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Glow-to-Arc Transition

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The conditions for the glow-to-arc transition at moderately high pressures (50 to 1300 mm Hg) have been studied experimentally. It is found that over the whole pressure range the transition is certain when the field at the cathode reaches a critical value. This result is shown to be consistent with a field emission mechanism for the transition. Calculations of the critical field based on this process are found to be in good agreement with the fields which are determined experimentally.

INTRODUCTION

IT is quite certain that there is no single mechanism which can account for all the experimental conditions under which a glow discharge changes abruptly into an electric arc. Druyvesteyn,[1] for example, has shown that thermionic emission from the cathode of a discharge can lead to a continuous transition. The thermionic emission may be produced by direct heating of the cathode by positive ion bombardment or the same effects may be achieved by raising the temperature of the cathode with an auxiliary heater. There are many experiments, however, which show that a transition can take place in such a short time that all gross heating effects are precluded, and moreover this mechanism cannot operate with a cathode which has a low melting point. The number of possible alternative mechanisms which have been suggested is unfortunately large.[2] Many of these involve extraneous effects, i.e., loose particles or various impurities on the cathode. Perhaps these should not be considered as fundamental mechanisms for a gas discharge. Mackeown[3] has suggested that the high fields at the cathode of a glow discharge may induce field emission and that this may lead to an instability in the glow. It is the purpose of this paper to develop this field emission theory on a semiquantitative basis and to show experimentally that it is responsible for the transitions observed in moderately high-pressure discharges as described below.

The normal glow discharge in air between noble metal electrodes is quite stable over a wide range of pressures. When the cathode has been cleaned by sputtering, it is only by restricting the area of the cathode and increasing the current so that discharge becomes an abnormal glow that a transition to an arc takes place. The field at the cathode increases in the abnormal glow and it will be shown that the glow to arc transition is certain only when this field has reached a critical value, and that this transition field is the same over the whole pressure range investigated.

This in itself is not sufficient to establish the field emission mechanism. Further measurements, however, on field emission currents from similar electrodes establish the field emission constants for this surface. When these constants are used in the field emission transition theory which is developed below good agreement between the calculated and observed critical fields is obtained.

THEORY

The process by which a small field emission current from the cathode can lead to instability in the glow discharge is described as follows. Because of the large fields in the cathode fall region, every electron liberated from the cathode produces a small Townsend avalanche. The positive ions from this avalanche increase the field at the cathode by their space charge. This increased field at the cathode may, if there is a large field already present, extract another field emission electron and an unstable cycle results. The criterion for this process to lead to breakdown has been developed in a recent paper[4] and its extension to the glow discharge is presented in the second part of this section. The first part immediately below presents the method used in calculating the field at the cathode of the abnormal glow.

It is well established experimentally that the field in the cathode fall region decreases approximately linearly[5] with the distance from the cathode, becoming zero at the edge of the negative glow.[6] It follows then that

$$j_+ = (V_c/2\pi d_c^2)u_+, \tag{1}$$

where j_+ is the positive ion current density at the cathode, V_c the cathode drop, u_+ the ion velocity at the cathode, and d_c the thickness of the cathode fall. The space charge effects of the electrons have been neglected because of their high mobility.

The drift velocities of ions in oxygen and nitrogen have been measured by Varney[7] and he obtains $u_+ = 1.23 \times 10^4\ (E/p_0)^{0.5}$ for O_2^+, and $u_+ = 0.94 \times 10^4$

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[1] M. S. Druyvesteyn, Z. Physik **73**, 727 (1932).

[2] J. M. Meek and J. D. Graggs, *Electrical Breakdown in Gases* (Oxford University Press, Oxford, 1953), p. 472.

[3] S. S. Mackeown, Elec. Eng. **51**, 386 (1932).

[4] W. S. Boyle and P. Kisliuk, Phys. Rev. **97**, 255 (1955).

[5] L. Loeb, *Fundamental Processes of Electrical Discharges in Gases* (John Wiley and Sons, Inc., New York, 1939), p. 581.

[6] Roger Warren [Phys. Rev. **98**, 1658 (1955)] concludes that the field varies as the $\frac{2}{3}$ power of the distance from the negative glow. Such a variation changes only the constant of Eq. (2) below, increasing it by about 9%.

[7] R. M. Varney, Phys. Rev. **89**, 708 (1953).

