this case at about 16 centimeters. Figure 6 represents the pairs with angular divergence less than 20°. A maximum is definitely marked at 17.5 cm where the rate is at least four standard errors above the nearest two points on either side.

The results displayed in the last column of Table I agree with what others in the field have reported: the maximum is not indicated by the total intensity of the particles. The upward trend suggested by the last two points may be discounted because of an inadequate number of counts.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. Levinger of the Louisiana State University Physics Department faculty for discussion, to Mr. Ed Keel for the excellent machine shop work, and to Dr. Leo Broussard and Dr. R. W. Krebs of Esso Laboratories.

PHYSICAL REVIEW VOLUME 102, NUMBER 4 MAY 15, 1956

Influence of Geomagnetic Quadrupole Fields upon Cosmic-Ray Intensity*†

F. S. Jory‡
Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois
(Received August 24, 1955)

The effect upon cosmic-ray particles of the quadrupole part of the earth's magnetic field has been calculated, using the results of the magnetic survey of 1945. The effect predicted from the zonal quadrupole term is a northern shift of the cosmic-ray latitude curve. The 1945 magnetic center was calculated, using Schmidt's method, and it is 0.0629 earth radii from the center of the earth, as compared to 0.0536 earth radii in 1922. The shift in magnetic center results in an increase in the predicted longitude effect. The residual or sectorial quadrupole effect upon cosmic-ray intensity is predicted to be a two-period sine curve in the longitude effect.

I. INTRODUCTION

THE latitude effect in cosmic-ray intensity has been explained by Fermi and Rossi.[1] They used the angular integral of motion for a charged particle moving in a dipole magnetic field, i.e., Stoermer's theorem,[2] and assumed an isotropic cosmic-ray flux at infinity. The longitude effect has been shown to be consistent with an off-center position for the earth's magnetic dipole.[3] The present investigation is concerned with the influence upon cosmic-ray intensity of all the quadrupole terms of the earth's magnetic field. The calculations made are based on the 1945 magnetic survey by Vestine and Lange.[4]

During a magnetic survey, measurements of the earth's magnetic field are made at a large number of stations all over the world. For the analysis of the measurements, a magnetic potential function is introduced and expanded in spherical harmonics, the coefficients of the expansion being called Gauss coefficients. The coefficients are adjusted so that the gradient of the magnetic potential renders a best fit to the experimentally measured field.[5]

In the study of geomagnetic effects upon cosmic-ray intensity, two different dipole approximations to the earth's magnetic field have been used. The first approximation is a dipole located at the geographic center of the earth, which can be represented mathematically by the first three Gauss coefficients. The second and higher approximation is an off-center dipole located at the magnetic center. The equations for the magnetic center involve the first eight Gauss coefficients and are given by Schmidt.[6] A line along the centered dipole determines the magnetic axis of the earth. A quadrupole which is symmetric about this axis, i.e., a zonal quadrupole, can be added to the centered dipole and the motion of charged particles in the combined dipole and zonal quadrupole fields can be studied, following a suggestion by S. B. Treiman. The cylindrical symmetry leads to an angular integral of motion which is similar to Stoermer's theorem, and critical magnetic rigidities or cutoff rigidities are defined analogous to the ordinary Stoermer case.

The location of the off-center dipole has been computed by Schmidt[6] from the magnetic survey of 1922. To find the effect of the off-center dipole upon cosmic-ray intensity, ordinary Stoermer theory can be applied

* Assisted in part by the Office of Scientific Research and the Geophysics Research Directorate, Air Force Cambridge Research Center, Air Research and Development Command, U. S. Air Force.

† Based on a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics, University of Chicago.

‡ Now at the University of Maryland, College Park, Maryland.

[1] E. Fermi and B. Rossi, Rend. reale accad. nazl. Lincei **17**, 346 (1933).

[2] C. Stoermer, Astrophys. Norv. **1**, 1 (1936).

[3] M. S. Vallarta, Phys. Rev. **74**, 1837 (1948).

[4] E. H. Vestine and I. Lange, Carnegie Inst. Wash. Publ. No. 578 and No. 580 (1947).

[5] S. Chapman and J. Bartels, *Geomagnetism* (Oxford University Press, New York, 1951).

[6] A. Schmidt, Beitr. angew. Geophys. **41**, 346 (1934).

TABLE I. Gauss coefficients of V in 10^{-4} cgs.

