of 10^9 sec^{-1}. Since the attenuation maximum was observed at a frequency of 10^7 sec^{-1} it would seem that the barrier effect, corresponding to $\frac{1}{2}E_F$ in Eq. (18), which is neglected in the calculation of the maximum transition frequency cannot be ignored. On the other hand Friauf's rather arbitrary estimate of the barrier appears too high. On the basis of the above result one might hope to observe the transition from relaxed to unrelaxed behavior at a frequency of about 10^8 sec^{-1} at 400°C. Our attenuation measurements were not precise, however, and the remarks based upon them should be regarded as suggestive only. No attenuation maximum was observed at 58 kc/sec.

In Fig. 4 we have plotted the adiabatic compressibility of AgBr as a function of the absolute temperature. Using Eq. (1) we have computed the isothermal compressibility[12] which is also shown in Fig. 4. The isothermal curve is based on the specific heat data of Pochapsky.[6] Since his data are lower than the data of Christy and Lawson,[5] it is possible that K_T is overestimated near the melting point. The adiabatic compressibility of NaCl as determined by Hunter and Siegel[10] is included for comparison. It should be noted that the point of inflection in K_S found by Hunter and Siegel in NaCl does not appear in AgBr nor does the turnover near the melting point. On the whole the behavior of AgBr appears to be much more nearly what one might expect for this type of lattice than appears to be the case for NaCl.

[12] The value stated for K_T in reference (7) was incorrect because of an error in calculation. K_T should have read 2.69×10^{-12} (dyne/cm^2).

ACKNOWLEDGMENTS

The authors wish to acknowledge their indebtedness to Mr. N. D. Nachtrieb and to Mr. K. Lindholm for assistance given in design and construction of the specimen holders and other apparatus. Thanks are due also to Mr. D. Zimmerman for advice and assistance in the operation of the electronic equipment.

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Temperature Dependence of the Hall Coefficients in Some Copper Nickel Alloys*

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The two Hall coefficients and the resistivity of Cu–Ni alloys containing 80%, 70%, and 60% Ni have been measured from 4°K to 400°K. These alloys were chosen because early measurements had indicated that conduction in the 3*d* bands of these ferromagnetic alloys could be neglected. The values of R_0*, from precise Hall measurements at high fields, are in excellent agreement with the predictions of the band model that divides the conduction band into parallel and antiparallel bands. The data on all three alloys fit the predicted curve at temperatures below their Curie points and also fit the predicted Curie point anomaly due to $\partial M/\partial B\neq0$ at high fields in this temperature region. The dependence of R_1* upon resistivity is in agreement with the law $R_1^*=A\rho^n$. For 80% and 70% Ni the coefficient A and the exponent n agree with the values obtained for pure Ni. A significant departure from the pure Ni value is observed, however, in the 60% Ni alloy. Evidence is presented that indicates this law should be extrapolated through the temperature region above the Curie point where the alloys are paramagnetic.

It is now well known that the Hall effect in ferromagnetic materials is described by the equation

$$e_H=E_y/i_x=R_0H_z+4\pi R_1M_z. \quad (1)$$

In a recent paper Smit[1] suggests that this equation accurately describes the effect only when the material is magnetically saturated, otherwise the equation is only approximate. It should be pointed out that Eq. (1) was first suggested[2] to explain data taken far below saturation in the region where $4\pi M_z\gg H_z$. Since also $R_1\gg R_0$, the first term of Eq. (1) could not be detected, even though precise measurements were made simultaneously of the Hall voltage and of both B and H in bars of material where such measurements were possible. In those tests several different[3] materials were measured. Observations made at the time, but not published, showed that the first hysteresis loops, of materials having no previous magnetic history, did not close for either M_z *versus* H_z or E_y *versus* H_z. In all cases the E_y was so exactly proportional to M_z that the coefficient R_1 could be obtained within a few percent by dividing the closing discrepancy in E_y by the

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† Submitted by one of the authors (FEA) in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Carnegie Institute of Technology.
[1] J. Smit, Physica **21**, 877 (1955).
[2] Emerson M. Pugh, Phys. Rev. **36**, 1503 (1930).
[3] E. M. Pugh and T. W. Lippert, Phys. Rev. **42**, 709–710 (1932).

closing discrepancy in M_z. More recently[4] it has been shown that the existence of a domain structure should not hinder but in fact should contribute to the proportionality between macroscopically measured Hall voltages and magnetization below saturation. We know of no evidence that Eq. (1) is not an exact relationship at all fields.

