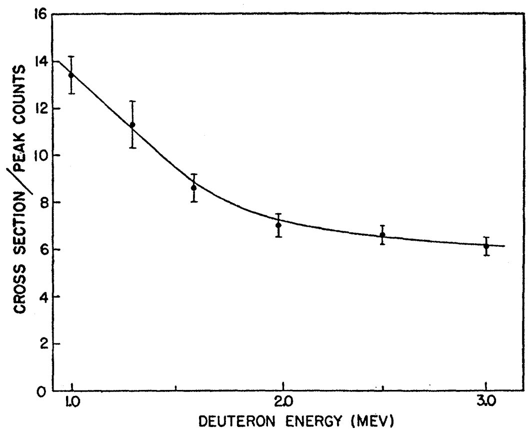

FIG. 2. Empirical correction curve relating the peak counts in a momentum profile to the differential cross section. The curve is for the ground-state α particles and tritons from $C^{13}+d$ and was obtained with the foil target.

B. Target Composition and Thickness

Targets enriched in C^{13} were prepared by cracking methyl iodide onto hot tantalum strips, following the method of Seagrave[12] and Milne.[13] The manufacturer's[14] determination of the C^{13} enrichment was 60%. In order to check this figure, the ratio of the $C^{13}(d,p)$ to the $C^{12}(d,p)$ cross section was measured in the same geometry and at the same energy with foils made from the enriched sample and from a sample containing natural carbon (1.1% C^{13}). Owing to the low counting rate from the $C^{13}(d,p)$ reaction in the natural carbon target, this measurement was accurate only to about 15%. The value obtained for the enrichment was 54±8%, in satisfactory agreement with the manufacturer's figure. The 60% value was used in the calculation of the cross sections.

Two targets were used in these experiments, a self-supporting foil, stripped from the tantalum, and a thinner target on a tantalum backing. The thicknesses of these targets to 1.00-Mev deuterons was measured by observing the displacement of the elastic scattering edge between a tantalum blank and the tantalum backing of the targets; a piece of clean tantalum was placed behind the foil for this measurement. The measured thicknesses at 45° to the beam direction were 38.4±1.0 kev and 8.8±1.0 kev for the foil and thin target, respectively. Using a value of $(7.38\pm0.15)\times10^{-15}$ ev-cm^2 for the stopping cross section of carbon for 500-kev protons,[15] it was calculated that the foil target presented $(3.12\pm0.10)\times10^{18}$ C^{13} atoms/cm^2 when oriented at an angle of 45° to the beam.

For 2.2-Mev deuterons, these targets had thicknesses of 27 kev and 6 kev. A less precise but confirmatory check on these thicknesses was obtained in the following manner. The relative counting rates from the two targets were measured at the same bombarding energy, and then the width of a narrow resonance in the $C^{13}(d,\alpha)$ reaction at 2.20 Mev was obtained for both targets. These three measurements allowed the target thicknesses to be calculated; the values obtained were 28 kev and 7 kev.

C. Spectrometer Constants and Differential Cross-Section Calculations

Direct measurements of the solid angle and resolution of the magnetic spectrometer have previously been made[16] by scattering protons from copper. For the geometry employed in these experiments, the solid angle was 62.4×10^{-4} steradian and the resolution $p/\Delta p$ was 452. An indirect check of these spectrometer constants was obtained by measuring the $C^{13}(d,p)$ and $C^{12}(d,p)$ differential cross sections at the same bombarding energy and angle, using the foil target, with both the magnetic spectrometer and an angular distribution chamber (described later), the solid angle of which was precisely known. The measurements were carried out at $E_d=2.00$ Mev, $\theta(\text{lab})=30°$ and the

[12] J. D. Seagrave, Phys. Rev. 85, 197 (1952).
[13] E. A. Milne, Phys. Rev. 93, 762 (1954).
[14] Eastman Kodak Company, Rochester, New York.
[15] Reynolds, Dunbar, Wenzel, and Whaling, Phys. Rev. 92, 742 (1953).
[16] W. Whaling, private communication.

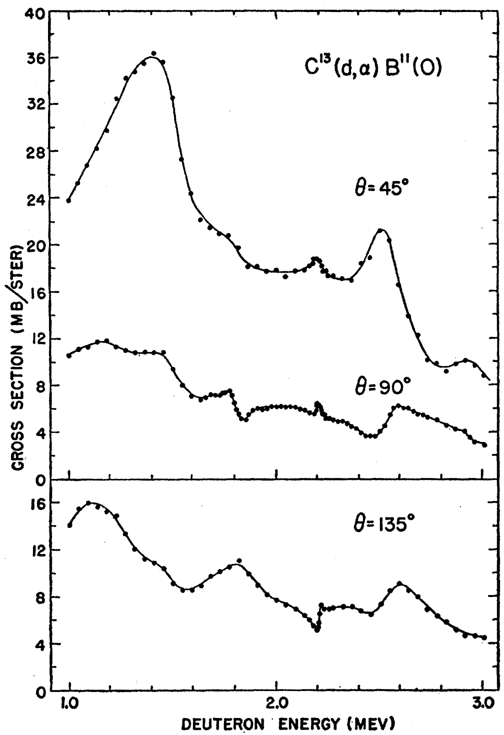

FIG. 3. Excitation curves for the $C^{13}(d,\alpha)B^{11}$ (ground state) reaction as a function of the bombarding energy at observation angles of 45°, 90°, and 135°. The foil target was used and the energy scale is uncorrected for target thickness.

values obtained are shown in Table I. The indicated errors are not independent since both include the uncertainties in target thickness, stopping cross section, and current integrator calibration. Since there is no significant difference between these measurements, it was concluded that the spectrometer constants used could not be seriously in error. A further check on these cross section measurements is afforded by the determination of the $C^{12}(d,p)$ differential cross section at $E_d=2.00$ Mev, $\theta(\text{lab})=30°$, made by the Rice group,[2] who obtained a value of 26 ± 4 mb/sterad (lab), in excellent agreement with the present value.