		Dipole			Quadrupole				
Source	Epoch	g_1^0	g_1^1	h_1^1	g_2^0	g_2^1	h_2^1	g_2^2	h_2^2
Dyson and Furner	1922	−3095	−226	592	− 89	299	−124	144	84
Vestine and Lange	1945	−3057	−211	581	−127	296	−166	164	54

to the motion of particles in the off-center dipole field. Then proper account must be taken of the position of the observer relative to the magnetic center. As discussed by Vallarta[3] for the 1922 survey, a correction is added to the centered dipole cut-off rigidity to give the off-center dipole cutoff.

Using the method of Schmidt, the magnetic center for 1945 has been computed and is found to be 59 km farther from the earth's center than it was in 1922. The correction to be added to the centered dipole cut-off rigidity to give the off-centered cutoff has been computed for the 1945 survey by a method slightly different from that used by Vallarta.

The dipole and quadrupole terms of the earth's magnetic potential are equivalent to the off-centered dipole plus a residual or sectorial quadrupole.[6] The off-centered dipole has cylindrical symmetry in the limited sense that Stoermer theory of a single dipole can still be applied to compute vertical cut-off rigidities, provided the latitude and radius vector of the observer relative to the magnetic center are used. The addition of a sectorial quadrupole to the off-center dipole, however, destroys the cylindrical symmetry of the dipole altogether. Stoermer's theorem is then no longer valid. The corresponding equation now contains a variable term which depends explicitly on the charged particle trajectory. The magnitude of this variable term is computed for two cases by a perturbation calculation.

II. ZONAL QUADRUPOLE

A. Modification of Stoermer's Theorem

The earth's magnetic field can be approximated by a centered dipole. To obtain a better approximation, higher order poles must be included. It is the purpose of this section to consider the model of centered dipole plus zonal quadrupole. A line along the centered dipole defines the magnetic axis and a quadrupole symmetric about this line permits an integral of the motion (in addition to the energy integral) similar to the integral of the dipole case. The effect upon cosmic-ray intensity is, of course, independent of magnetic longitude, and it will be shown that the intensity in the geomagnetic equatorial plane remains the same. However, the northern hemisphere intensity is different from that in the southern hemisphere at the same geomagnetic latitude.

A description of the earth's magnetic field conventionally consists of an internal and an external part. An example of an external field can be found in the great magnetic storms which have been described by a ring current external to the earth[5] and their effect upon cosmic-ray intensity has been calculated by several workers.[7,8] We are assuming here, however, that the earth's main field is entirely of internal origin. This is in line with the Vestine and Lange analysis which does not reveal the existence of any permanent external source of field. Accordingly, the earth's magnetic potential is written[5]

$$\frac{V}{a}=\sum_{l,m}\frac{a^{l+1}}{r^{l+1}}(g_l^m\cos m\Phi+h_l^m\sin m\Phi)P_l^m(\cos\Theta).$$

Here a is the earth radius, Φ is geographic longitude east from Greenwich, Θ is the geographic colatitude, and g_l^m, h_l^m are the Gauss coefficients. The $P_l^m(\cos\Theta)$ are associated Legendre polynomials. Vestine and Lange computed the Gauss coefficients up to sixth order for the epoch 1945. The 1922 and 1945 coefficients for the dipole and quadrupole orders are given in Table I. The octopole terms are seven in number. They are of less interest, however, first because they are about half the size of the quadrupole terms, and second, because their magnetic field falls off with increasing r like the inverse fifth power of r, as compared to inverse fourth power for quadrupole field and inverse third power for dipole field.

There is a way of estimating the error in some of the Gauss coefficients, when they can be derived separately from the northern and eastern components of the earth's magnetic field. A coefficient derived from the northern component should check with that derived from the eastern component. If there is disagreement, a weighted average is taken. The disagreement in the dipole and quadrupole coefficients has been reported by Vestine and Lange. g_1^0 and g_2^0 are derived from the northern components alone so there is no test for consistency. g_1^1, which is derived separately, shows disagreement of 19×10^{-4} gauss from the average. h_2^2 shows departures of 8×10^{-4} gauss from average. The other coefficients disagree by only 1 or 2×10^{-4} gauss.

We consider the magnetic potential of the earth's centered dipole plus zonal quadrupole, calling it V'.

