Superconductivity and Electronic Specific Heat

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An empirical correlation between critical field and critical temperature of a class of superconducting elements and intermetallic compounds is described. It is shown that for all those which do not have a partly filled d or f shell, $H_0 \sim T_c^{1.37}$. This implies a similar relation involving the electronic specific heat parameter γ. Some further implications are discussed.

RECENTLY, having convinced ourselves that the perturbation treatment of the electron-lattice interaction theory[1] of superconductivity is better than had previously been thought, we undertook to apply this method to a more realistic case than the free electron gas treated by Fröhlich. It is not proposed here to discuss the theoretical work, which will be published in full later, but to report an interesting correlation among superconductors, to which we were led by this work.

Briefly, it was predicted that the critical field H_0 of a superconductor at absolute zero should vary approximately as a power of the electronic specific heat constant, γ. Since, according to the thermodynamic treatment of superconductivity, $H_0^2/8\pi = \gamma T_c^2/2$, where T_c is the critical temperature at zero field, this means that H_0 should vary as a power of T_c. This is expected to be true for all elements and intermetallic compounds with a relatively simple band structure. Specifically, there should not be a partly filled d or f shell, since that would require a serious modification of the theoretical treatment. (This modification is not difficult in principle, but has not yet been made.)

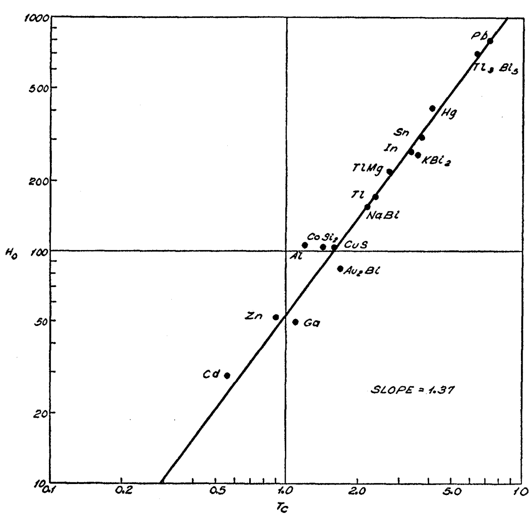

FIG. 1. Critical field at absolute zero *versus* critical temperature at zero field. All data taken from D. Schoenberg, *Superconductivity* (Cambridge University Press, Cambridge, 1952). All elements and intermetallic compounds without partly filled d or f shells, for which measurements of H_0 and T_c are available.

In any case, we were led to plot H_0 against T_c on log-log paper, for all elements and intermetallic compounds which do not have a partly filled d or f shell, and for which decent measurements of H_0 are available. In cases where differing values of H_0 have been measured by different workers, we have uncritically taken the average, and where only $(dH_c/dT)_{T=T_c}$ has been measured, we have used a parabolic extrapolation. The resulting curve is shown in Fig. 1, and is a surprisingly good straight line, whose slope is approximately 1.37. Therefore, combining this result with $H_0^2/8\pi = \gamma T_c^2/2$, we get $T_c \sim \gamma^{1.35}$ and $H_0 \sim \gamma^{1.85}$. We feel that this correlation is sufficiently good to warrant its publication independently of the theoretical work that led to it. A somewhat different correlation between critical temperature and specific heat has been proposed by Daunt[2] for the superconducting elements.[3]

There remain only two brief comments on the interpretation of this curve. In the first place, as an empirical fact, the electronic specific heat cannot be as unilaterally responsible for superconductivity as might be inferred from the curve. There are nonsuperconductors (like copper) whose electronic specific heat per unit volume is large enough to put them on the curve. Further, it is certain that nonelectronic properties of the material, such as the isotopic mass, are important. The correlation here isolates one particular feature of the electronic system.

In the second place, it may be of some interest to note that, if the curve is taken seriously, and extrapolated, a critical field of one gauss is reached for $T_c \approx 0.05°$K. Consequently, one would not expect to find any superconductors below this temperature without taking the appropriate precautions to shield out the earth's magnetic field. Indeed, the susceptibility of the paramagnetic salt used to reach such temperatures, and the residual field (if any) of the demagnetization apparatus, strengthen this argument.

[1] H. Fröhlich, Phys. Rev. **79**, 845 (1950); J. Bardeen, Phys. Rev. **80**, 567 (1951).

[2] J. G. Daunt, Phys. Rev. **80**, 911 (1950).

[3] I am indebted to Professor Daunt for an interesting conversation about the implications of such correlations as these.