In view of the uncertainties in the target thickness (3%), in the stopping power of carbon (2%), in the C^{13} enrichment (assumed 3%), in the spectrometer constants (1.5%), in the current integrator calibration (1%), in the determination of the peak counting rates (3%), and in the application of the correction curve (8%), the absolute differential cross-section measurements for the $C^{13}(d,\alpha)$ and $C^{13}(d,t)$ reactions are probably accurate to about 10%.

TABLE I. Differential cross sections for $C^{12}(d,p)$ and $C^{13}(d,p)$ at $E_d=2.00$ Mev, $\theta(\text{lab})=30°$.

Reaction	Cross section, mb/sterad (lab): Magnetic spectrometer	Cross section, mb/sterad (lab): Angular distribution chamber
$C^{12}(d,p)$	27 ± 3	28.6 ± 1.4
$C^{13}(d,p)$	4.4 ± 0.4	4.26 ± 0.21

Values of the $C^{13}(d,\alpha)B^{11}$ and $C^{13}(d,t)C^{12}$ differential cross sections at $E_d=1.00$ Mev, $\theta(\text{lab})=90°$, have previously been obtained by Li and Whaling.[17] Their values are compared with the present results in Table II. There is good agreement between the two determinations for both reactions. The published differential cross sections for the $C^{13}(d,t)C^{12}$ reaction measured at 2.19 Mev by the Minnesota group[3] are larger than the present determination by approximately a factor of 2. However, the method by which their target thickness was measured probably underestimated the average thickness by about 30%.[18] This would tend to reduce the cross sections and bring them into satisfactory agreement with the present results.

RESULTS

When C^{13} is bombarded with deuterons, the following reactions may take place[7]:

$$C^{13}(d,p)C^{14},\quad Q=5.944 \text{ Mev};$$
$$C^{13}(d,n)N^{14},\quad Q=5.317 \text{ Mev};$$
$$C^{13}(d,\alpha)B^{11},\quad Q=5.163 \text{ Mev};$$
$$C^{13}(d,t)C^{12},\quad Q=1.309 \text{ Mev}.$$

The corresponding reactions in C^{12} all have much smaller Q-values, so that the highest energy particles of a given type produced in a carbon target enriched in C^{13} result from the $C^{13}+d$ reactions.

By using the foil target, excitation functions were measured for the range of bombarding energy from 1 to 3 Mev at observation angles of 45°, 90°, and 135° for the $C^{13}(d,\alpha)B^{11}$ and $C^{13}(d,t)C^{12}$ reactions. These curves are shown in Figs. 3 and 4. The differential cross sections in millibarns per steradian (lab) are plotted against incident deuteron energy (i.e., uncorrected for target thickness). At $\theta=45°$, the (d,t) curve could not be extended above 2.1 Mev since for these energies the triton momentum exceeds the limit of the spectrometer.

In addition to a number of broad, overlapping resonances, several pronounced peaks were observed in both reactions. The only resonances which appear to be common to both reactions occur at 1.4 Mev and at 1.80 Mev. The 1.4-Mev peak occurs at all three angles

TABLE II. Comparison of the differential cross sections for the $C^{13}(d,\alpha)B^{11}$ and $C^{13}(d,t)C^{12}$ reactions at $E_d=1.00$ Mev, $\theta(\text{lab})=90°$.

Reaction	Cross section, mb/sterad (lab): Li and Whaling[17]	Cross section, mb/sterad (lab): Present work
$C^{13}(d,\alpha)B^{11}$	7 ± 2	10.6 ± 1.1
$C^{13}(d,t)C^{12}$	1.7 ± 0.4	1.5 ± 0.2

[17] C. W. Li and W. Whaling, Phys. Rev. **82**, 122 (1951); W. Whaling, private communication.
[18] J. M. Blair, private communication.

for the (d,α) reaction, whereas the (d,t) reaction shows only a weak effect at 90° and none at 45°. Both reactions show the most pronounced effects in the forward direction and both show a tendency for the total cross section to decrease with increasing bombarding energy.

One of the more interesting features of these excitation curves is the occurrence of the two relatively narrow resonances in the (d,α) reaction at 1.80 Mev ($\Gamma=55\pm10$ kev) and at 2.20 Mev ($\Gamma=22\pm4$ kev). The corresponding N^{15} states are located at 17.71 ± 0.01 and 18.06 ± 0.01 Mev; such narrow levels have not been observed at comparable excitation energies in other nuclei.[7] Since both of these resonances show rather striking interference effects with the nonresonant background, the region from $E_d=1.6$ to 2.3 Mev was investigated at a number of other angles. At angles of 72.5°, 80°, 90°, 100°, 110°, 120°, 130°, and 140°, this region was studied with the thin target; these data are shown in Fig. 5. In the forward direction the foil target was used and observations were made at angles of 25°, 35°, 45°, 55°, and 65°; Fig. 6 shows these results. At an angle of 35°, the excitation function for the α-particle group leaving B^{11} in the first excited state (2.14 Mev) was measured for this same energy range. This curve is presented in Fig. 7 and it shows the 1.80- and 2.20-Mev resonances as well.

