Weak Field Magnetoresistance of *n*-Type Germanium

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The magnetoresistance of 11 ohm-cm *n*-type germanium has been measured from 77°K to 320°K. Analysis of the results indicates that the data are consistent with the model which assumes that the energy surfaces are (111) ellipsoids. The most consistent analysis indicates that the mass ratio is approximately constant (about 11.9) in this temperature range but that the scattering mechanism is temperature-dependent. The relaxation time may be approximated by $\tau=l\epsilon^s$, where $s=s(T)$.

I. INTRODUCTION

THE magnetoresistance effects in *n*-type germanium have been found to be anisotropic.[1,2] Abeles and Meiboom[3] have shown that the existing anisotropy agrees with a model which assumes that the energy surfaces are ellipsoids oriented along the (111) directions in *k*-space. By assuming that the ellipsoids are prolate with the masses characterizing the ellipsoids being in the ratio 20:1 and by assuming lattice scattering ($\tau\propto\epsilon^{-\frac{1}{2}}$), they were able to obtain quantitative agreement with the room temperature measurements of Pearson and Suhl.[1] The assumption that the energy surfaces are ellipsoids oriented along the (111) directions has been verified by low-temperature cyclotron resonance experiments. Kittel[4] finds a mass ratio of about 15:1 while Lax, Zeiger, and Dexter[5] report a mass ratio of about 17:1.

Abeles and Meiboom found that the magnetoresistance effects observed at liquid nitrogen temperature by Pearson and Suhl did not agree with their assumptions as well as the room temperature measurements. This could be due to a temperature dependence of the mass ratio or the scattering mechanism. The measurements of the temperature dependence of the magnetoresistance effects being reported here were undertaken in an attempt to determine whether the mass ratio or the scattering mechanism was temperature dependent.

The current carried by a cubic semiconductor in the presence of weak magnetic fields is[6]

$$\mathbf{i}=\sigma_0\mathbf{E}+\alpha\mathbf{E}\times\mathbf{H}+\beta\mathbf{E}H^2+\gamma\mathbf{H}(\mathbf{E}\cdot\mathbf{H})+\delta M\mathbf{E}, \quad (1)$$

where M is a diagonal tensor with elements H_1^2, H_2^2, and H_3^2; the subscripts 1, 2, and 3 refer to the axes of cubic symmetry. The coefficients σ_0, α, β, γ, and δ are integrals which can be evaluated, in principle, if assumptions are made concerning the equilibrium distribution function and the dependence of relaxation time and energy upon the wave vector $\mathbf{k}$. The reciprocal of Eq. (1) is[1]

$$\mathbf{E}=\rho_0[\mathbf{i}+A\mathbf{i}\times\mathbf{H}+B\mathbf{i}H^2+C\mathbf{H}(\mathbf{i}\cdot\mathbf{H})+DM\mathbf{i}], \quad (2)$$

where

$$\rho_0=1/\sigma_0, \quad (3a)$$
$$A=-\alpha\rho_0, \quad (3b)$$
$$B=-(\beta+\rho_0\alpha^2)\rho_0, \quad (3c)$$
$$C=-(\gamma-\rho_0\alpha^2)\rho_0, \quad (3d)$$
$$D=-\delta p_0. \quad (3e)$$

It can be readily shown[1] that the fractional change of resistivity in the presence of a weak magnetic field can be expressed in terms of the coefficients of Eq. (2):

$$(\rho_H-\rho_0)/\rho_0=\Delta\rho/\rho_0=H^2[B+C(\Sigma\iota\eta)^2+D\Sigma\iota^2\eta^2], \quad (4)$$

where ι_1, ι_2, ι_3 and η_1, η_2, η_3, are the direction cosines with respect to the axes of cubic symmetry of the current and the magnetic field respectively.