Following the first[5] accurate identification[4,6] of the R_0 term, the majority of our efforts have been concentrated upon obtaining accurate values for R_0. This has been done for two reasons: first, values of R_0 provide much information concerning band structures; second, due to the relative ease with which it can be obtained, considerable information is already available concerning R_1. Our efforts have also been concentrated largely on alloys of ferromagnetic elements whose atomic numbers differ by 1 or 2, since the band structures of these have been presumed to be well established. If the Hall effects in these can be understood, less well understood band structures can be investigated with more confidence.

Recent measurements[5,7] of the Cu–Ni alloys at 4°K, 20°K, 77°K, and room temperature show such large negative R_0's for the ferromagnetic alloys at the low temperatures that there can be only 0.3 electron per atom sufficiently mobile to influence the Hall effect. A band model proposed by one of us[8] following ideas given by Mott[9] appears to explain this data. At low temperatures the parallel d band is full and the antiparallel d band is partly empty. Therefore the parallel s electrons cannot be scattered into the d band while the antiparallel s electrons can. Thus the 0.3 electron per atom in the s band which have spins parallel to the field have much higher mobility. The band model[8] can be used to make detailed predictions for the temperature dependence of the R_0's in any of these alloys in which the R_0 at low temperature corresponds to 0.3 electron per atom.

The extraordinary effect is much more strongly temperature dependent than the ordinary effect, and until recently its origin had remained obscure. In 1950 Samoilovich and Kon'Kov[10] proposed that the extraordinary effect was due to a spin-orbit interaction, but it was not until 1954 that Karplus and Luttinger[11] demonstrated that the spin-orbit interaction could account for the strong temperature dependence. The theory predicts that the extraordinary Hall coefficient will vary with temperature as ρ^2, where ρ is the electrical resistivity. Using the data of Jan and Gijsman,[12] Karplus and Luttinger showed that R_1 varies as ρ^n, with $n=1.94$ for Fe and $n=1.42$ for Ni.

The present investigation was undertaken to examine in more detail the temperature dependence of both Hall coefficients in those Cu–Ni alloys for which the low temperature data indicated an effective number of electrons of the order of 0.3 per atom.

EXPERIMENTAL METHOD AND DATA ANALYSIS

The samples that were used in this investigation are the same samples that were used in the earlier work.[5,7] These samples are nominally 2 cm wide and 1 mm thick with 4.5 cm exposed between copper lugs that are soldered to each end. Small holes were drilled in the copper lugs for wires and thermocouples to measure the longitudinal electric field and temperature gradient. The resistivity was then determined by $\rho=E_xA/I$, and, although this is not the most ideal experimental arrangement for this determination, it is sufficiently accurate for the present purpose.

To obtain the detailed temperature dependence, R_0 must be determined in the temperature range from 77°K to room temperature. For accuracy the intermediate temperatures must be maintained for a considerable length of time. This requirement was satisfied by using baths of liquid methane, ethylene, and propane, which have normal boiling points of 111.6°K, 169.2°K, and 230.9°K, respectively. Measurements were first made with a liquid nitrogen bath. When the nitrogen was removed, the Dewar and sample were cold, so that methane could be transferred with a minimum loss of liquid, and a correspondingly small amount of gaseous methane in the laboratory to present an explosive hazard. This process was repeated with the other liquids until room temperature was reached.

When measurements above room temperature were desired, a furnace was made by sealing a Pyrex tube at the bottom end and a heating element of Nichrome ribbon was wound around the bottom half. This element was wound noninductively to eliminate the magnetic field that would normally be produced by the heating element. The sample was set inside and covered with a low-viscosity silicone oil (Dow Corning D.C. 200, 1.5 centistokes). Since a temperature gradient can be produced in the oil bath when the top is heated more than the bottom, the windings of the heating element were made closer together at the bottom.

The measuring techniques have been described[13] previously. The Hall coefficients are calculated from the data by means of Eq. (1). Since the quantity that is normally measured is the magnetic induction B instead of the magnetic field H, it is more convenient to take account of the demagnetizing factor for a thin

[4] Pugh, Rostoker, and Schindler, Phys. Rev. **80**, 680 (1950).
[5] A. I. Schindler and E. M. Pugh, Phys. Rev. **89**, 295 (1950).
[6] Some fine measurements of the Hall effect in ferromagnetic materials were made before 1920 from which good values of R_0 can now be obtained, though their significance was not recognized at the time. See A. W. Smith, Phys. Rev. **30**, 1 (1910).
[7] P. Cohen, Office of Naval Research Technical Report, June, 1955 (unpublished); thesis, Carnegie Institute of Technology, 1955 (unpublished).
[8] E. M. Pugh, Phys. Rev. **97**, 647 (1955).
[9] N. F. Mott, Proc. Roy. Soc. (London) **A153**, 699 (1936).
[10] A. Samoilovich and U. Kon'Kov, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 782 (1950).
[11] R. Karplus and J. M. Luttinger, Phys. Rev. **95**, 1154 (1954).
[12] J.-P. Jan, Helv. Phys. Acta **25**, 677 (1952); J.-P. Jan and H. M. Gijsman, Physica **18**, 339 (1952).
[13] S. Foner and E. M. Pugh, Phys. Rev. **91**, 20 (1953).