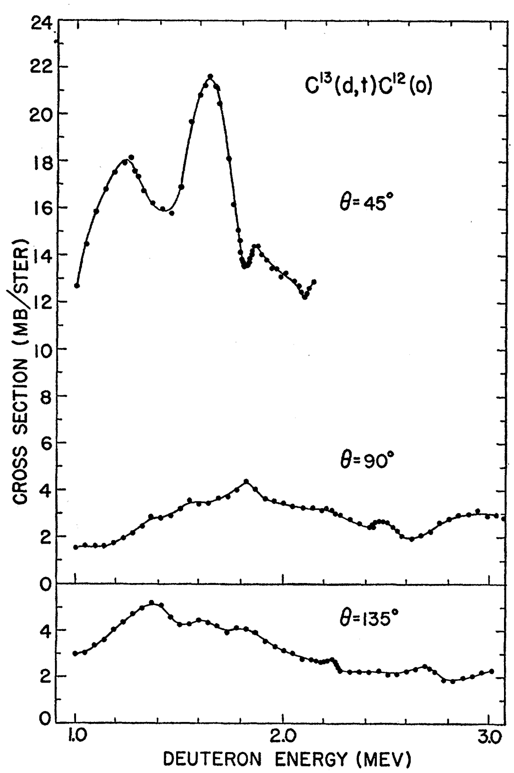

FIG. 4. Excitation curves for the $C^{13}(d,t)C^{12}$ (ground state) reaction as a function of the bombarding energy at observation angles of 45°, 90°, and 135°. The foil target was used and the energy scale is uncorrected for target thickness.

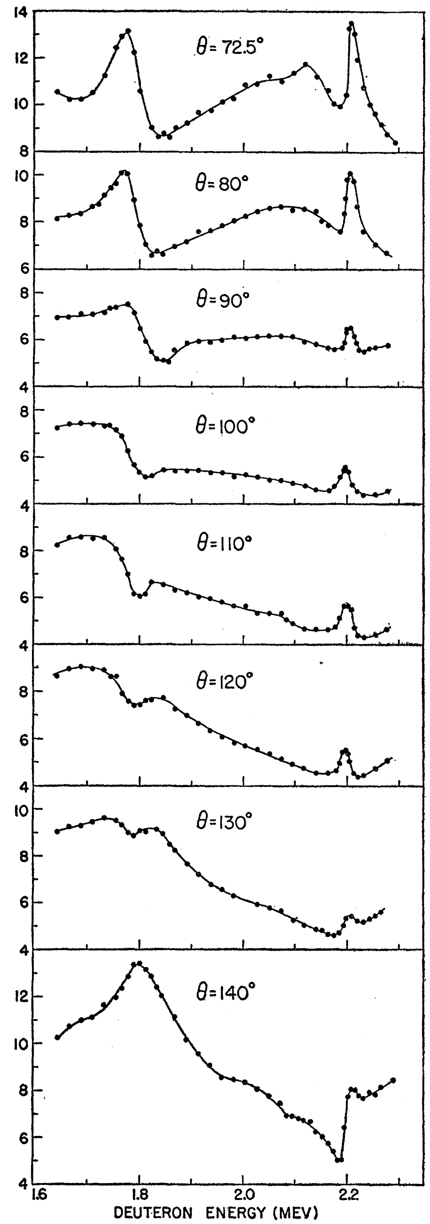

FIG. 5. Excitation curves for the $C^{13}(d,\alpha)B^{11}$ (ground-state) reaction as a function of the bombarding energy in the vicinity of the 1.80- and 2.20-Mev resonances at 8 observation angles from 72.5° to 180°. The thin target was used so that the corrections for target thickness are negligible.

In the (d,t) excitation curve at $\theta=90°$ and 135°, there is a weak maximum near 2.2 Mev. In order to determine if this effect was due to the same compound nuclear state that gives rise to the narrow resonance in the (d,α) reaction, the energy region near 2.20 Mev was investigated with the thin target at angles of 90°, 120°, and 140°. These curves are shown in Fig. 8 in which the (d,α) curves have been included for comparison. The cross sections given are for the $C^{13}(d,t)$ reaction. It appears that there is a weak resonance for

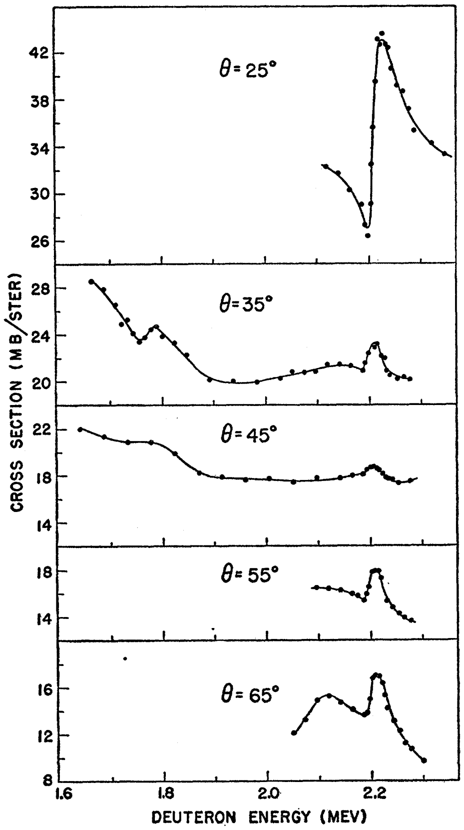

FIG. 6. Excitation curves for the $C^{13}(d,\alpha)B^{11}$ (ground state) reaction as a function of the bombarding energy in the vicinity of the 2.20-Mev resonance at 6 observation angles in the forward directions. The foil target was used and the energy scale is uncorrected for target thickness.

the (d,t) reaction in this energy range, but the resonance energy is about 25 kev greater than that for the narrow (d,α) resonance. Furthermore, the width is about twice that for the (d,α) peak. Therefore, these resonances appear to be distinct and if the (d,t) reaction has a resonance corresponding to the 2.20-Mev (d,α) resonance, it is certainly very weak.

