Mobility in Zinc Blende and Indium Antimonide

WALTER A. HARRISON
Department of Physics, University of Illinois, Urbana, Illinois
(Received October 19, 1955)

CRYSTALS of the sphalerite structure are piezoelectric.[1] Therefore, there will be an electric polarization associated with the acoustical modes of vibration in zinc blende and indium antimonide. This polarization may lead to local charge accumulation and a periodic electrical potential. The contribution to the electronic scattering by this potential has been calculated for these two cases. It can be shown that there is a phase difference of 90° between the matrix element for scattering by this mechanism and the matrix element found by deformation potential theory; hence it is valid to consider the mechanisms independently.

Normal modes of vibration are developed in the form of traveling waves, assuming the crystal to be elastically isotropic as a working approximation. From the strain associated with each of the modes, the electric polarization is found by using the form of the piezoelectric tensor determined by the crystal symmetry.[1] Use of the measured piezoelectric constant, however, gives the polarization in the absence of an electric field, and an induced polarization equal to $(k_s-1)/4\pi\mathbf{E}$ must be added, where $\mathbf{E}$ is the internal electric field and k_s is the static dielectric constant. By making this correction and using Poisson's equation, the electrical potential is calculated. From this potential the matrix elements for scattering an electron of wave number $\mathbf{k}$ into a state of wave number $\mathbf{k}'$ by longitudinal waves and by each of the transverse waves are found, assuming that the lattice vibrations of importance are fully excited and that an effective mass approximation is valid. The squares of the three matrix elements are then added. In the determination of the relaxation time for electron scattering, τ, the integral over $\mathbf{k}'$ depends upon the choice of initial electron wave number $\mathbf{k}$. The integration has been carried out for $\mathbf{k}$ lying along the three directions [001], [011], and [111]. The results are

$$\frac{1}{\tau}=\frac{\alpha\pi me^2C^2}{\rho c_l^2\hbar^3k_s^2}\cdot\frac{KT}{k},$$

where C is the measured piezoelectric constant, ρ is the density, c_l is the acoustic velocity for longitudinal waves, and α equals 15.8, 15.6, and 15.1 for the three directions, respectively. An average value of α equal to 15.5 was used. In the evaluation of α, the ratio of longitudinal to transverse acoustical velocities is determined from Poisson's ratio which is assumed to be 0.3. Finally an average value of k appropriate for drift mobility is found[2] for low temperatures where the conduction electrons are not degenerate, giving $k=8(2mKT)^{\frac{1}{2}}/3\pi^{\frac{1}{2}}\hbar$. The resulting expression for mobility is

$$\mu=\frac{0.044\rho c_l^2\hbar^2k_s^2}{eC^2m^{\frac{3}{2}}(KT)^{\frac{1}{2}}}.$$

The temperature dependence will be modified if the electrons are degenerate; further, other parameters may vary with temperature, notably m and C.

In zinc blende, $\rho c_l^2=1.14\times10^{12}$ dynes/cm²,[3] $k_s=8.3$,[4] $C=4.2\times10^4$ statcoul/cm²,[5] and m is taken equal to the electron mass. At 300°K, these values give 2700 cm²/volt-sec. The data of Lenz[6] indicates a mobility of about 300 cm²/volt-sec, but corrections for polarization suggested by Klick and Maurer[7] might raise this considerably. Further, the importance of impurity scattering in the measurement by Lenz is not known.

Neither C nor k_s is known for indium antimonide. Since m is very small and k_s is presumably quite large, an anomalously large piezoelectric constant would be required if this mechanism is to be important in indium antimonide.

If the optical dielectric constant, equal to 14,[8] is taken as the lower limit for k_s, $m=0.013m_e$,[9] and $\rho c_l^2=0.75\times10^{12}$ dynes/cm²,[10] then a piezoelectric constant of $C=2.4\times10^5$ statcoul/cm² is required for a mobility of 10^5 cm²/volt-sec at 300°K.

The author is indebted to Professor F. J. Blatt and Professor F. Seitz for guidance and helpful discussions on this work.