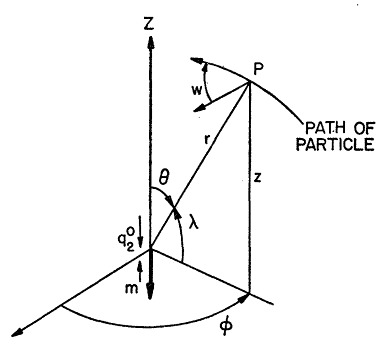

FIG. 1. Illustration of the coordinates that are used for the centered dipole plus zonal quadrupole model. Also shown is the vector pointing toward magnetic west and the path of the particle.

[7] Hayakawa, Nishimura, Nagata, and Sugiura, J. Sci. Research Inst. (Tokyo) 44, 121 (1950).
[8] S. B. Treiman, Phys. Rev. 89, 130 (1953).

As coordinate system we choose geomagnetic coordinates with the z axis along the northern magnetic axis (Fig. 1). The earth's dipole will then be directed along the negative z axis. The zonal quadrupole can be represented as two dipoles lined up along the z axis with north poles together or south poles together, depending on a choice of sign. The magnetic longitude φ and magnetic colatitude θ are in small letters to distinguish them from the geographic longitude and colatitude Φ, Θ which appeared above. It is evident that the magnetic potential V' has the form

$$V' = \frac{-a^3 m}{r^2} P_1^0(\cos\theta) + \frac{a^4}{r^3} q_2^0 P_2^0(\cos\theta).$$

The value of m and the direction of the magnetic axis are derived from the first three Gauss coefficients.[5] The magnetic axis intersects the northern hemisphere at the geomagnetic pole with coordinates $\Theta_0 = 11°\ 26'$, $\Phi_0 = -70°\ 2'$. The geomagnetic coordinates refer then to this pole. The dipole strength $m = 3119 \times 10^{-4}$ gauss. To evaluate the zonal quadrupole strength q_2^0, we take the quadrupole terms of the earth's magnetic potential and expand them in terms of spherical harmonics with geomagnetic coordinates. q_2^0 is the coefficient of P_2^0 in the expansion. We find that for 1945, $q_2^0 = -38 \times 10^{-4}$ gauss.

The relative strength of q_2^0 with respect to m is 1.2%. For comparison, the quadrupoles which are combined with the centered dipole to give the off-center dipole are about 10% of m. The residual or sectorial quadrupole is about 7% of m. We can thus expect that the zonal effect will be less prominent than either the longitude or the sectorial effect. It must be emphasized, however, that a secular change in the geomagnetic field would change the relative strength of the various quadrupoles, and thus a discussion of their effect upon cosmic-ray intensity seems justified.

We take the motion of a particle of charge e in the combined magnetic field of centered dipole and zonal quadrupole. We are interested in the modified Stoermer's theorem and the modified vertical cutoff. It will be recalled that the integral of motion in the simple dipole case is[2]

$$R \cos\lambda \cos w + (\cos^2\lambda)/R = B,$$

where R is the radius vector from the dipole measured in Stoermer units: $R = (N/M)^{\frac{1}{2}} r$ (N is the magnetic rigidity, $M = ma^3$ is the dipole moment). λ is geomagnetic latitude, w is the angle between the velocity vector and the west, and B is the impact parameter in the ordinary sense, provided distances are measured in Stoermer units. From the Lagrangian for the motion of a charged particle in the combined magnetic fields, it is evident that the equation of motion in geomagnetic longitude can be integrated at once. We then have the following result:

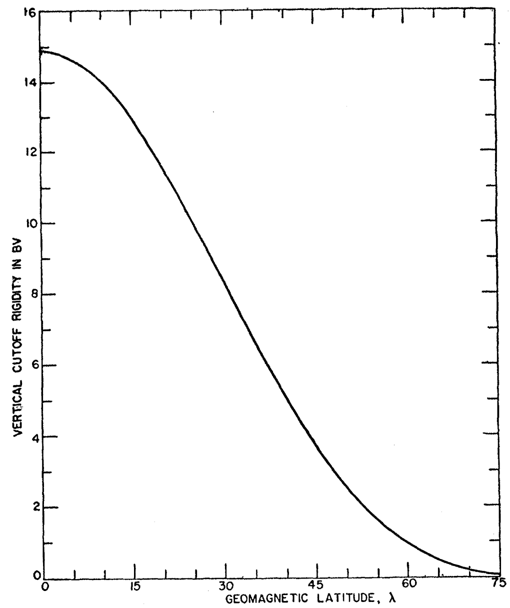

FIG. 2. Vertical cut-off rigidity for the centered dipole model neglecting penumbra effects.