Table III summarizes the results obtained thus far on resonances in $C^{13}+d$ reactions. The (d,n) resonance at 2.45 Mev is probably the same as the 2.55-Mev resonance for the (d,p) and (d,α) reactions. Similarly, the 1.55-Mev resonance in the (d,n) reaction is probably to be identified with the 1.40-Mev resonance which appears in the (d,α) and (d,t) reactions and in the integrated cross section for the (d,p) reaction,[10] since in the latter case the 90° yield shows the peak shifted to 1.55-Mev.[10,19] Figure 5 shows that the 1.80-Mev resonance becomes quite weak at the most backward angles investigated. At 140° the effect appears to have almost disappeared and an underlying resonance becomes discernible. This peak probably corresponds to the 1.78-Mev resonance observed in the (d,n) reaction.

TABLE III. Resonances in $C^{13}+d$.

E_d (Mev)[a]	Emitted particles	Γ (kev)	N^{15} (Mev)[b]	References
0.64	n,p		16.70	c, d, e
0.85	n		16.89	c
1.10	α	broad	17.10	f
1.24±0.04	t	≈200	17.22	f
1.40±0.04	p, α, t	≈400	17.35	e, f, g
1.55	n			c
1.64±0.04	t	≈200	17.57	f
1.78±0.05	n, α	≈600	17.69	c, f, h
1.80±0.01	$\alpha, t, (p)$	55±10	17.71	f
2.20±0.01	α	22± 4	18.06	f
2.23±0.02	p, t	≈ 50	18.08	e, f
2.45	n	≈400	18.3	h
2.55	p, α	270±70		e, f
3.46±0.03	n	≈150	19.15	h

[a] Corrected for target thickness.
[b] Uncorrected for barrier penetration.
[c] (d,n) data from J. E. Richardson, Phys. Rev. **80**, 850 (1950).
[d] (d,p) data from Koudijs, Valckx, and Endt, Physica **19**, 1133 (1953), and C. D. Curling and J. O. Newton, Nature **165**, 609 (1950).
[e] (d,p) data from reference 10.
[f] (d,α) and (d,t) data from this work.
[g] (d,p) data from reference 19.
[h] (d,n) data from Marion, Bonner, and Cook, Phys. Rev. **100**, 847 (1955).

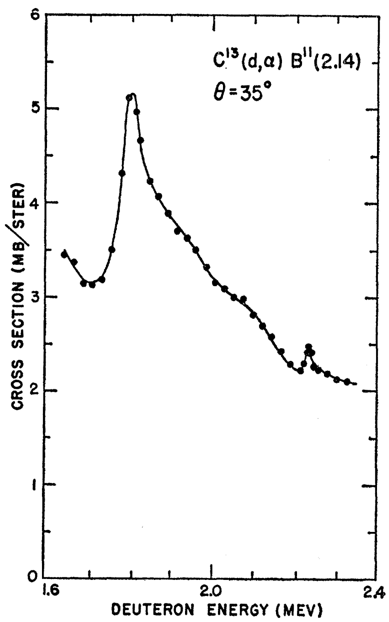

FIG. 7. Excitation curve for the $C^{13}(d,\alpha)B^{11}$(2.14-Mev state) reaction as a function of the bombarding energy in the vicinity of the 1.80- and 2.20-Mev resonances. The foil target was used and the energy scale is uncorrected for target thickness.

At a bombarding energy of 2.28 Mev, some additional data were obtained on the differential cross section for the (d,α) reaction at angles near the forward direction. These data were combined with the differential cross-section data of Figs. 3, 5, and 6, and are presented in the form of an angular distribution in Fig. 9. Measurements at 17 center-of-mass angles from 11.2° to 144.6° are shown and the data are summarized in Table IV. Two curves, one containing powers of $\cos\theta$ up to $\cos^2\theta$ and the other up to $\cos^4\theta$ are shown in Fig. 9. The solid curve is of the form $\sigma(\theta)=6.0+6.2\cos\theta+3.1\cos^2\theta+2.8\cos^3\theta+21.9\cos^4\theta$ and the dashed curve is $\sigma(\theta)$

[19] Bennett, Bonner, Hudspeth, Richards, and Watt, Phys. Rev. **59**, 781 (1941).

$=4.0+9.4\cos\theta+22.0\cos^2\theta$. It appears that terms at least as high as $\cos^4\theta$, and probably higher, are necessary to give an adequate description of the angular distribution. The indicated errors apply to the absolute differential cross section.