We shall adopt the notation $\Delta\rho/\rho_0=M_{hkl}{}^{h'k'l'}$, where the subscript hkl refers to the crystallographic direction of the current and the superscript $h'k'l'$ refers to the crystallographic direction of the magnetic field. Using (4), we find that

$$M_{110}{}^{001}=BH^2, \quad (5a)$$
$$M_{110}{}^{1\bar{1}0}=(B+D/2)H^2, \quad (5b)$$
$$M_{110}{}^{110}=(B+C+D/2)H^2. \quad (5c)$$

All of the above equations are phenomenological and no assumptions are involved other than assuming weak magnetic fields. Because of this assumption, $(M_{hkl}{}^{h'k'l'}/H^2)$ means $\lim_{H\to0}(M_{hkl}{}^{h'k'l'}/H^2)$.

If one assumes that the energy surfaces are ellipsoids oriented along the (111) axes in *k*-space and that the relaxation time is a function of the energy alone, i.e. $\tau=\tau(\epsilon)$, then the magnetoresistance coefficients of Eq. (1) are

$$\beta=cK(2K+1)(K+2)I, \quad (6a)$$
$$\gamma=-\beta, \quad (6b)$$
$$\delta=2cK(K-1)^2I, \quad (6c)$$

where K is the mass ratio, c is independent of K, and

$$I=\int_0^\infty \tau^3\epsilon^{\frac{3}{2}}\frac{\partial f_0}{\partial\epsilon}d\epsilon, \quad (7)$$

f_0 being the equilibrium distribution function. The

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[1] G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).
[2] C. Goldberg and R. E. Davis, Phys. Rev. **94**, 1121 (1954).
[3] B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954).
[4] C. Kittel, Physica **20**, 829 (1954).
[5] Lax, Zeiger, and Dexter, Physica **20**, 818 (1954).
[6] F. Seitz, Phys. Rev. **79**, 372 (1950).

consequences of (6b) are $B=-C$ and

$$M_{110}{}^{1\bar{1}0}=M_{110}{}^{001}+M_{110}{}^{110}. \quad (8)$$

If it is assumed that $\tau=l\epsilon^s$, the integral (7) can be evaluated and from Eqs. (6), (3), and (5) the magnetoresistance effects can be evaluated.

$$(M_{110}{}^{001}/H^2)/(R_0\sigma_0)^2 =[b_1(2K+1)^2/3K(K+2)]-1, \quad (9a)$$

$$(M_{110}{}^{110}/H^2)/(R_0\sigma_0)^2 =b_1(K-1)^2(2K+1)/3K(K+2)^2, \quad (9b)$$

where R_0 is the zero-field Hall coefficient and

$$b_1=\frac{\Gamma\left(\frac{6s+5}{2}\right)\Gamma\left(\frac{2s+5}{2}\right)}{\left[\Gamma\left(\frac{4s+5}{2}\right)\right]^2}. \quad (10)$$

The quantity $M_{110}{}^{1\bar{1}0}$ is, of course, still given by (8).

Abeles and Meiboom analyzed the data of Pearson and Suhl by use of Eqs. (8) and (9), assuming $s=-\frac{1}{2}$. The same procedure will be followed here but an additional method will also be used.[7] From (6) it is seen that

$$\delta/\beta=2(K-1)^2/(2K+1)(K+2). \quad (11)$$

This relation, which assumes only that $\tau=\tau(\epsilon)$, can be used to determine the mass ratio experimentally and (9a) and (9b) can then be used to determine s if the data can be consistently described by a relaxation time of the form $\tau=l\epsilon^s$.