sample and write this equation in the form

$$e_H = R_0 B + (R_1 - R_0) 4\pi M = R_0 B + R_s 4\pi M. \quad (2)$$

The derivative with respect to B of the above equation is

$$de_H/dB = R_0 + R_s 4\pi \partial M/\partial B, \quad (3)$$

which gives the ordinary Hall coefficient directly when the term involving $\partial M/\partial B$ can be neglected, as it can at sufficiently high fields and at temperatures well below the Curie point. Coefficients calculated by Eq. (3) will be designated by $R_0^* = de_H/dB$. The R_0^* may not be even approximately equal to R_0 when the sample is not magnetically saturated because $\partial M/\partial B$ is multiplied by R_1 which is normally much larger than R_0. Fields up to 15.5 kilogauss were used for the Cu–Ni alloys studied in this investigation. Even fields this large are not sufficient at temperatures in the neighborhood of the Curie point, and great care must be exercised in interpreting the results. The effect of this term on R_0^* can be clearly observed in the experimental results.

The extraordinary Hall coefficient can be obtained from the experimental data by extrapolating the high-field linear region to $B=0$. If the intercept at $B=0$ is designated by $(e_H)_0$, Eq. (2) shows that

$$(e_H)_0 = (R_1 - R_0) 4\pi M_s, \quad (4)$$

where $4\pi M_s$ is the saturation inductance. In most cases of interest R_0 can be neglected in comparison with R_1, and coefficients computed from experimental data by this method will be denoted by $R_1^* = (e_H)_0/4\pi M_s$.

It is well known[13] that accurate Hall coefficients cannot be obtained without taking account of the spurious transverse potential produced by the combined Ettingshausen and Seebeck effects. An exact opposite set of effects also exist, and errors due to them apparently have not been investigated. As shown in Fig. 1, the sample S is normally connected between two heavy copper leads. In practice, the sample is never in contact with an infinite heat reservoir, and the two junctions I and II will be heated unequally by the Peltier effect. A temperature gradient will then exist between the ends of the sample, and a transverse potential due to the Nernst effect will be superimposed upon the Hall potential. Just as the potential due to the Ettingshausen and Seebeck effects reverses with reversal of either the magnetic field or the primary current, so also does the potential due to the Peltier and Nernst effects. It should be noted, however, that in ferromagnetic materials the Ettingshausen and Nernst effects exhibit a behavior similar to that of the Hall effect and should therefore be formulated in terms of an ordinary and extraordinary coefficient. Recently Genkin and Priporawa[14] have made measurements on the Nernst effect that were very similar to those made by Pugh[2] on the Hall effect. From their results they conclude that the Nernst potential must contain a contribution that is proportional to the intensity of magnetization. On this basis K. Meyer[15] proposed that the Nernst effect should obey the relation

$$E_y = -(Q_0 H + Q_1 M)\partial T/\partial x. \quad (5)$$

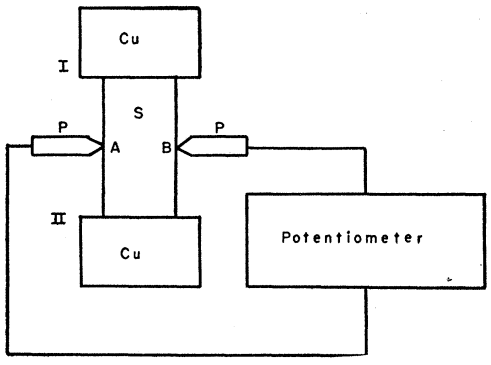

FIG. 1. Experimental arrangement for measuring the Hall effect.

TABLE I. Summary of the Hall coefficients and resistivity data.