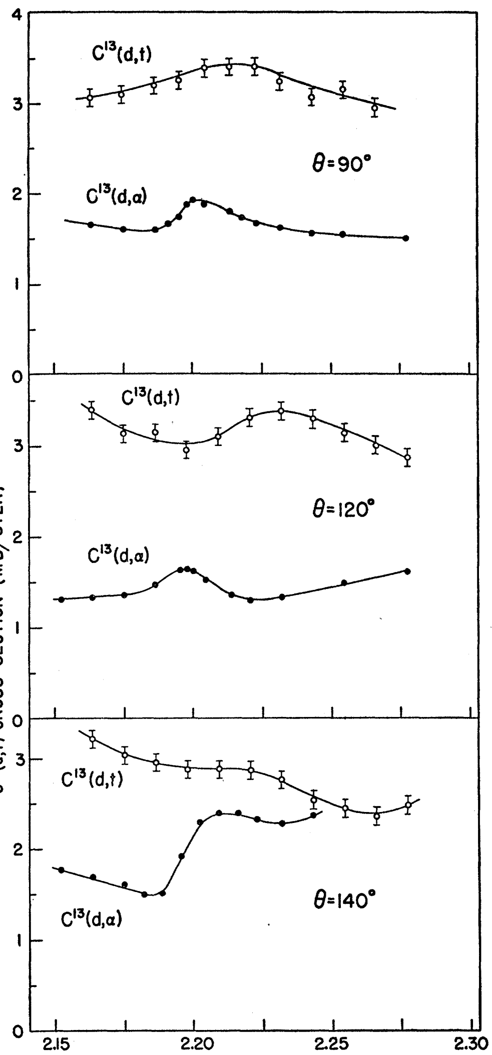

FIG. 8. Excitation curves for the $C^{13}(d,t)C^{12}$ (ground-state) reaction as a function of the bombarding energy in the vicinity of the 2.20-Mev resonance. The curves for the $C^{13}(d,\alpha)$ reaction are the same as those shown in Fig. 5 and have been included for purposes of comparison. The thin target was used so that corrections for target thickness are negligible. The indicated differential cross sections are for the (d,t) reaction only.

DISCUSSION

A. Narrow Resonance

It is clear from the manner in which the shape of the (d,α) excitation function changes with the angle of observation at the 2.20-Mev resonance that there are strong interference effects between the resonant and nonresonant contributions to the cross section. In order

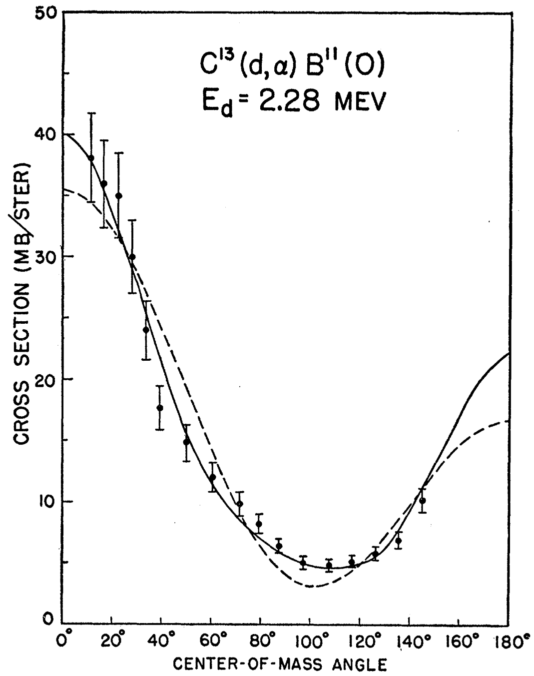

FIG. 9. Differential cross section for the $C^{13}(d,\alpha)B^{11}$ (ground-state) reaction at a bombarding energy of 2.28 Mev. The dashed curve is a least-squares fit to the experimental points of the form $a+b\cos\theta+c\cos^2\theta$; the solid curve includes terms up to $\cos^4\theta$. See text for the coefficients.

to obtain an angular distribution for the resonant portion, the two effects were separated according to the method of Bonner, Eisinger, Kraus, and Marion,[2] in which the total differential cross section, $\sigma(E,\theta)$, is written as the square of the sum of two parts, a nonresonant or "background" term and a resonant term which obeys the Breit-Wigner relation:

$$\sigma(E,\theta)=\left|\sigma_B^{\frac{1}{2}}(E,\theta)e^{i\delta(\theta)}+\tfrac{1}{2}i\Gamma\frac{\sigma_R^{\frac{1}{2}}(\theta)}{E-E_r+\frac{1}{2}i\Gamma}\right|^2,$$

where $\sigma_B^{\frac{1}{2}}(E,\theta)$ = amplitude of the nonresonant background, $\sigma_R^{\frac{1}{2}}(\theta)$ = amplitude of the resonant contribution, $\delta(\theta)$ = relative phase between $\sigma_B^{\frac{1}{2}}$ and $\sigma_R^{\frac{1}{2}}$, Γ = total width, E = bombarding energy, E_r = resonance energy, and θ = c.m. angle.

TABLE IV. Differential cross sections for the $C^{13}(d,\alpha)B^{11}$ reaction at $E_d=2.28$ Mev.

θ (c.m.)	σ, mb/sterad (c.m.)	θ (c.m.)	σ, mb/sterad (c.m.)
11.2°	38.0±3.8	71.5°	9.8±1.0
16.8°	36.0±3.6	79.3°	8.2±0.8
22.5°	34.9±3.5	87.1°	6.4±0.6
28.0°	30.0±3.0	97.2°	5.0±0.5
33.6°	24.1±2.4	107.1°	4.8±0.5
39.1°	17.6±1.8	116.8°	5.1±0.5
50.1°	14.8+1.5	126.2°	5.8±0.6
60.9°	12.0±1.2	135.5°	6.9±0.7
		144.6°	10.2±1.0