$$R \cos\lambda \cos w - \frac{\cos^2\lambda}{R}\left[1 - \frac{3}{2}\frac{q_2^0}{m}\frac{A}{R}\sin\lambda\right] = B.$$

Requiring $\cos w \leq 1$ defines an allowed region of the $R-\lambda$ plane. As in ordinary Stoermer theory, at some value of B the allowed region splits up into an inner and an outer region. Since in the equatorial plane the above integral reduces to the Stoermer integral, we find that the value of B at which the allowed regions separate is $B = 2$, as in ordinary Stoermer theory. We note therefore that the intensity in the centered dipole equator is unchanged.

B. Modified Vertical Cutoff

It is well known that for a centered dipole there is a minimum rigidity for which a particle coming from infinity can arrive at the earth's surface at latitude λ and angle with the west w. For particles which arrive in the geomagnetic meridian plane, the lowest rigidity, i.e., the vertical cut-off rigidity N, is given by

$$N = 14.9 \cos^4\lambda \text{ Bv}$$

(see Fig. 2). It is clear that the vertical cutoff is of particular interest because particles arriving from the west have a lower critical rigidity and those from the east have a higher critical rigidity, but on the average over zenith and azimuth for a given latitude, the mean effective cutoff is that of the vertical.

The effect of adding a zonal quadrupole to the centered dipole is to modify Stoermer's theorem, as mentioned above. From the modified theorem, and

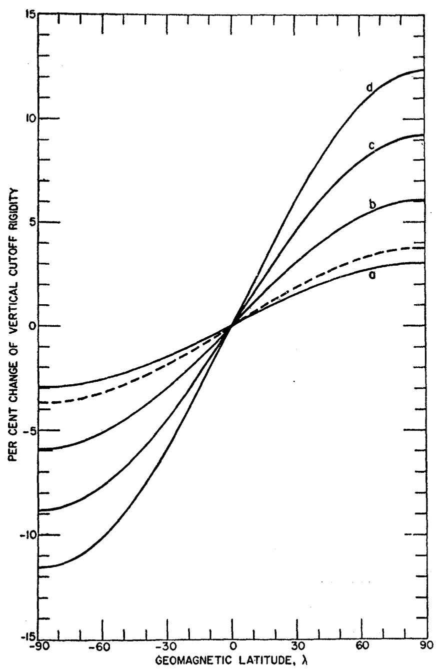

FIG. 3. Percent change of vertical cut-off rigidity for the centered dipole plus zonal quadrupole model. The dotted line is for $q_2^0/m = -1.2\%$. The curves a, b, c, d, are for values of $q_2^0/m = -1$, -2, -3, -4% respectively.

from the 1945 computed value of the zonal quadrupole, we get for the vertical cut-off rigidity

$$N' = 14.9 \cos^4\lambda[1+0.018 \sin\lambda]^2 \text{ Bv}.$$

We note that vertical here is the true geographic vertical, i.e., the line joining observer to the center of the earth.

For convenience in discussing the physical meaning of the zonal effect, we introduce the percent change of vertical cut-off rigidity, $(N'-N)/N$. This is the correction which must be added to the centered dipole vertical cutoff to account for the addition of the zonal quadrupole. $(N'-N)/N$ is plotted as a function of latitude for several values of q_2^0/m in Fig. 3. The dotted curve corresponds to the value of q_2^0/m as computed from the 1945 survey.

It is also convenient to introduce the quantity $[N'(\lambda)-N'(-\lambda)]/N(\lambda)$. This is the north vertical cutoff minus the south vertical cutoff in percent. It is given in Table II. We would like to point out that $(N'-N)/N$ is positive, which means that the northern hemisphere cutoff N' is greater (and hence the cosmic-ray intensity less) than the N' at the corresponding latitude in the southern hemisphere. The result is that the intensity at a latitude just north of the centered dipole equator will be less than that at the equator, so that the minimum of the latitude curve is shifted northward. It is to be remembered that these results depend on $q_2^0/m = -0.012$. If q_2^0 were positive instead of negative, for example, a southward shift of the cosmic-ray latitude curve would be predicted.

TABLE II. North vertical cutoff minus south vertical cutoff, in percent.