T(°K)	$R_0^* \times 10^{13}$ (volt cm/amp gauss)	$R_1^* \times 10^{12}$ (volt cm/amp gauss)	$\rho \times 10^6$ ohm-cm
	60% Ni–40% Cu		
64	20.3	34.86	34.12
77	18.7	36.00	35.34
112	17.5	39.19	37.76
169	18.0	44.89	42.00
210	19.7	...	44.57
231	22.2	...	45.33
250	21.6	...	...
271	17.4	...	...
281	16.7	...	...
300	...	...	46.91
310	15.0	...	47.29
313	15.5	...	...
325	14.9	...	...
342	15.5	...	...
	70% Ni–30% Cu		
64	19.6	32.0	24.89
77	18.6	34.4	25.31
112	16.9	38.6	27.55
169	14.8	47.1	31.40
206	14.1	53.7	34.41
230	14.7	57.3	36.22
269	14.0	69.4	39.55
291	15.5	79.1	41.48
295	16.5	81.6	41.89
304	19.3	92.7	42.55
316	23.2	203	43.71
327	25.1	...	44.29
341	26.6	...	44.90
354	25.9	...	45.23
364	25.5	...	45.77
378	22.8	...	...
399	18.3	...	46.48
	80% Ni–20% Cu		
77	20.6	18.5	16.22
112	19.7	20.6	17.55
169	18.5	25.4	20.26
231	16.7	31.5	23.83
298	13.7	47.4	...
302	...	...	29.27

[14] N. M. Genkin and G. P. Priporawa, J. Exptl. Theoret. Phys. (U.S.S.R.) **26**, 323 (1954).
[15] K. Meyer, Z. Naturforsch **10a**, 166 (1955).

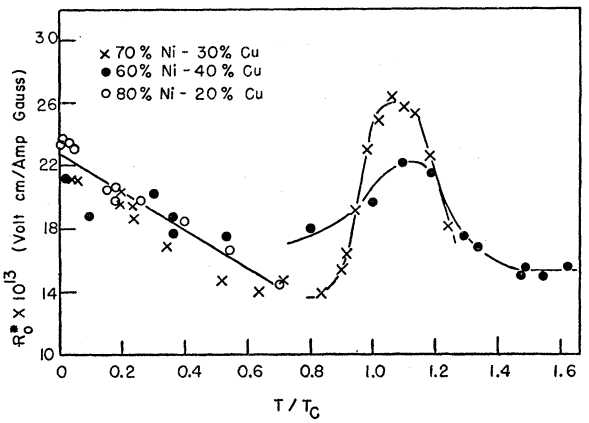

FIG. 2. Observed values of R_0^* *versus* T/T_c showing the temperature dependence of R_0 and the anomaly near the Curie point.

A similar relation was also proposed a short time later, although independently, by the authors.[16]

The total transverse electric field is the sum of that due to the Hall and Nernst effects. For ferromagnetic materials this will be given by

$$E_y=(R_0H+R_1 4\pi M)j_x+(Q_0H+Q_1 4\pi M)q_x, \quad (6)$$

where j_x and q_x are electric and thermal current densities respectively. The transverse electric field per unit electrical current density will be

$$e_y=(R_0+Q_0q_x/j_x)B +[(R_1-R_0)+(Q_1-Q_0)q_x/j_x]4\pi M, \quad (7)$$

where $B-4\pi M$ has been substituted for H. The derivative of e_y with respect to B is given by

$$de_y/dB=(R_0+Q_0q_x/j_x) +[(R_1-R_0)+(Q_1-Q_0)q_x/j_x]4\pi\partial M/\partial B. \quad (8)$$

Above saturation where terms involving $\partial M/\partial B$ can be neglected, the above equation reduces to

$$de_y/dB=R_0+Q_0q_x/j_x. \quad (9)$$

This shows that de_y/dB, which is normally associated with R_0^*, is a linear function of q_x/j_x whose intercept is R_0 and whose slope is Q_0. In the limit of $B=0$, Eq. (7) reduces to

$$(e_y)_0=[(R_1-R_0)+(Q_1-Q_0)q_x/j_x]4\pi M_s. \quad (10)$$

Again R_0 and Q_0 can usually be neglected in comparison with R_1 and Q_1 so that the above equation reduces to

$$(e_y)_0/4\pi M_s=R_1+Q_1q_x/j_x. \quad (11)$$

Thus $(e_y)_0/4\pi M_s$, which is normally associated with R_1^*, is also a linear function of q_x/j_x, but whose intercept is R_1 and whose slope is Q_1. To investigate the influence of the longitudinal temperature gradient, accurate measurements were made on one sample (80% Ni) for different values of q_x/j_x at room temperature. The anticipated effect, as outlined above, was easily observed in the experimental results.

[16] F. E. Allison and E. M. Pugh, Phys. Rev. **99**, 1642 (1955).