It is necessary to choose the three parameters $\sigma_B^{\frac{1}{2}}$, $\sigma_R^{\frac{1}{2}}$, and δ to fit the experimental points. This is done by first estimating what the excitation function would be if the resonance were not present; i.e., a curve is drawn through the resonance region which joins smoothly on to the experimental curve a few half-widths away from E_r. Such curves must be drawn for the data at each angle and therefore $\sigma_B^{\frac{1}{2}}$ is a function of E and θ. Then $\sigma_R^{\frac{1}{2}}$ and δ are chosen to make $\sigma(E,\theta)$ agree with the experimental curve. This process is carried out graphically and the parameters giving the best fit are said to characterize the resonance contribution at that angle. This procedure is an oversimplification of the actual situation since it neglects the effects of spin. If such effects were taken into account, both terms in the expression for $\sigma(E,\theta)$ would require averages over the projections of the incoming channel spins and sums over the projections of the outgoing channel spins. If the compound nucleus state is assumed to have a definite J-value and parity, it is reasonable to consider only the smaller of the two deuteron angular momenta which may form this state; however, the nonresonant background contains contributions from many broad levels with differing J-values and parities and many deuteron angular momenta are effective.

It is just this latter effect which introduces a great complication into the analysis since the background contains contributions from deuteron angular momenta at least as large as 2, and probably 3. Furthermore, the data of Figs. 5 and 6 indicate that the resonance still appears in the total cross section and is asymmetric. This could occur only if the interference term, (which contains $\sigma_B^{\frac{1}{2}}\sigma_R^{\frac{1}{2}}$) does not vanish when integrated over angles, and therefore implies that $\sigma_B^{\frac{1}{2}}\sigma_R^{\frac{1}{2}}$ contains a $P_0(\cos\theta)$ term. If $\sigma_R^{\frac{1}{2}}$ contains angular terms up to $P_l(\cos\theta)$, if $\sigma_B^{\frac{1}{2}}$ contains terms up to $P_{l'}$, and if $\sigma_B^{\frac{1}{2}}\sigma_R^{\frac{1}{2}}$ contains terms P_L then $|l-l'|\leq L\leq|l+l'|$. The nonvanishing of the integrated interference term implies that the minimum value of L is zero. Consequently, the nonresonant background must contain contributions from deuteron angular momenta at least as large as that required to form the 2.20-Mev resonance. Therefore, $l\leq l'_{\max}$. Figure 10 shows the angular distribution of the interference term obtained from the analysis of the data of Figs. 5 and 6. Terms at least as high as $\cos^6\theta$ are necessary to give a description of such a complicated angular dependence. Therefore, $l+l'\geq 6$ and since $l'\geq 2$ and probably ≥ 3, it seems reasonable to conclude that $l=l'\geq 3$.

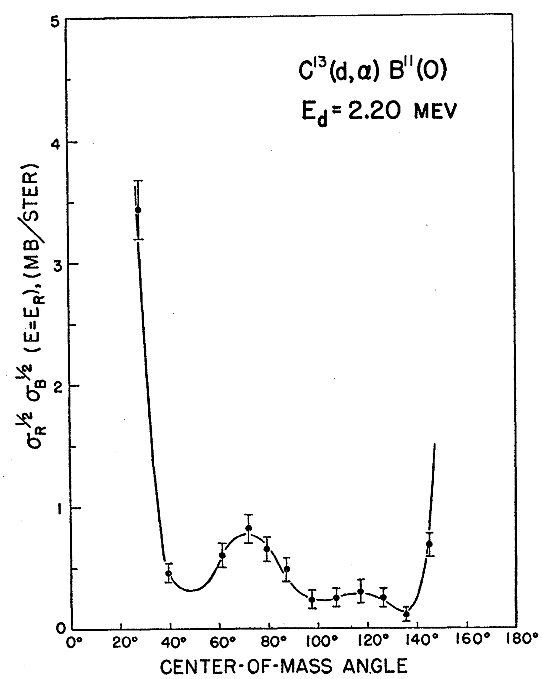

FIG. 10. Angular distribution of the interference term, $\sigma_B^{\frac{1}{2}}\sigma_R^{\frac{1}{2}}$, at the 2.20-Mev resonance in the $C^{13}(d,\alpha)B^{11}$ (ground-state) reaction.

TABLE V. Reduced widths for deuterons and α particles at the 2.20-Mev resonance in $C^{13}(d,\alpha)B^{11}$.

l	Deuterons[a] $\gamma_d^2\Big/\left(\frac{3}{2}\frac{\hbar^2}{\mu R}\right)$	α-Particles[b] $\gamma_\alpha^2\Big/\left(\frac{3}{2}\frac{\hbar^2}{\mu R}\right)$
0	6.7×10^{-5}	3.6×10^{-3}
1	1.2×10^{-4}	3.8×10^{-3}
2	4.4×10^{-4}	4.2×10^{-3}
3	3.8×10^{-3}	5.1×10^{-3}
4	6.4×10^{-2}	7.1×10^{-3}
5	3[c]	

[a] $R(C^{13}+d)=4.40\times10^{-13}$ cm.
[b] $R(B^{11}+\alpha)=5.60\times10^{-13}$ cm.
[c] Extrapolated.

Figure 11 shows the relative phase, $\delta(\theta)$, between the resonant and nonresonant parts of the differential cross section at the 2.20-Mev resonance. That the interference term does not vanish when integrated over angle is also shown by the shape of the function $\delta(\theta)$, which would tend to have as many positive as negative values if the integral were to vanish.