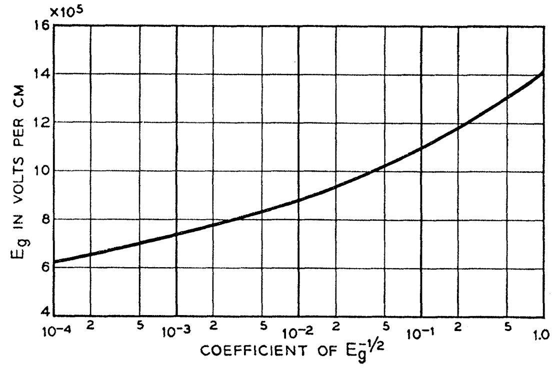

FIG. 1. Solution of the equation for the critical field E_g plotted as a function of the coefficient of $E_g^{-\frac{1}{2}}$ on the right-hand side of the equation.

$(E/p_0)^{0.6}$ for N_2^+. Substituting this expression in (1) and eliminating d_c from the fact that the field at the cathode E_c is equal to $2V_c/d_c$ for a linear variation in field, one obtains in practical electrical units and mm Hg, for either O_2^+ or N_2^+,

$$E_c \doteqdot 5100 p_0^{0.2}(V_c j_+)^{0.4}. \tag{2}$$

In Eq. (2), p_0 is the pressure corrected to standard conditions. In the present experiments, a normal glow discharge is maintained over the full area of the electrodes and then pulse currents are applied to reach into the abnormal glow. At the higher pressures the normal glow produces considerable heating of the gas and the electrodes. It is necessary therefore, to make a correction for this heating effect. This is done by using the similarity law, i.e., j_n/p_0^2 is a constant,[8] where j_n is the normal glow current density. This constant is measured at low pressures where heating is unimportant and then subsequent measurements of the normal glow current density at higher pressures give appropriate values of the reduced pressure p_0.

We now proceed to develop an expression for the glow to arc transition due to field emission. The electron field emission current density j_- is given by the Fowler-Nordheim equation as $j_- = 1.54\times10^{-6}(E^2/\varphi)\times\exp(-6.8\times10^7\varphi^{\frac{3}{2}}/E)$, where φ is the work function of the surface and E is the field. This may be written approximately as $j_- = A\exp(-b/E)$. The field E is the sum of the field due to the glow discharge E_G and the field E_+ due to the extra ionization produced by the field emission current. Because of the steepness of the Fowler-Nordheim equation, it can be assumed that $E_+ \ll E_G$ up to the point of transition whenever E_G is large enough to produce appreciable field emission current by itself. The above approximate form of the Fowler-Nordheim equation can then be expanded to give

$$j_- = A[\exp(-b/E_G)][\exp(bE_+/E_G^2)]. \tag{3}$$

In this expression $E_+ = (dE/dj_+)j_+'$, where j_+' is the extra positive ion current density at the cathode arising from j_-, and dE/dj_+ is a constant G = approximately $2\times10^3 p_0^{0.2}V_c^{0.4}/j_+^{0.6}$ from Eq. (2).

Let $j_+' = \delta M j_-$, where M is the total number of ions formed in a single avalanche in the cathode fall and δ is the spreading factor giving the ratio of the area of the emitting point on the cathode to the area covered by the ions as they move back to the cathode. Then

$$E_+ = GM\delta j_-. \tag{4}$$

As pointed out in reference 4, the process becomes unstable when the quantity bE_+/E_G^2 in Eq. (3) is equal to the reciprocal of the exponent of j_- in Eq. (4). This instability represents transition into an arc, and occurs when

$$bE_+/E_G^2 = 1. \tag{5}$$

Eliminating E_+ and j_- from Eqs. (3), (4) and (5) gives an explicit equation for determining the critical value of E_G at which a glow becomes unstable and breaks down into an arc,

$$\exp(-b/E_G) = E_G^2/AM\delta Geb. \tag{6}$$

The evaluation of each of the parameters b, δ, M, and A in Eq. (6) is discussed below:

(1) b.—This is determined from the slope of a field emission plot taken in vacuum with electrodes which had been previously sputtered clean in a glow discharge.

(2) δ—The diameter of the emitting point is assumed to be about 200 A. This is based on measurements made by Boyle, Kisliuk, and Germer,[9] which gave approximately the same value of b which was obtained in the present measurements. It can then be shown that the dominant factor in the spreading arises from scattering of electrons in the cathode fall. The average lateral displacement d_f of electrons in crossing the fall region is given by $d_f = (\pi/2)d_c(\bar{V}/V_c)^{\frac{1}{2}}$, where $\bar{V}$ is the average electron energy expressed in volts. This is based on the assumption of simple diffusion in the linearly decreasing field of the cathode fall. In terms of

TABLE I. Summary of data when transition is certain.