[1] W. Voigt, *Lehrbuch der Kristallphysik* (B. G. Teubner, Leipzig and Berlin, 1910), p. 832.
[2] The author is indebted to Dr. Conyers Herring for critical comments on the averaging procedure.
[3] Averaged from the elastic constants measured by W. Voigt, given in Walter G. Cady, *Piezoelectricity* (McGraw-Hill Book Company, Inc., New York, 1946), p. 159.
[4] K. Höjendahl, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **16**, No. 2, 91 (1938).
[5] Reference 3, p. 229.
[6] H. Lenz, Ann. Physik **382**, 449 (1925).
[7] C. C. Klick and R. Maurer, Phys. Rev. **81**, 124 (1951).
[8] Avery, Goodwin, Lawson, and Moss, Proc. Phys. Soc. (London) **B67**, 761 (1954).
[9] Dresselhaus, Kip, Kittel, and Wagoner, Phys. Rev. **98**, 556 (1955).
[10] Estimate by R. W. Keyes, Phys. Rev. **99**, 490 (1955).

Possible Source of Line Width in Ferromagnetic Resonance

A. M. CLOGSTON, H. SUHL, L. R. WALKER, AND P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received November 28, 1955)

THE dissipation of power in ferromagnetic resonance is considered to be a two-stage process.[1] The spin wave state with wave number, k, equal to zero

decays into states of higher wave number, which in turn are degraded by interaction with the lattice. However, the relaxation times estimated from various processes for the initial relaxation time are far too long and have a temperature dependence unlike that observed. Usually the coupling to higher k is considered to be effected by those terms in the Hamiltonian which involve three or more spin operators. The form of the third-order terms allows no transitions conserving energy from the state, $k=0$, if the spin wave dispersion relation for an infinite medium is used. These terms, then, do not give true damping, yielding lines remote from the main line, which account for the moment calculated by Keffer.[2] The fourth-order terms, however, as evaluated by Kasuya,[3] give a genuine relaxation time, rapidly decreasing with temperature.

For a finite sample on the other hand, a dispersion relation may properly be established only for spin wave disturbances short compared to the sample size, but it is important to realize that it then differs markedly from that of an infinite medium. In a spheroid magnetized along the axis of rotation, the dispersion relation is

$$\omega(k)=\gamma[(H_0-N_zM_0+H_{ex}l^2k^2)\times(H_0-N_zM_0+H_{ex}l^2k^2+4\pi M_0\sin^2\theta)]^{\frac{1}{2}}, \quad (1)$$

provided that $2\pi/k\gg l$, the lattice spacing. On the other hand, for $k=0$, one has the exact relation

$$\omega(0)=\gamma[H_0-(N_z-N_t)M_0]. \quad (2)$$

Here H_0 is the applied dc field; M_0, the saturation magnetization; N_z and N_t, the axial and transverse demagnetizing factors; H_{ex} is an exchange field; l, the lattice spacing; and θ, the angle between $\mathbf{k}$ and the axis of magnetization. In the infinite medium the formula[4] for $\omega(\mathbf{k})$ lacks the term, N_zM_0, arising from the demagnetization of the external field. Comparison of $\omega(\mathbf{k})$ with $\omega(0)$ now shows that for all spheroids, save the infinitely thin disk, there will be a closed surface in $\mathbf{k}$-space on which $\omega(\mathbf{k})=\omega(0)$. On this surface, k is of order $(1/l)[4\pi M_0/H_{ex}]^{\frac{1}{2}}$, its maximum value being $(1/l)[(4\pi M_0/H_{ex})(N_t/4\pi)]^{\frac{1}{2}}$. One then has $2\pi/k$ of order $2\pi l(H_{ex}/4\pi M_0)^{\frac{1}{2}}$ so that (1) is valid. As a result of this degeneracy, a genuine relaxation time may be found from a variety of processes which cause transitions from $k=0$ to the degenerate manifold. Such mechanisms are effective also when the motion is described classically. Here they will normally give dissipation, but at high signal levels the degeneracy combined with level dependent coupling terms arising from the nonlinear motion of the magnetization yields a situation in which the spin waves with $\omega(\mathbf{k})=\omega(0)$ grow indefinitely at the expense of the $k=0$ state.[5]

The quadruple terms now give for finite samples a relaxation term which depends linearly upon the excitation of the $k=0$ spin wave and linearly upon temperature; in addition to one of the type found by Kasuya which is little affected by degeneracy. The triple terms may also contribute to damping if the field is sufficiently small for the equality, $\omega(0)=2\omega(\mathbf{k})$, to be fulfilled; for a sphere, this requires $H_0<8\pi M_0/3$. The decay constant for this process is independent of excitation and varies linearly with temperature.