λ in degrees	0	15	30	45	60	75
$\frac{N'(\lambda)-N'(-\lambda)}{N(\lambda)}$, in percent	0	1.9	3.7	5.2	6.4	7.1

There is a way of checking the validity of Table II. It is a computational check, that is, a test of the numerical work. The north vertical cutoff minus the south vertical cutoff in percent was found by starting with the model: dipole plus zonal quadrupole. In the next section, a closer approximation to the earth's field is reviewed: the off-center dipole. Schmidt[6] has shown that the off-center dipole is equivalent to the centered dipole plus the zonal and tesseral quadrupoles. We expect, therefore, that the closer approximation will give roughly the same north minus south vertical cutoff as Table II, since it is mainly the zonal quadrupole which causes a north-south asymmetry. In fact, the off-center dipole does give approximately the same north vertical cutoff minus south vertical cutoff, as shown in Fig. 7.

A northern shift in the cosmic-ray latitude curve would be most easily observed with a neutron monitor, which is a latitude-sensitive and high-count rate detector.[9] We consider the following experiment: A measurement of cosmic-ray intensity has been made at a given latitude in the northern hemisphere. Then at what southern latitude should the same intensity be expected? If corrections are made for time variations, the same intensity should be observed where the vertical cutoff rigidity is the same. Thus if the northern measurement is at 45 degrees, we would expect the same intensity at 44.2 degrees in the southern hemisphere, or a northward shift of 0.4 degree is predicted. In particular, the minimum is predicted to occur at latitude 0.6 degree north. The amount of the shift decreases near the knee of the latitude curve. Such a shift is just at the threshold of observability now. If a secular change of the earth's field were to bring about a zonal quadrupole two or three times greater, however, a north or south shift would probably be observable.

III. OFF-CENTER DIPOLE

The off-center dipole has been discussed extensively in the literature.[3,10] The discussions have been based upon the magnetic survey of 1922, however, and it was felt worth while to calculate the magnetic center using the survey of 1945. The results of this calculation are given in this section together with a treatment of the vertical cut-off rigidities differing slightly from that given previously.

[9] Simpson, Fonger, and Treiman, Phys. Rev. **90**, 934 (1953).
[10] T. H. Johnson, Revs. Modern Phys. **10**, 193 (1938).

It is well known that the centered dipole and five quadrupole terms of the earth's field can be replaced by a single dipole at the magnetic center having the same strength and direction, together with two quadrupole terms specifying a sectorial quadrupole. The equations for the magnetic center have been derived by Schmidt.[6] He had calculated the magnetic center for the 1922 magnetic survey and several previous surveys. Using his method, we have calculated the 1945 magnetic center and it is compared with the 1922 center in Table III. (See also Fig. 4.) δ is the radius vector from the center of the earth to the magnetic center in units of earth radii; it is directed towards geographic latitude 14 N, geographic longitude 154 E.

A result of the slow change in magnetic center is that a corresponding slow change in knee of the cosmic-ray latitude curve would be seen. In making measurements of the knee over periods of several years, one would like to know what maximum shift in off-center dipole latitude can be expected, at the present velocity of the magnetic center. The average velocity of the magnetic center from 1922 to 1945 is 4 km per year. In the latitude airborne flights of Simpson[11] in 1948 and Simpson and Meyer[12] in 1954, a northward shift of the knee of about 3 degrees is observed. A change in magnetic center can only account for at most $\frac{1}{4}$ degree of shift between 1948 and 1954.

TABLE III. Off-center dipole.

Epoch	Coordinates of magnetic center: Along 0°E	Along 90°E	Toward geographic north	δ Distance to center of earth	Distance from centered dipole equator
1922	−0.0509	0.0168	0.0061	0.0536 = 341 km	10 km S
1945	−0.0549	0.0267	0.0152	0.0629 = 400 km	40 km N

It should be noted that the motion of a cosmic-ray particle can be referred to the case of the simple dipole *provided the sectorial quadrupole is neglected* and account is taken of the observer's position relative to the off-center dipole. Thus, following previous treatments, it will be assumed here that the sectorial quadrupole is zero. In the next section, however, we will consider what the effects are of including a nonzero sectorial quadrupole.