The total resonance cross section may be estimated in the following manner:

$$\int\sigma_R d\Omega=\left(\int\sigma_B^{\frac{1}{2}}\sigma_R^{\frac{1}{2}}d\Omega\right)^2\left(\int\sigma_B d\Omega\right)^{-1}.$$

The value obtained was $\int\sigma_R d\Omega\cong 1$ mb. This is to be compared with a value of about 150 mb for the total cross section at this energy. Even though the resonance cross section is less than one percent of the total cross section, the resonance is rendered observable by virtue of the much larger ($\approx 16\%$) ratio of the interference term to the total cross section. The total width, Γ, is 22 ± 4 kev. It is known that Γ_t is small; furthermore, no narrow resonance at this energy is found in either the

(d,p) reaction[10] or in the (d,n) reaction,[20] so that $\Gamma_p+\Gamma_n$ must also be small. Since the angular distribution indicates that deuteron angular momenta of at least 3 are involved, Γ_d is probably much smaller than Γ_α, and therefore, $\Gamma_\alpha \cong \Gamma$. Consequently, Γ_d may be obtained from the Breit-Wigner formula if we take for the statistical factor

$$\omega=\frac{(2J+1)}{(2s+1)(2I+1)(2l+1)}=\frac{1}{6}\frac{(2J+1)}{(2l+1)}\cong\frac{1}{6}.$$

The partial width obtained for the deuteron is $\Gamma_d \cong 0.2$ kev. By using the tables of Coulomb wave functions of Breit and his co-workers,[21] the penetrabilities for the deuterons and α-particles were computed. The ratios of the reduced widths to the sum-rule limit were then calculated and are shown in Table V. For a deuteron angular momentum of 4, the reduced width is only 6% of the limit. The angular distribution indicates that $l_d \geq 3$, while the reduced widths require that $l_d \leq 4$. Therefore, 3 or 4 units of angular momentum must be supplied by the incoming deuteron in the formation of the 18.06-Mev state in N^{15}. Another possibility that could account for the very small width of this level is that this state is rendered narrow by violation of isotopic spin conservation. This possibility can be considered since the region of excitation near 18–20 Mev in N^{15} probably contains the first $T=5/2$ levels of that nucleus. If the reaction were to proceed through such a state it would require $\Delta T=1$, and consequently a small width.

It was not possible to analyze the 1.80-Mev resonance by the method used on the 2.20-Mev resonance because of the larger width and the more complicated nature of the background (there is an additional resonance at 1.78-Mev). It is evident, however, from the excitation curves of Figs. 5 and 6 that the resonance contribution is peaked near 90°. The fact that the effect of this resonance almost disappears at the 130° and 140° (Fig. 5) and is quite small at 45° (Fig. 3) indicates that the isotropic term in the angular distribution is probably small.

B. (d,α) Angular Distribution

The angular distribution of the α particles taken at a bombarding energy of 2.28 Mev (Fig. 9) shows a pronounced forward peaking which requires terms at least up to $\cos^4\theta$ to give an adequate representation of the experimental data. It is clear from the excitation functions measured at observation angles of 45°, 90°, and 135° (Fig. 3), that this forward peaking persists over the entire deuteron energy range studied (1 to 3 Mev).

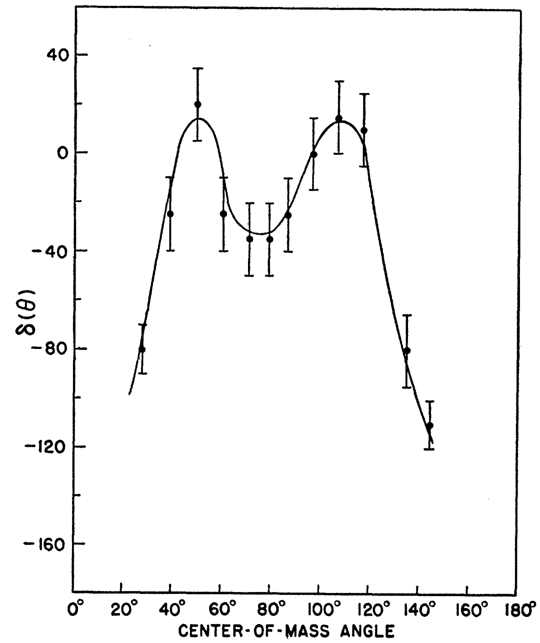

FIG. 11. Relative phase between the resonant and nonresonant parts of the cross section at the 2.20-Mev resonance in the $C^{13}(d,\alpha)B^{11}$ (ground-state) reaction.

Wolfenstein[22] has shown that, in reactions proceeding through a compound nucleus which is sufficiently highly excited to allow the use of the statistical theory, the emitted particle groups are distributed symmetrically about 90°. This result requires the assumption that the interference terms between states with differing J-values and parity (which would destroy the symmetry) tend to cancel when averaged over many levels because the outgoing waves have random phases. This is equivalent to the assumption that the $\gamma_{\lambda c}$, the square roots of the reduced widths of the levels λ and the channels c, which occur in the R-matrix formulation of nuclear reactions,[23] are uncorrelated in sign. Since (d,α) reactions are thought to proceed through compound nucleus formation and since the regions of excitation reached in deuteron-induced reactions contain many overlapping levels, on the basis of random phases (i.e., uncorrelated $\gamma_{\lambda c}$), the angular distribution should be symmetric about 90°. Since the present experiments have shown that this is not the case for a wide range of excitation energies in the $C^{13}(d,\alpha)$ reaction, it is possible that a highly excited compound nucleus can be formed in such a way that the $\gamma_{\lambda c}$ are correlated.