p (mm Hg)	j_n (amp/cm²)	p_0 (mm Hg)	j (amp/cm²)	V_c (volts)	E_c (10^5 v/cm) (exp)	E_c (10^5 v/cm) (calc)
50	0.122	25	123	730	9.3	9.3
140	0.44	47	118	575	9.4	9.0
440	1.90	98	117	460	9.9	8.8
750	3.30	129	109	400	9.6	8.7
1300	4.50	150	98	380	9.4	8.6

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[8] Engel, Seeliger, and Steenbeck [Z. Physik **85**, 144 (1933)] have shown that the reported departure from the similarity principle with $j \propto p^{4/3}$ at the pressures used here is caused by the heating of the gas. When they allow for this the similarity principle is closely followed. One can, therefore, calculate the temperature correction with considerable confidence, except for a thin layer of gas cooled by the cathode. The effect of the latter is to increase the field at the cathode by a small amount above that which is calculated here. Further heating of the gas during the pulsed excursions into the abnormal glow is estimated to be small. This is indicated experimentally by an essentially constant cathode fall voltage throughout a single pulse.

[9] Boyle, Kisliuk, and Germer, J. Appl. Phys. **26**, 720 (1955).

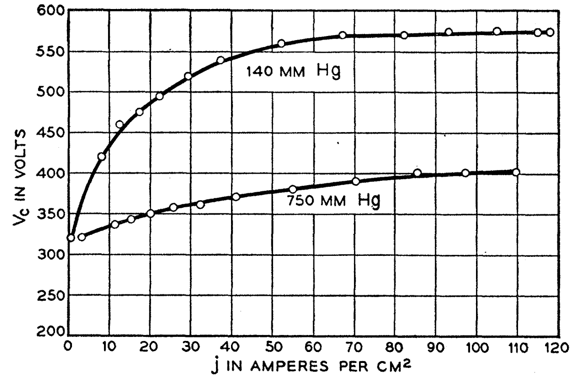

FIG. 2. Typical curves of the variation of the cathode voltage drop V_c with the current density j. The point at lowest current density on each curve is for the normal glow discharge, and that at highest current density is when arcing is certain.

the radius ρ of the emitting point the spreading factor δ is therefore given by

$$\delta=(\rho/d_f)^2=4\rho^2V_c/\pi^2d_c^2\bar{V}. \tag{7}$$

(3) M.—If the field in the cathode fall were such as to give a maximum in value to the ionization coefficient, approximately 200 ion pairs would be produced in a single avalanche. Because of the nonuniform field the value is probably considerably smaller than this, and we shall use a value of 100 for our calculations.

(4) A.—The Fowler-Nordheim equation gives for A, $(1.54E^2/\phi)\times10^{-6}$. The appropriate value of E to use is not E_c but βE_c, where β is the field multiplication due to surface irregularities. The value of β can be calculated from the b value of the field emission plot since $b=6.8\times10^7\varphi^{\frac{3}{2}}/\beta$.

Inserting the values for A, G, and δ and eliminating β, Eq. (6) becomes

$$\begin{aligned}&\exp(-b/E_G)\\&\quad=2.5\times10^{-6}b\bar{V}(V_cj_+)^{0.6}/p_0^{0.2}M\varphi^2E_G^2\rho^2.\end{aligned} \tag{8}$$

Since E_G is very closely equal to E_c, this expression can be further simplified by using Eq. (2) to eliminate the product V_cj_+, giving finally:

$$\exp(-b/E_G)=(7\times10^{-12}b\bar{V}/M\varphi^2\rho^2p_0^{\frac{1}{2}})E_G^{-\frac{1}{2}}. \tag{9}$$

In deriving this equation several crude approximations have been made in determining δ and M. The exponential form of the left-hand side of Eq. (9), however, reduces the corresponding uncertainty in E_G. This is illustrated in Fig. 1 where the solution of Eq. (9) for E_G is plotted against the coefficient of $E_G^{-\frac{1}{2}}$ on the right-hand side of the equation. Particular solutions corresponding to the experimental values of p_0 are presented in the last column of Table I. For these calculations, we have taken $\varphi=4.8$ volts, $\bar{V}=30$ volts, an experimental value of $b=10^7$ volts/cm which is given later, and M and ρ values as given above.

EXPERIMENTAL DETAILS

A steady glow discharge in air at pressures varying from 50 to 1300 mm Hg is established between the electrodes. The current is adjusted until the negative glow just covers the cathode, time being allowed for it to come to thermal equilibrium. The current density of the normal glow is derived from measurement of this current. The spacing of the electrodes is set so that the anode is in the Faraday dark space (approximately 0.01 to 0.05 cm), the voltage of the discharge then being about equal to the cathode drop.