Vallarta has shown[3] that the effect upon cosmic-ray intensity of the off-center dipole is that a vertical cutoff is defined which depends upon longitude, thus qualitatively explaining the cosmic-ray longitude effect. It should be pointed out that vertical now refers to magnetic vertical, that is, the line between the observer and the magnetic center. Geographic vertical is the line between observer and the center of the earth and will therefore be different from the magnetic vertical. We introduce then the relative change: off-center dipole cutoff minus centered dipole cutoff, or $(N'-N)/N$. In changing from a centered dipole to the off-centered dipole, there is a change in the radius vector, which is most important in the equatorial plane, and becomes less important at higher latitudes. There is also a change in latitude which is more prominent at higher latitudes. One finds

$$\frac{N'-N}{N}=\frac{1}{r'^2}\cdot\frac{\cos^4\lambda'}{\cos^4\lambda}-1,$$

where r' is the off-center radius vector in units of earth radii and λ' is the off-center latitude. $(N'-N)/N$ is computed for several latitudes and is given in Figs. 5 and 6. We notice that the northern and southern hemispheres are slightly different. As a measure of the north-south asymmetry, we consider the quantity $[N'(\lambda)-N'(-\lambda)]/N$. It is plotted as a function of latitude in Fig. 7 and it is seen that there is approximate agreement with the north-south asymmetry, obtained from the zonal quadrupole alone. That there is not an exact agreement is due to the fact that the magnetic vertical for the off-center dipole does not coincide with the geographic vertical.

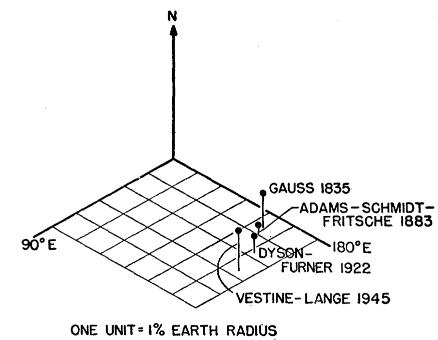

FIG. 4. Position of the magnetic center for various epochs. The axes are geographic.

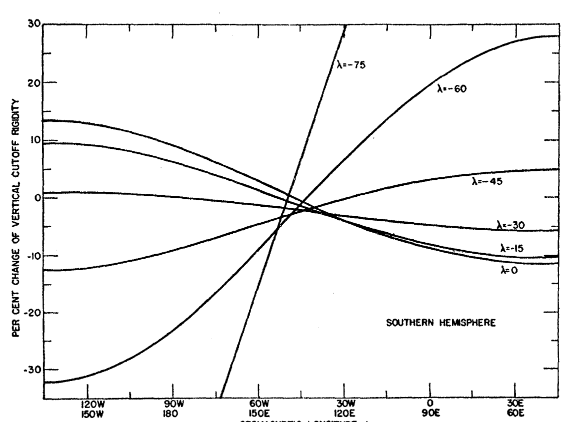

FIG. 5. Off-center vertical cutoff minus centered vertical cutoff in percent. The equatorial curve has its maximum at the point of nearest approach to the magnetic center which is approximately at 135 W magnetic longitude.

[11] J. A. Simpson, Phys. Rev. 83, 1175 (1951).
[12] J. A. Simpson and P. Meyer (to be published).

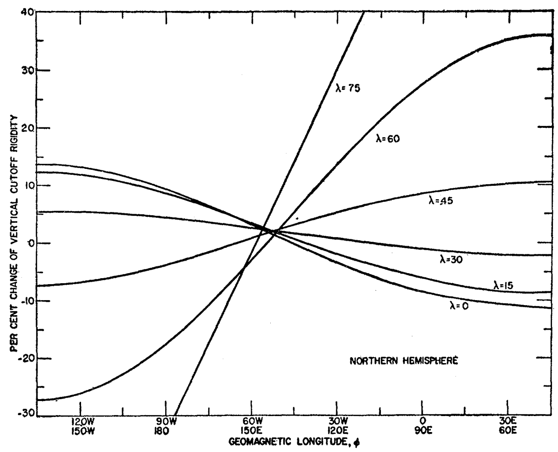

FIG. 6. Same as Fig. 5, but for the northern hemisphere. Except for the equator, the two hemispheres are different.

IV. SECTORIAL QUADRUPOLE

We would like to point out that up until now models of the earth's field have been considered which either could be referred to the case of a single dipole, or else had axial symmetry. With the off-center dipole and sectorial quadrupole, however, there is no longer the possibility of referring to a single dipole, nor is there axial symmetry.