There is an additional mechanism, again giving a small line width in the infinite medium, which, in the presence of degeneracy, probably gives a larger contribution than those mentioned above. If the regularity of the lattice is disturbed in some random fashion, the resulting perturbation of the spin interactions makes possible many transitions between levels of the spin wave system which were forbidden by the regular perturbations so far discussed. In fact, double terms in the perturbing Hamiltonian are already sufficient to induce transitions connecting the state, $k=0$, to the manifold of states with the same energy. A line width can thus be calculated on the basis of double terms alone. Impurities, vacancies, and lattice defects are probably not numerous enough to make this line width appreciable. However, in ferrites with the inverted spinel structure, the random distribution of ions on the octahedral sites gives rise to a fluctuation in the magnetic interactions throughout the lattice. Disregarding sublattice effects the relaxation time, τ, obtained from a model in which the interaction between the spins varies in a random manner and without spatial correlation throughout the crystal, is given by

$$\frac{1}{\tau}=\gamma\frac{S^2}{2\pi}\frac{(4\pi M_0)^{\frac{1}{2}}}{H_{ex}^{\frac{3}{2}}}\cdot H_p^2aF, \quad (3)$$

where S is the spin; H_p^2 is the mean square field corresponding to the fluctuations in the pseudo-dipolar interaction energy of pairs of spins (fluctuations in the exchange energy do not contribute); a is roughly the probability of occurrence of a disordered pair of neighbors; F is a pure number of order unity arising from an integration over the degenerate manifold. F varies rapidly with sample shape through N_t, slowly through changes in $H_0/4\pi M_0$, and depends upon the angle between H_0 and the crystal axes. For a spherical sample, with the values $4\pi M_0/H_{ex}\sim10^{-3}$, $H_p\sim10^5$ oersteds, $S=1$, one finds a line width comparable with measured values. Since the area of the degenerate manifold goes to zero with N_t, F must do the same. The formula (3) thus predicts a marked shape dependence.

It is to be noted that the τ calculated here refers to a decay of the components of magnetization transverse to H_0 into the degenerate manifold in such a manner as to preserve M_z; thus τ will only represent the relaxation time for M_z if the further decay from the degenerate manifold to the lattice is fast compared to τ.

The double terms in this model also give rise to a frequency shift which is proportional to H_p^2/H_{ex}; the

actual magnitude is not markedly affected by the form of the dispersion relation and is roughly that of the observed anisotropy.

[1] C. Kittel and E. Abrahams, Revs. Modern Phys. **25**, 233 (1953).
[2] F. Keffer, Phys. Rev. **88**, 686 (1952); N. Bloembergen and S. Wang, Phys. Rev. **93**, 72 (1954).
[3] T. Kasuya, Progr. Theoret. Phys. (Japan) **12**, 802 (1954).
[4] C. Herring and C. Kittel, Phys. Rev. **81**, 869 (1951).
[5] P. W. Anderson and H. Suhl, Phys. Rev. **100**, 1789 (1955).

Relativistic and Screening Effects in Radiative Electron Capture

R. J. Glauber and P. C. Martin,
Harvard University, Cambridge, Massachusetts,
T. Lindqvist, *University of Uppsala, Uppsala, Sweden,*
and
C. S. Wu, *Columbia University, New York, New York*
(Received October 26, 1955)

THE continuous gamma-ray spectrum (inner bremsstrahlung) accompanying orbital electron capture was recently investigated theoretically.[1] This theory takes the p-electron capture as well as the s-electron capture and the Coulomb effects into consideration and gives a better agreement with experiments in the low-energy region than earlier theories.[2,3] A check of the theory was subsequently made[4] by investigating the inner bremsstrahlung from A^{37} by the method of scintillation spectroscopy with particular emphasis on the low-energy region.