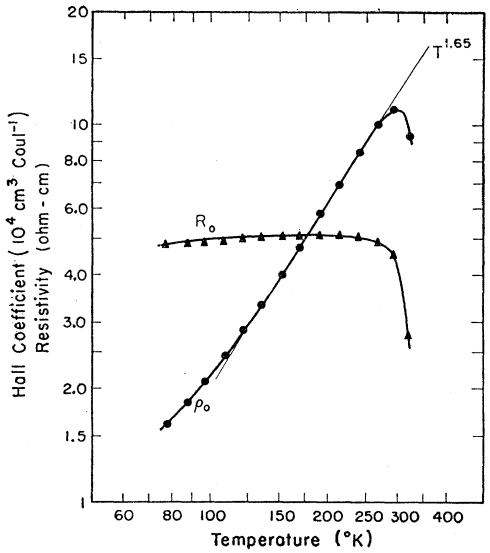

FIG. 1. Temperature dependence of zero-field resistivity and Hall coefficient.

[7] This method is, in part, that used by Herring in analyzing the magnetoresistance of n-type silicon [C. Herring, Bell System Tech. J. **34**, 237 (1954); G. L. Pearson and C. Herring, Physica **20**, 975 (1954)].

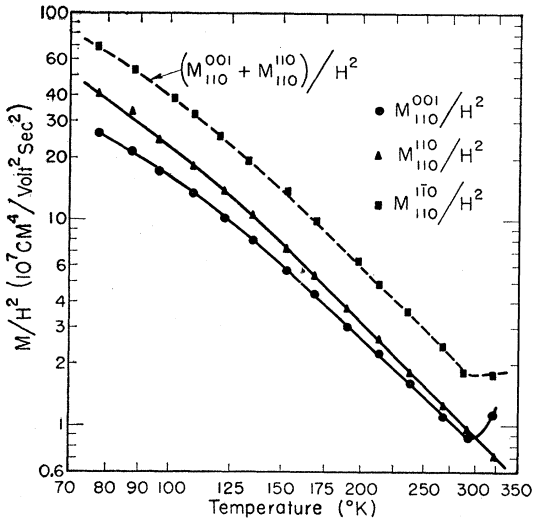

FIG. 2. Temperature dependence of magnetoresistance. The dashed line is the sum of the two solid lines. See Eq. (8).

II. EXPERIMENTAL RESULTS

n-type rectangular samples were cut from an antimony doped germanium ingot grown by the method of Teal and Little.[8] The crystals were cut so that the current would be carried in either the (100) or (110) directions. The resistivity was approximately 11 ohm cm at room temperature. Measurements were made of ρ_0, the resistivity in the absence of a magnetic field, R_0, and $M_{110}{}^{110}$, $M_{110}{}^{110}$, $M_{110}{}^{001}$, $M_{100}{}^{100}$, and $M_{100}{}^{001}$ from 77°K to 320°K. Measurements of G, the planar Hall coefficient,[2] were also made.

Measurements were made as a function of field from 500 to 3700 gauss. All values reported here have been obtained by extrapolation to zero field.

Figure 1 shows the Hall coefficient and resistivity for a sample carrying current in the (110) direction. The Hall measurements were made with the field in the (001) direction. Figure 2 shows the magnetoresistance effects for the same sample. The dashed line through the $M_{110}{}^{0\bar{1}1}$ data has been calculated from the $M_{110}{}^{001}$ and $M_{110}{}^{110}$ data by means of Eq. (8).

The good agreement with (6c) is indicative of the presence of (111) ellipsoids. The effects of the intrinsic holes present around room temperatures can be seen in the Hall coefficient and magnetoresistance effect data. The observed effect due to the holes agrees qualitatively with the magnitude and sign of the room temperature magnetoresistance coefficients for p-type germanium.[2] The observed temperature dependence of the resistivity in the extrinsic region may be expressed as approximately $T^{1.65}$. This is in agreement with previous measurements of resistivity[9] and drift mobility.[10]

The data given in Fig. 2 is sufficient to determine the three magnetoresistance coefficients of Eq. (2) so that

[8] G. K. Teal and J. B. Little, Phys. Rev. **78**, 647 (1950).
[9] P. P. Debye and E. M. Conwell, Phys. Rev. **93**, 693 (1954).
[10] M. Prince, Phys. Rev. **92** 681 (1953).