Thomas[24] has pointed out that in the surface region of nuclei, where the interactions are not as strong as in the interior, the $\gamma_{\lambda c}$ can become correlated and that therefore by increasing the channel radii to include this

[20] Marion, Bonner, and Cook, Phys. Rev. **100**, 847 (1955).
[21] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).
[22] L. Wolfenstein, Phys. Rev. **82**, 690 (1951).
[23] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[24] R. G. Thomas, Phys. Rev. **97**, 224 (1955).

surface region, the R-matrix theory can account for such processes as pickup and stripping reactions. This procedure requires the explicit use of wave functions and a weak-interaction potential. It does not seem realistic to try to account for the forward peaking in a (d,α) reaction using the customary approximations for the treatment of surface phenomena (i.e., pickup or direct interaction). Nevertheless, it appears that some sort of a direct interaction process holds the only hope as a basis for calculations.

ACKNOWLEDGMENTS

The authors would like to express their appreciation for several helpful discussions with Professor R. F. Christy, Professor W. A. Fowler, and Professor T. Lauritsen. They would also like to thank the National Science Foundation and the International Cooperation Administration for research fellowships held during the course of this work, and the members of the Kellogg Radiation Laboratory staff for extending to them the use of the laboratory's facilities.

PHYSICAL REVIEW VOLUME 102, NUMBER 5 JUNE 1, 1956

Anomaly in Energy Level Density Measurements*

G. Igo and H. E. Wegner
Brookhaven National Laboratory, Upton, New York

(Received January 23, 1956)

An analysis of nuclear reactions in the intermediate energy range has been made in terms of the statistical theory of nuclear reactions using the Fermi gas level density, $\text{const}\times\exp[(aE)^{\frac{1}{2}}]$, where E is the energy of excitation. The quantity, a, determined from excitation function experiments and from the (α,p) reaction at 40 Mev shows an anomalous behavior when plotted against atomic number, A. On the other hand, the values of a determined from several inelastic scattering and reaction experiments show the correct dependence on A. The effects of noncompound-nucleus processes are discussed.

The dependence of the density of energy levels on excitation energy can be determined[1] from reaction and inelastic scattering experiments and from a study of excitation functions for reactions in which neutrons are emitted from the compound nucleus produced by bombardment with energetic particles or photons. Data from reaction, inelastic scattering, and excitation function measurements has been analyzed in terms of the statistical model of nuclear reactions employing the Fermi gas level density formula, $\omega(E) = C\exp[(aE)^{\frac{1}{2}}]$. The quantity, $\omega(E)$, is the energy level density; C and a are constants; and E is the nuclear excitation energy. Figure 1 shows a compilation of the values of a obtained from the analysis.

The anomaly is that the values of a obtained from the analysis of γ-ray, neutron, and charged-particle excitation function data[2] and from the 40-Mev (α,p) experiment[3] are reasonably independent of A and abnormally small for large A. These values of a are grouped about the line, $a = 8\ \text{Mev}^{-1}$. This is in disagreement with the Fermi gas prediction of $a = \text{const}\times A$.

On the other hand, values of a obtained from an analysis of some of the available reaction and inelastic scattering data[4] are in agreement with the Fermi gas prediction of $a = \text{const}\times A$.[5] Some experimental data, such as that from the (p,p') experiment at 31 Mev[6] and from the (n,p) reaction at 14 Mev,[7] have not been included in the analysis since they have been interpreted as being predominantly noncompound-nucleus processes.[8] Also, the values of a obtained from the inelastic scattering of 14-Mev neutrons[9] are not included in the present compilation because of the uncertainties which arise in correcting the spectra for the very large contribution of the second neutron from $(n,2n)$ events. This correction has been calculated in several ways,[10] and the values of a which result differ considerably. However, they are in rough agreement with the upper curve in Fig. 1 regardless of how the corrections are made.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] See for instance, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 365.

[2] K. G. Porges, Phys. Rev. **101**, 225 (1956); E. Kelly and E. Segré, Phys. Rev. **75**, 999 (1949); G. Temmer, Phys. Rev. **76**, 424 (1949); Brolley, Fowler, and Schlacks, Phys. Rev. **88**, 618 (1952); Bleuler, Steffins, and Tendam, Phys. Rev. **90**, 460 (1953); R. W. Hayward, University of California Radiation Laboratory Report UCRL-3093 (unpublished); P. R. Byerly, Jr., and W. E. Stephens, Phys. Rev. **83**, 54 (1951).

[3] Eisberg, Igo, and Wegner, Phys. Rev. **100**, 1309 (1955).

[4] J. M. B. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) **A67**, 586 (1954); P. C. Gugelot, Phys. Rev. **81**, 51 (1951); P. C. Gugelot, Phys. Rev. **93**, 425 (1954).

[5] J. M. B. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) **A67**, 586 (1954). These authors have also pointed out that level densities determined from fission neutron measurements have the same dependence on A.

[6] R. M. Eisberg and G. Igo, Phys. Rev. **93**, 1039 (1954).

[7] E. B. Paul and R. L. Clarke, Can. J. Phys. **31**, 267 (1953); L. Rosen and L. Stewart, Phys. Rev. **99**, 1052 (1955).

[8] "Statistical Aspects of the Compound Nucleus", Brookhaven National Laboratory Report, 1955 (unpublished), p. 68.

[9] E. R. Graves and L. Rosen, Phys. Rev. **89**, 343 (1953).

[10] L. Rosen and L. Stewart, LA-1560, Los Alamos Scientific Laboratory Report, 1953 (unpublished); A. Tomasini, Nuovo cimento **12**, 134 (1954); and K. J. LeCouteur, Birmingham Conference Notes, 1953 (unpublished).