EXPERIMENTAL RESULTS

The experimental results are summarized in Table I for all three alloys. Since it was anticipated that the temperature dependence of R_0^* below the Curie point would arise from the ferromagnetic properties of the samples, R_0^* is shown as a function of T/T_c in Fig. 2. In order to complete the curve at low temperatures, the results obtained by Cohen[7] are also included. There are three features of special interest:

(i) For both alloys in which measurements were completed above the Curie temperature, a peak in R_0^* is observed at $T/T_c\approx 1.05$. Such a behavior was anticipated and is due to the fact that the term involving $\partial M/\partial B$ in Eq. (3) is not negligible in comparison with R_0^* when the sample temperature is near its Curie point.

(ii) The peak for the 60% Ni is considerably broader than the peak for the 70% Ni; and, although the peaks are not sharply defined, the peak for 60% Ni appears to be shifted toward a slightly larger value of T/T_c.

(iii) The ordinary Hall coefficient, R_0^*, as a function of T/T_c below 0.7 does not vary greatly with alloy composition for the three alloys measured. Although there is considerable scatter, the agreement among the three alloys is sufficient to indicate that the hypothesis of a ferromagnetic origin for the temperature dependence in this region is correct.

The intercept at absolute zero for R_0^* is 23×10^{-13} which corresponds to an effective number of electrons $n^*=1/R_0^*Nec$ of about 0.3. The minimum value of R_0^* of 14×10^{-13} corresponds to an n^* of about 0.5. These are in good agreement with predictions in the band-model paper[8] where it was pointed out that gyromagnetic ratios now indicate that the 4*s* band electron/atom ratio may be as low as 0.54 for the whole band and 0.27 for the parallel half of this band.

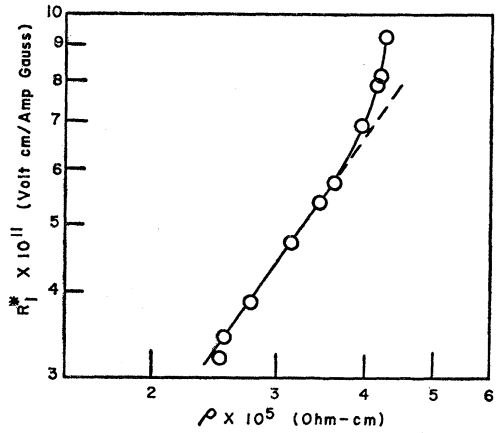

FIG. 3. Extraordinary Hall coefficients plotted against the resistivity for 70% Ni–30% Cu on a log log scale showing the apparent deviation from linearity beyond the Curie point.

The dependence of R_1^* upon ρ for 70% Ni is shown on a log-log plot in Fig. 3. The dependence is similar for the other two alloys. A linear dependence is not observed between $\log R_1^*$ and $\log\rho$ at the high-temperature end. This is not of fundamental significance, but rather a consequence of the method used in calculating R_1^*. In calculating R_1^*, the magnetization of the sample was assumed to follow a reduced magnetization curve that corresponds to the limit $H=0$. This condition is not satisfied by the measurements of the Hall effect, and, if one wishes to calculate R_1^*, one must know $4\pi M_s$ in fields of the order of 12 kilogauss. Since such data is not readily available, it was decided to assume that the linear relation between $\log R_1^*$ and $\log\rho$ would be obeyed at these higher temperatures and to compute the reduced magnetization curve for $H\neq 0$. This was done for 70% Ni and 60% Ni. The results are shown in Fig. 4. The "tail" on the magnetization curve appears to be quite reasonable when compared with a similar curve obtained by Weiss and Forrer[17] for pure Ni, thus substantiating the hypothesis.

The general character of the Curie point anomalies in R_0^* are in qualitative agreement with the magnetization curves in Fig. 4. The relatively broader peak at a slightly higher T/T_c for the 60% Ni alloy corresponds to the somewhat longer "tail" in the magnetization curve for this alloy.

It has recently been pointed out by Schindler and Salkovitz[18] that, when the room-temperature data for the Cu–Ni alloys are plotted as $\log R_1^*$ *versus* $\log\rho$, the result agrees with the pure Ni data obtained by Jan and Gijsman.[12] Such a plot of the data obtained in this investigation is shown in Fig. 5. The results for 80% Ni and 70% Ni do indeed agree with the pure Ni curve; however, 60% Ni, which is normally not ferromagnetic at room temperature and therefore not included in the analysis of Schindler and Salkovitz, shows a significant departure from the pure Ni curve. If the extraordinary

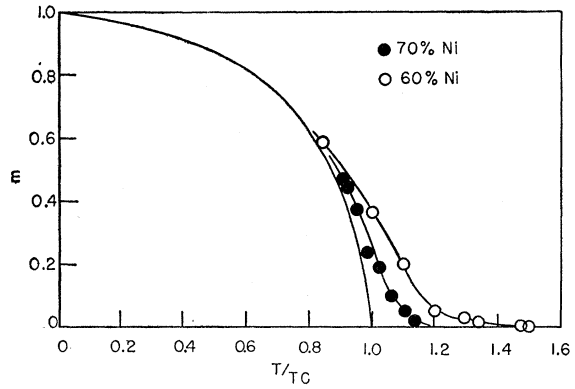

FIG. 4. Reduced magnetization curves for two Cu–Ni alloys showing the "tails" due to $H\neq 0$ calculated with the aid of the extraordinary Hall coefficients.