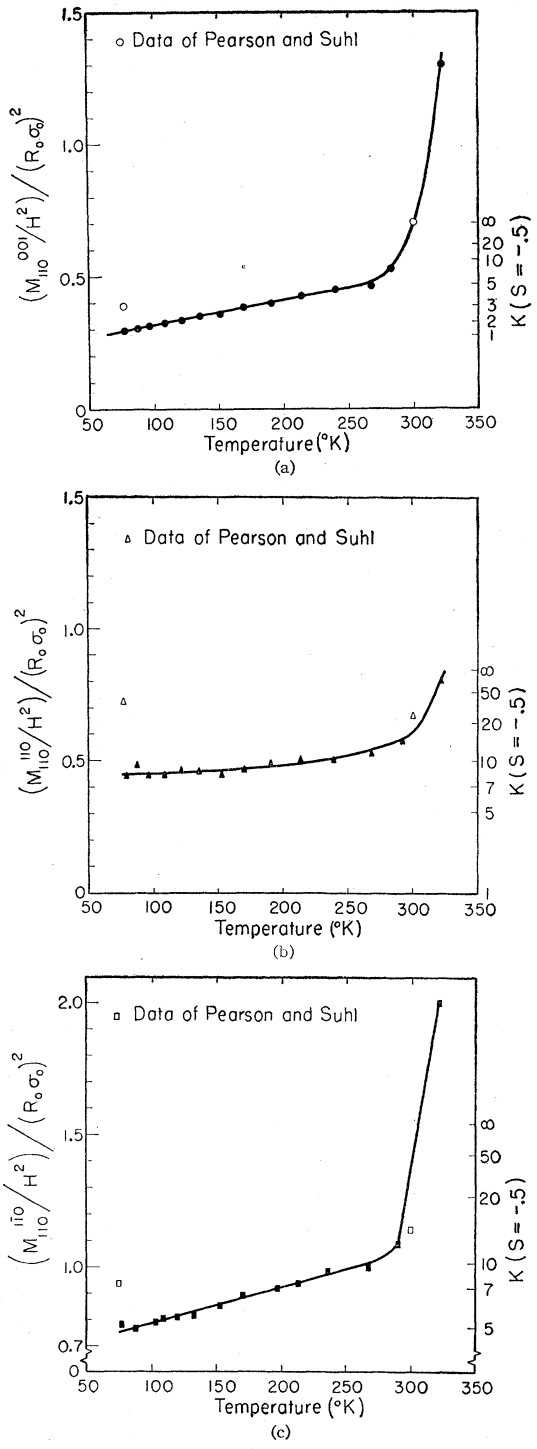

FIG. 3. Temperature dependence of the quantities $(M/H^2)/(R_0\sigma_0)^2$, (a) for M_{110}^{001}, (b) for M_{110}^{110}, and (c) for $M_{110}^{1\bar{1}0}$. The values of the mass ratio K given on the right of each figure have been calculated from Eqs. (8) and (9) for $\tau=l\epsilon^{-0.5}$.

additional measurements will not give new information but can only be used to check the accuracy of the measurements shown in Fig. 2. The planar Hall coefficient can be expressed in terms of M_{110}^{110}, M_{110}^{001}, and $M_{110}^{1\bar{1}0}$. This is also true of the magnetoresistance of a sample with current being carried in the (100) direction. Measurements have been made of G_{110}^{100}, G_{100}^{110}, M_{100}^{100}, and M_{100}^{010}. The results of these measurements are in agreement with the data shown in Fig. 2.