Square-wave current pulses of 10 microseconds duration are now superimposed on the steady current. The current pulses may drive the discharge far enough into the abnormal glow to force it into an arc. Following each pulse the discharge lapses back into the glow.

The electrodes are small palladium rods 0.074 cm in diameter. They are mounted end to end, the discharge thus taking place between two opposed ends which are ground flat and then polished. The cylindrical side of the cathode is surrounded by a close fitting tube of Al_2O_3, which restricts the area of the negative glow to that of the flat end.

Measurements were made of the cathode voltage drop as a function of current density at five gas pressures from 50 to 1300 mm Hg. The measurements plotted in Fig. 2 are typical. The point at lowest current density on each curve is for the normal glow discharge, and that at highest current density is when arcing is certain.

The field emission constant b was determined for the palladium surface with the same experimental technique as described in reference 4.

RESULTS

The values of the normal glow current density at low pressures are shown in Fig. 3, where j_n is plotted against p_0^2. The slope of this curve yields a value of 2.0×10^{-4}

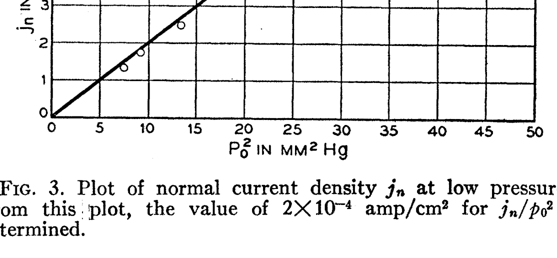

FIG. 3. Plot of normal current density j_n at low pressures. From this plot, the value of 2×10^{-4} amp/cm² for j_n/p_0^2 is determined.

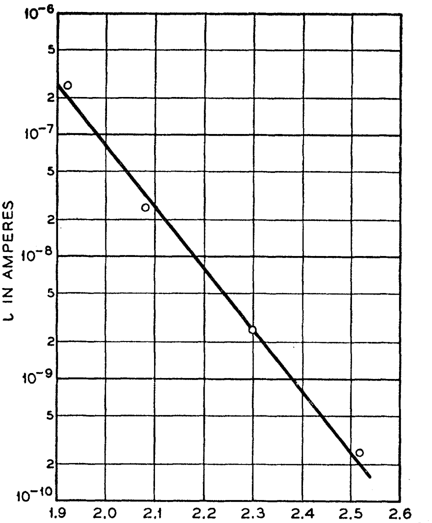

FIG. 4. Field emission current i plotted against the reciprocal of the apparent field E. This plot yields a value of 1.1×10^7 for the empirical field emission constant b.

amp/cm^2 mm^2 Hg. From this, the reduced pressure in the high-pressure range of the experiment was calculated in terms of j_n. The latter data are presented in Table I together with the measured values of V_c and j when arcing is certain. The second to last column shows the critical field for a transition as calculated from Eq. (2).

A typical field emission plot of the current against the reciprocal of the apparent field is shown in Fig. 4. This gives a value for b of 1.1×10^7 volts/cm. Other values obtained in different runs with freshly sputtered surfaces were 1.2, 1.3, 2.7, 0.9, and 1.4×10^7. Clearly the transition will always take place from a point on the surface which has the smallest b value. With this in mind, the above values indicate that a figure near 10^7 volts/cm is appropriate.

DISCUSSION

The agreement between the experimental and calculated field for a transition, as shown in the last two columns of Table I, is certainly better than one could expect from the approximations. One may safely assign the following factors of uncertainty to the parameters, $\rho(\times5)$, $M(\times2)$ and $\bar{V}(\times2)$, found in Eq. (9) for the transition field. Reference to Fig. 1 then shows that if all the errors are in the same sense the resulting factor of 100 leads to an error of about 35% in the predicted field. This still leaves sufficiently good agreement between experimental and theoretical values to establish field emission as the cause of the transition from a glow to an arc.

In conclusion, two points of a general nature may be made concerning the stability of a glow discharge if we exclude gross heating of the electrodes (which can be accomplished either by pulse operation or by appropriate cooling of the electrodes):

1. Since the normal glow current density always increases with pressure and the cathode field increases both with pressure and current density, there must always be an upper limit to the pressure at which a stable glow can be maintained.
2. From the equation for the field emission transition process one can see that the field emission constant b determines very largely the critical field for a transition. Thus, small differences either in the work function or in the surface roughness have a large effect on the stability of the glow at high pressures, or when the calculated cathode field approaches 10^6 volts/cm.

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