[17] P. Weiss and R. Forrer, Compt. rend. **178**, 1670 (1924).
[18] A. I. Schindler and E. I. Salkovitz, Phys. Rev. **99**, 1251 (1955).

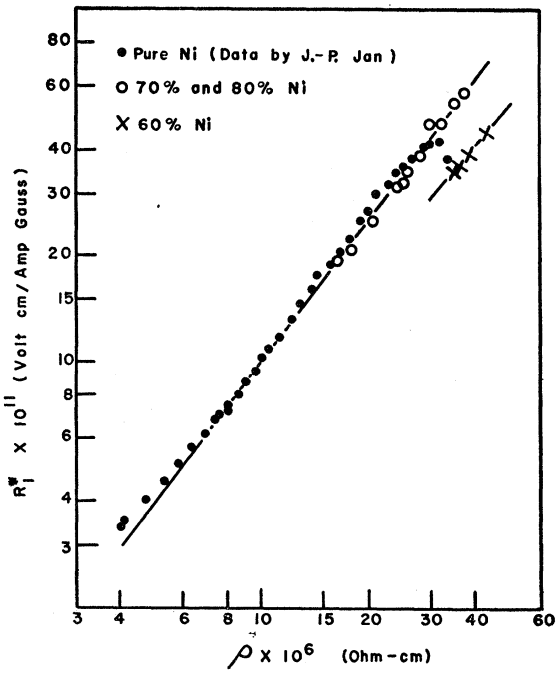

FIG. 5. Extraordinary Hall coefficients for Ni and three Cu–Ni alloys plotted against the resistivity on a log log scale.

coefficient is written as $R_1^*=A\rho^n$, it is apparent that neither A nor n changes appreciably when Cu is alloyed with Ni up to at least 30% Cu. It is not immediately apparent why this should be the case, and, in fact, similar results are not obtained when Co is alloyed with Ni. As Co is added to Ni, the resistivity does increase but the extraordinary coefficient actually decreases.[13]

The effect of the longitudinal temperature gradient on the Hall potential is illustrated in Fig. 6. The Hall coefficients computed from these data are presented in Table II, and the anticipated linear dependence of R_1^* and R_0^* upon q_x/j_x is shown in Fig. 7. The results clearly demonstrate that the Peltier and Ettingshausen effects can give rise to an experimental error that must be taken into consideration when precise determinations of the Hall coefficients are to be made. In this particular case they introduce an error of about 5% under conditions that were previously thought to be favorable. This error is not easily eliminated experimentally, and, since it is not too large, no attempt was made to correct any of the other data.

It has been demonstrated that the extraordinary coefficients for the Hall, Ettingshausen, Nernst, and Righi-Leduc effects obey to within a factor of three the relationships that are theoretically predicted for the ordinary coefficients.[19] Since the agreement is no better than this for the ordinary coefficients themselves, these results strongly suggest that the extraordinary effects bear a constant ratio to the ordinary effects. According

[19] E. H. Butler and E. M. Pugh, Phys. Rev. **57**, 916 (1940).

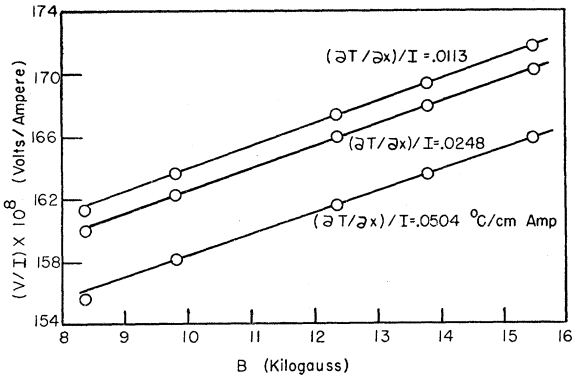

FIG. 6. Transverse potential per unit current plotted against the magnetic induction for 80% Ni–20% Cu.

to the results presented in Table II, $4\pi M_s(R_1/R_0) = 10.8\times10^5$ gauss and $4\pi M_s(Q_1/Q_0) = 8.8\times10^5$ gauss. The difference between these two values is about 20%. Since the Nernst potential was only approximately 5% of the Hall potential, (Q_1/Q_0) is subject to considerable experimental error, which could account for the observed discrepancy.