III. DISCUSSION

As previously stated, the measurements of $M_{110}^{1\bar{1}0}/H^2$ are seen to be in good agreement with Eq. (8) which is based on the (111) ellipsoid model. In Figs. 3(a), 3(b), and 3(c) the temperature dependence of the quantity $(M/H^2)/(R_0\sigma_0)^2$ is plotted for M_{110}^{110}, $M_{110}^{1\bar{1}0}$, and M_{110}^{001}. The values of K shown on the right-hand side of these figures have been calculated from Eqs. (8) and (9) for $s=-\frac{1}{2}$ and indicate the values of the mass ratio that correspond (for this relaxation time assumption) to the values of $(M/H^2)/(R_0\sigma_0)^2$ shown on the left hand side of each of these figures. The measurements of Pearson and Suhl are also shown. At 77°K, there is an appreciable difference between their measurements and the ones being reported here.

The large values of $(M/H^2)/(R_0\sigma_0)^2$ found at 320°K are due mainly to the fact that at this temperature the intrinsic holes present have caused an appreciable decrease in the Hall coefficient (see Fig. 1). Despite this fact, or perhaps because of it, the values found around 300°K are in approximate agreement with a mass ratio of 20 as found by Abeles and Meiboom.

Below room temperature our data could not be consistent with their model unless the mass ratio slowly decreases as the temperature is lowered. Moreover, in any case, at any given temperature the three mass ratios indicated by the three parts of Fig. 3 are not mutually consistent. For example, at 77°K a mass ratio of 1.5 is obtained from (3a), 8 from (3b), and 5 from (5c). It would seem that a changing mass ratio alone cannot account for the variation with temperature of the quantities $(M/H^2)/(R_0\sigma_0)^2$ since the measurements cannot be explained consistently at a given temperature by use of a single mass ratio.

It is more likely that the difficulty is with the relaxation time assumptions. The model assumes, e.g.,

TABLE I. Experimental values of the quantity δ/β and the values of K deduced from these measurements by means of Eq.(11).

T(°K)	δ/β	K
77.3	0.686	11.6
87.3	0.682	11.4
96.5	0.681	11.4
109.0	0.675	11.2
121.6	0.695	12.1
134.8	0.682	11.4
152.4	0.664	10.9
168.8	0.680	11.4
189.4	0.705	12.5
213.3	0.711	13.0
238.9	0.700	12.3
267.7	0.726	13.9
292.3	0.753	15.5

that there is no intervalley scattering[7] (scattering from one ellipsoid to another) and no ionized impurity scattering. If intervalley scattering is present, its relative importance probably changes with temperature so that the relaxation time does not follow the same simple power law at all temperatures. As indicated in Sec. I, the ratio δ/β should be independent of the scattering mechansim as long as the relaxation time is a function of the energy only. In Table I are shown the experimental values of the ratio δ/β and the mass ratios deduced therefrom by means of Eq. (11). It is seen that throughout the temperature range from 77°K to room temperature the data are consistent with a mass ratio of about 12. There is an indication in Table I that the mass ratio increases as room temperature is approached. This may be caused by a change in the ratio δ/β due to the presence of intrinsic holes. (The effect of these holes can be seen most clearly in Fig. 1 in the decrease in the Hall coefficient as room temperature is approached.) Furthermore, excluding the measurements at $T=292.3$°K, the ratio δ/β given in Table I varies only a few percent throughout the temperature range. It is felt that this variation is not large enough, considering experimental uncertainties in the necessary extrapolations to zero field of the quantities M/H^2, to state definitely that the mass ratio changes with temperature.

Excluding the value given for $T=292.3$°K, the values of K given in Table I have a mean value of 11.9 with a standard deviation of 0.8. By assuming that this mean value is the correct value at all temperatures the data given in Fig. 3 can be used to evaluate the parameter s as a function of temperature for a relaxation time $\tau=l\epsilon^s$. For this mass ratio (9a) yields

$$(M_{110}{}^{001})/(R_0\sigma_0)^2=1.236b_1-1, \qquad (12)$$

where b_1 is given by (10). The values of s determined in this way are shown in Fig. 4. This curve can be used to determine $(M_{110}{}^{1\bar{1}0}/H^2)/R_0\sigma_0)^2$ and $(M_{110}{}^{110}/H^2)/(R_0\sigma_0)^2$ by means of (9b) and (8). The solid lines in Fig. 5 show the values of these quantities calculated using a mass ratio of 11.9 and a relaxation time of the form $\tau=l\epsilon^s$ with s being given by Fig. 4. Also shown are the experimentally measured values previously