A general formulation of the problem of radiative capture from the various electronic states has been made.[5] According to this theory, the general expression for the radiative events ω_k per $1S$-electron capture $(\omega_c)_{1S}$ can be written, in the absence of relativistic and screening effects, as

$$\frac{\omega_k dk}{(\omega_c)_{1S}}=\frac{\alpha}{\pi}\sum_l\left(\frac{Z^2\alpha^2}{2}\right)^2\left[1-\frac{x-(E_l-E_{1S})}{x_{\max}}\right]^2 I_l(x)dx, \quad (1)$$

where x is the photon energy given in units of Z^2 ry $=Z^2\times 13.5$ ev; $x_{\max}$ is the upper limit of the photon energy; E_l is the ionization potential for the l-electron, and $I_l(x)$ is a tabulated function. The summation has been carried out for $l=1S$, $2S$, $2P$, and $3P$. In Fig. 8 of reference 4 is shown the application of this theory to the inner bremsstrahlung from A^{37}. The agreement down to 100 kev is excellent. As predicted by the theory, the experimental distribution does show a sudden increase at around 30 kev. However, the experimental points below 100 kev lie below the theoretical curve. This discrepancy was found also for the inner bremsstrahlung from Fe^{55} by Madansky and Rasetti (Fig. 9, reference 4).

In order to explain these discrepancies, the calculations have recently been performed[5] taking account of the relativistic effects for the S-state spectrum, and including correction factors for screening. Both the

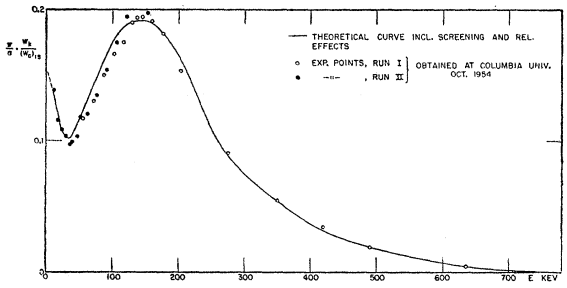

FIG. 1. Inner bremsstrahlung from A^{37}.

relativistic and screening corrections are energy- and charge-dependent, but the variation of the latter with energy is very small. The general expression (1) can therefore be written

$$\frac{\omega_k dk}{(\omega_c)_{1S}}=\frac{\alpha}{\pi}\sum_l\frac{S_l(Z)}{S_{1S}(Z)}R_l(x,Z)\left(\frac{Z^2\alpha^2}{2}\right)^2 \times\left[1-\frac{x-(E_l-E_{1S})}{x_{\max}}\right]^2 I_l(x)dx, \quad (2)$$

where $S_l(Z)$ is the screening correction factor for the lth electron and $R_l(x,Z)$ is the relativistic correction factor for the energy x and charge Z. The numerical values for these correction factors were determined.

The experimental data[4] on A^{37} have been used in a comparison with the theory given by formula (2). The treatment of the data is exactly the same as before,[4] i.e., the experimental points are corrected only for background and the theoretical curve is corrected for the various effects in the NaI scintillation counter. Figure 1 shows the result. The agreement is now excellent over the whole energy region.

[1] R. J. Glauber and P. C. Martin, Phys. Rev. **95**, 572 (1954).
[2] P. Morrison and L. I. Schiff, Phys. Rev. **58**, 24 (1940).
[3] J. M. Jauch, Oak Ridge National Laboratory Report ORNL-1102, 1951 (unpublished).
[4] T. Lindqvist and C. S. Wu, Phys. Rev. **100**, 145 (1955).
[5] R. J. Glauber and P. C. Martin (to be published).

Decay of Co^{61} and Cu^{61}†

R. H. Nussbaum,* A. H. Wapstra, W. A. Bruil,
M. J. Sterk, G. J. Nijgh, and N. Grobben
Instituut voor Kernphysisch Onderzoek, Amsterdam, Holland
(Received November 28, 1955)

THE decay of 1.66-hr Co^{61} and 3.33-hr Cu^{61} was investigated in this institute with β- and γ-scintillation spectrometers in various coincidence setups, with