DISCUSSION

When d-band conduction is neglected and the $4s$ band regarded as consisting of two sub-bands, the Hall coefficient will be given by[8]

$$\sigma^2 R_0 = \sum_j \sigma_j^2 R_{0j}, \tag{12}$$

and the conduction in both s bands will be electronic. Thus the Hall coefficient can be written as

$$R_0 = -\frac{1}{Nec}\left[\left(\frac{\sigma_p}{\sigma}\right)^2 \frac{1}{\nu_p} + \left(\frac{\sigma_a}{\sigma}\right)^2 \frac{1}{\nu_a}\right]. \tag{13}$$

The subscripts p and a refer to electrons whose magnetic moments are aligned respectively parallel and antiparallel with the field. For conduction (s band) electrons, the two orientations are equally probable, and hence

$$\nu_a = \nu_p = n_s/2. \tag{14}$$

TABLE II. Influence of a longitudinal temperature gradient upon the Hall coefficient.[a]

$(\partial T/\partial x)/I\times10^2$ (°C/amp cm)	$R_0^*\times10^{13}$ (volt cm/amp gauss)	$4\pi M_s R_1^*\times10^9$ (volt cm/amp)
5.04	13.80	147.8
2.48	14.16	151.7
1.13	14.38	152.9

Corrected $R_0^* = 14.54\times10^{-13}$ volt cm/amp gauss
Corrected $4\pi M_s R_1^* = 154.6\times10^{-8}$ volt cm/amp
$\kappa wt Q_0^* = 14.95\times10^{-13}$
$\kappa wt 4\pi M_s Q_1^* = 131.3\times10^{-8}$

[a] κ is the thermal conductivity, w and t the sample width and thickness respectively.

Under these conditions the Hall coefficient reduces to

$$R_0 = -\frac{2}{Nn_s ec}\left[1 - \frac{2\beta}{(1+\beta)^2}\right], \tag{15}$$

where β is σ_a/σ_p. From the last equation it is observed that the Hall coefficient depends only upon the ratio of the conductivities in the $4s$ sub-bands.

In discussing the temperature dependence of the resistivity of ferromagnetic material, Mott obtained the relation

$$\beta = \sigma_a/\sigma_p = (N_p/N_a)^{\frac{1}{3}},$$

where N_p and N_a are the number of holes in the parallel and antiparallel halves of the $3d$ band, respectively. These can be related to the total number of holes N_d

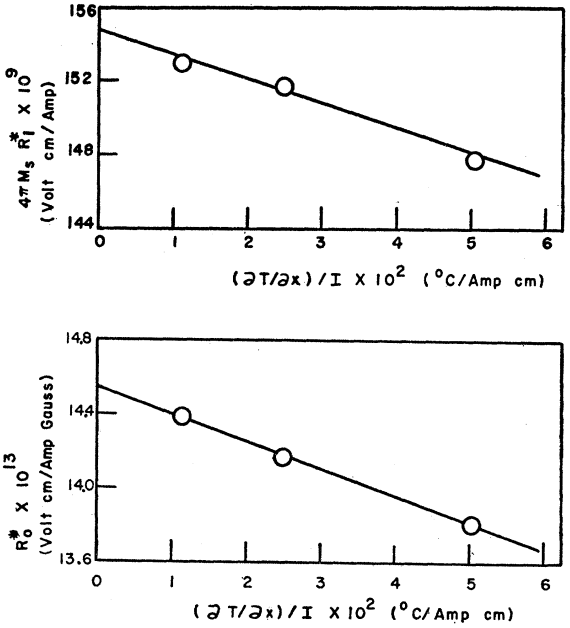

FIG. 7. Experimentally determined values of R_0^* and R_1^* plotted against the longitudinal temperature gradient per unit current for the 80% Ni–20% Cu sample at room temperature.

in the $3d$ band and the magnetization by the relations

$$\begin{aligned} N_p &= \tfrac{1}{2}N_d(1 - M/\mu N_d), \\ N_a &= \tfrac{1}{2}N_d(1 + M/\mu N_d). \end{aligned} \tag{16}$$

$$\beta = \left(\frac{1 - M/\mu N_d}{1 + M/\mu N_d}\right)^{\frac{1}{3}} = \left(\frac{1-\lambda m}{1+\lambda m}\right)^{\frac{1}{3}}, \tag{17}$$

where $m = M_s(T)/M_s(0)$ and $\lambda\mu N_d = M_s(0)$, i.e., $\lambda\mu$ is the effective magnetic moment associated with a hole in the $3d$ band.