A sectorial quadrupole can be represented by two equal and oppositely directed dipoles at an infinitesimal distance apart. The plane determined by the equal and opposite dipoles is perpendicular to the off-center dipole, as a consequence of the definition of magnetic center.[6] The cylindrical symmetry is destroyed (Fig. 8). The relative strength of the sectorial quadrupole to the dipole decreases with increasing distance from the earth. Stoermer's theorem will still be approximately valid. The question is, "how valid?"

Another way of stating the problem is as follows: Suppose we have a negative cosmic-ray particle leaving the earth and traveling out to some asymptotic latitude and longitude. (This is equivalent to a positive particle coming from infinity to the earth.) With the simple dipole, the position and velocity at every point along

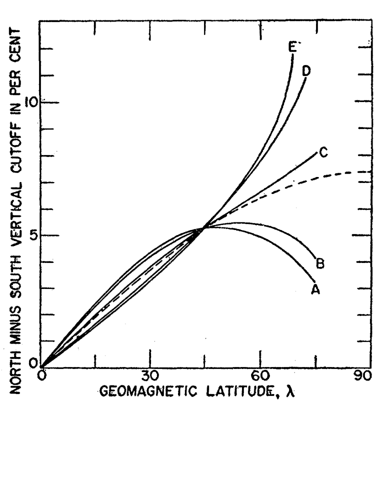

FIG. 7. The solid curves are the northern off-center vertical cutoff minus the southern off-center vertical cutoff. The dotted curve is north vertical cutoff minus south vertical cutoff for the centered dipole plus zonal quadrupole model. Curves A, B, C, D, E are for magnetic longitudes 135 W, 90 W, 45 W, 0 and 45 E respectively.

the trajectory will satisfy Stoermer's theorem for a negatively charged particle.

$$R\cos\lambda\cos w-(\cos^2\lambda)/R=B.$$

With the addition of the sectorial quadrupole, the effect upon the particle is that it will travel along a slightly different trajectory. If the coordinates of the particle along the new trajectory are denoted by a subscript q, they will satisfy the equation

$$R_q\cos\lambda_q\cos w_q-(\cos^2\lambda_q)/R_q=B+\delta B,$$

where δB is zero at the surface of the earth. δB increases in magnitude as the particle travels away from the earth. The problem is, then, "to what magnitude does δB increase?" A few earth radii away, the sectorial effect drops off in importance and ordinary Stoermer theory applies, but with the changed impact parameter.

We choose a system of coordinates with the magnetic center as the origin, the off-center dipole pointing downwards along the negative z axis. Referring to Fig. 8, the $\varphi'=0$ axis passes through the two north poles of

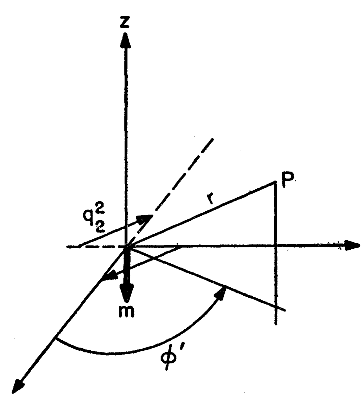

FIG. 8. Illustration of the coordinates used for the off-center dipole, plus sectorial quadrupole model.

the quadrupole couple. The magnetic potential, V'', of off-center dipole and sectorial quadrupole can be written

$$V''=\frac{-a^3m}{r^2}P_1^0(\cos\theta')+\frac{a^4q_2^2}{r^3}\cos2\varphi'P_2^2(\cos\theta').$$

By the method of Schmidt, q_2^2 is found to be 208×10^{-4} gauss and the $\varphi'=0$ axis is directed toward 17° E geographic longitude.

The equations of motion of a charged particle have been considered in this field and there are a few general statements that can be made about them. First, if the motion is entirely in the equatorial plane, we are reduced to the dipole case, except that there is a slight acceleration in the z direction tending to deflect the particle out of the equatorial plane. Second, as with the simple dipole equations, there is no solution in terms of known functions. The solution could be obtained by numerical integration. Rather, an approach has been made via Stoermer's theorem. In ordinary Stoermer theory, it is the equation containing the second derivative of the longitude that is integrated to give Stoermer's theorem. With the addition of a sectorial quadrupole, only a part of the equation can be integrated, and the rest of it is left as an indefinite integral, multiplied by the ratio of sectorial quadrupole strength to dipole strength, q_2^2/m, which is 7%. It is the indefinite integral multiplied by q_2^2/m which we have called δB. For two cases it has been evaluated by a first-order perturbation method. We have integrated two dipole orbits, that is, they are the unperturbed trajectories ($q_2^2=0$). They are inserted in the integral and it is evaluated numerically giving us δB to first order in q_2^2/m. δB for the two cases is given in Fig. 9 as a function of arc length (measured in units of earth radii). The two cases are negative particle orbits of magnetic rigidity 19 Bv and 6 Bv leaving a latitude of 45 degrees vertically with respect to the off-center dipole. The initial geographic longitude is 73 W or near the eastern seaboard of the United States.