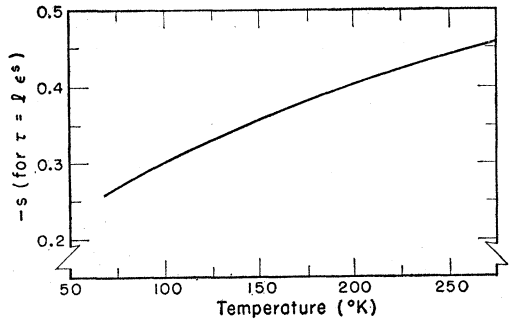

FIG. 4. Temperature dependence of the relaxation time assuming that $\tau=l\epsilon^s$ calculated from Fig. 3(a) by assuming $K=11.9$.

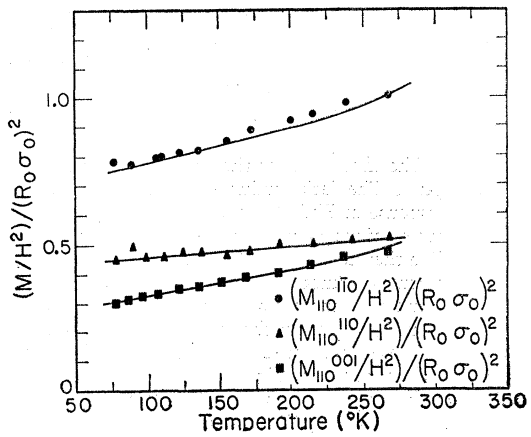

FIG. 5. Comparison of measured values of $(M/H^2)/(R_0\sigma_0)$ with values (solid lines) calculated by assuming $K=11.9$ with a temperature-dependent relaxation time described by Fig. 4.

given in Fig. 3. While the agreement between the calculated and experimental values is not perfect, it nevertheless indicates that the observed temperature dependence of the quantities $(M/H^2)/(R_0\sigma_0)^2$ can be explained by assuming that $s=s(T)$.

The slow decrease in R_0 as the temperature is lowered can also be shown to be consistent with a mass ratio of 11.9 and the relaxation time described by Fig. 4. For $\tau=l\epsilon^s$, the zero-field Hall coefficient is

$$R_0=(9\pi^{\frac{1}{2}}b_2/4)K(K+2)/(2K+1)^2, \qquad (13)$$

where

$$b_2=\Gamma\left(\frac{4s+5}{2}\right)\Big/\left[\Gamma\left(\frac{2s+5}{2}\right)\right]^2.$$

IV. SUMMARY

The measurements of the magnetoresistance of n-type germanium are in agreement with the model that assumes that the energy surfaces are ellipsoids oriented along the (111) directions in k-space. The most consistent agreement is found by assuming that the mass ratio is constant above 77°K but that the scattering mechanism changes with temperature.

Note added in proof.—The authors wish to call attention to two papers which have appeared since this manuscript was written. Benedek, Paul, and Brooks [Phys. Rev. **100**, 1129 (1955)] have pointed out the large effects that small amounts of impurity scattering will have on the integral I defined by Eq. (7). This could explain the temperature dependence of the relaxation time (Fig. 7) as being due to a small amount of impurity scattering which increases as the temperature decreases. C. Herring and E. Vogt [Phys. Rev. **101**, 944 (1956)] have shown that if the anisotropy of the relaxation time is considered, the quantity K is actually the mass ratio multiplied by the ratio of transverse to parallal relaxation times. This could explain the discrepancy between our experimental value of $K=11.9$ and the mass ratio as measured by cyclotron resonance.