Above the Curie point anomaly the Hall coefficient will be given by $R_0 = -1/Nn_s ec$, since the probability for scattering the s electrons will be independent of their spin orientation. This value of R_0 will be referred

to as the paramagnetic Hall coefficient and will be designated by R_0(para). Equation (15) can now be expressed as

$$R_0(T)/R_0(\text{para})=r=2[1-2\beta/(1+\beta)^2], \quad (18)$$

where β is given in terms of m by Eq. (17). The curves in Fig. 8 have been calculated by means of Eqs. (17) and (18) for $\lambda=1.0$ and $\lambda=0.9$. The experimental values of r for 70% Ni and 60% Ni are also shown in Fig. 8. Experimental values of R_0^*(para) were obtained by extrapolating the data across the Curie point anomaly; however, this should not be a serious limitation since R_0, as opposed to R_0^*, should vary only slowly with temperature in this region. Again, Cohen's[7]

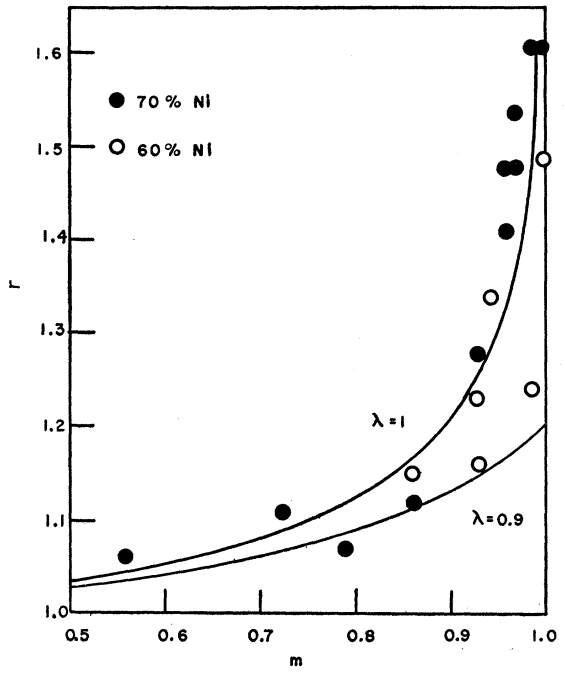

FIG. 8. Observed values of R_0^* relative to their paramagnetic values for two Cu–Ni alloys *versus* the reduced magnetization. The solid lines have been calculated from the band model. The curve for $\lambda=1$ assumes that at 0°K the parallel d band is completely full whereas the curve for $\lambda=0.9$ assumes it is 90% full.

results are included in order to complete the curve for low temperatures (large values of m).

The theoretical curve for $\lambda=1$ (no holes in the parallel half of the d band at absolute zero) is in rather remarkable agreement with the experimental results. The disagreement between the theoretical curve for $\lambda=0.9$ and the experimental results is sufficiently large to indicate that there is at most an extremely small fraction of the d band holes in the parallel half at absolute zero. For these alloys the simple band model (with $\lambda=1$) unambiguously predicts a temperature dependence of R_0^* that is in good agreement with experimental results.

Smit's paper[1] contains measurements on Ni and many Ni alloys at 20°K, 77°K, and 290°K. In most cases where the alloys are similar his results agree roughly with ours. However there are some discrepancies. In fact, every measurement on pure Ni seems to yield different R_0's with different temperature dependencies. This is what should be expected from the band model explanation, which presumes that the small R_0 found in Ni is caused by a positive Hall effect from the holes in the d band almost cancelling the negative Hall effect from the electrons in the s band. Small amounts of impurities could easily upset this delicate balance.

Smit's data[1] on "pseudo-nickel" (2.5% Fe, 92.5% Ni and 5.0% Cu) suggests that with the same total number of electrons per atom as in Ni but with sufficient impurities (2.5% Fe and 5.0% Cu) the contribution to the Hall effect from the holes in the d band can be ignored. His R_0's for this alloy correspond roughly to n^*'s of 0.5 and 0.3 at high and low temperatures, respectively, just the same as has been found here for the three Cu–Ni alloys in which the d-band contribution is negligible. One cannot be certain of the behavior for this pseudo-nickel sample until a complete temperature dependence curve has been obtained.

The only real discrepancy existing between the data obtained by Smit and that obtained here is in the R_0's at room temperature for three Co–Ni alloys, which have been mentioned by Foner.[20] It is our belief that some systematic error in either one or the other set of data must be responsible, for it is hard to see how small impurities in such alloys could produce results that differ this much.

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[20] S. Foner, Phys. Rev. **99**, 1080 (1955).