It must be emphasized that we have started with a known orbit and have assumed that the perturbed orbit was in the neighborhood of the known orbit. In reality, it could happen that the perturbed orbit is quite different from the unperturbed. For example, the unperturbed orbit might have a rigidity below the cutoff and hence return to the earth, whereas the addition of the sectorial quadrupole would perturb the orbit so that it could go on out to infinity.

Returning to a more qualitative discussion, we expect the sectorial effect upon cosmic-ray intensity to show up more at middle to high latitudes rather than at the equator, first because δB is in general larger for low energies, second because the equations revert essentially to the simple dipole case in the equatorial plane. The sectorial effect would be observed experimentally by a detector traveling around the earth at a constant middle geomagnetic latitude. The cosmic-ray intensity as a function of longitude would be a one-period sine curve due to the off-center dipole, and superimposed would be a two-period sine curve due to the sectorial quadrupole.

V. CONCLUSIONS

A study has been made of the motion of charged particles in the earth's combined quadrupole and dipole magnetic fields. The influence upon cosmic-ray intensity of a quadrupole axially symmetric about the centered dipole, is to shift the latitude curve northward or southward. In particular, the results of the 1945 magnetic survey indicate that the shift is northward, and such that the minimum in the cosmic-ray latitude curve should occur at 0.6 degree north of the centered dipole geomagnetic equator. The shift decreases to $\frac{1}{4}$ degree near the knee of the cosmic-ray latitude curve.

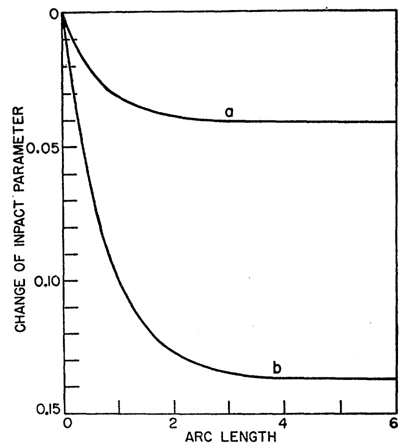

FIG. 9. Plot of the varying term δB which must be added to Stoermer's equation in the off-center dipole plus sectorial quadrupole model. Curve *a* is for a 19-Bv orbit, curve *b* for a 6-Bv orbit. Arc length is in units of earth radii.

The distance of the magnetic center from the geographic center is computed to be 0.0629 earth radii on the basis of the 1945 magnetic survey. This is compared to the 1922 value 0.0537. Cosmic-ray intensity in the off-center dipole field is taken into consideration by computing the off-center vertical cut-off rigidities for several geomagnetic latitudes. The expected longitude effect is larger in 1945 than in 1922.

The trajectory of a charged particle in the earth's sectorial quadrupole and off-center dipole field can be treated as a simple dipole trajectory which is perturbed by the asymmetric sectorial quadrupole. The effect of the sectorial quadrupole is described quantitatively by adding a new variable term to the impact parameter in Stoermer's theorem. For two trajectories the magnitude of this term is calculated by a first-order perturbation method. The physical interpretation attached to the variable term is an uncertainty in the asymptotic latitude and longitude of the two trajectories. Qualitatively, the effect upon cosmic-ray intensity of the sectorial quadrupole is predicted to be a two-period sine curve in the longitude effect at intermediate latitudes.

ACKNOWLEDGMENTS

The writer expresses his thanks to Professor J. A. Simpson for generous support and encouraging discussions. We would also like to thank Professor M. Schein for several interesting talks. For assistance of many kinds we are deeply indebted to M. Pyka, R. Vogt, and M. Baker of this laboratory. We are very grateful to S. Pao for preparing the graphs of the off-center vertical cutoff, and to M. Chen for integration of the 6-Bv cosmic-ray orbit.