with $f'(x)=f[\Lambda^{-1}(x-a)]$ in agreement with (1). For the anti-unitary $U(a,\Lambda)$ we have similarly:

$$U(a,\Lambda)\phi(f)\Psi_{g^{(n)}}=U(a,\Lambda)\Psi_{fg^{(n)}}=\Psi_{f^\times g^{\times(n)}}=\phi(f^\times)U(a,\Lambda)\Psi_{g^{(n)}}$$

Therefore,

$$U(a,\Lambda)\phi(f)=\phi(f^\times)U(a,\Lambda),$$

with $f^\times$ defined as in (27) in agreement with (1).

The local commutation rules of $\phi(f)$ can be verified as follows:

$$\phi(f_1)\phi(f_2)\Psi_{g^{(n)}}=\Psi_{f_1f_2g^{(n)}}=\Psi_{f_2f_1g^{(n)}}=\phi(f_2)\phi(f_1)\Psi_{g^{(n)}},$$

when $f_1(x)f_2(y)$ for all $(x-y)^2\geq 0$. The proof depends on the fact that for such f_1 and f_2,

$$f_1(x)f_2(y)[F^{(\cdots)}(\cdots x,y,\cdots)-F^{(\cdots)}(\cdots y,x,\cdots)]=0,$$

by virtue of Eq. (10).

To complete the reconstruction of the theory we need only show that the vacuum expectation values of products of $\phi(f)$'s are the $F^{(n)}$. This is an easy consequence of the formula $\phi(f_1)\phi(f_2)\cdots\phi(f_n)\Psi_0=\Psi_{f_1\cdots f_n}$. It implies

$$(\Psi_0,\phi(f_1)\cdots\phi(f_n)\Psi_0)=\int f_1(x_1)\cdots f_n(x_n)F^{(n)}(x_1,\cdots x_n)d^4x_1\cdots d^4x_n,$$

which was to be proved.

Our reconstruction remains valid in a theory in which the field $\phi(x)$ is not a complete description of the system, e.g., in a theory of interacting neutral mesons and nucleons. However, in such a case the reconstruction process given here will not recover the entire Hilbert space. If one were to define a theory by its analytic functions $F^{(n)}$, rather than by its field equations and commutation rules, then, to be sure that the theory was one of a single field $\phi(x)$, one would have to impose some kind of "completeness" requirement. For example one could require that the set of vectors $\phi(f_1)\cdots\phi(f_n)\Psi_0$ for $n=0, 1, 2, \cdots$ span the whole Hilbert space, where the f_j are testing functions which vanish outside of a spacelike slice of space time of arbitrarily small thickness Δt in the time direction.

7. CONCLUSION

A theory of a neutral scalar field can be reformulated as a theory of a denumerable set of analytic functions of complex variables, $F^{(n)}$, $n=0, 1, 2, \cdots$. Relativistic invariance implies that the $F^{(n)}$ are invariant under Lorentz transformation without time inversion and are therefore functions of certain complex variables z_{ij}^2. Local commutation rules of $\phi(x)$ imply $F^{(n)}(z_{ij}^2)=F^{(n)}(Pz_{ij}^2)$, where P is any of a certain set of permutations of the labels i, j; the positive definiteness of the scalar product implies a set of inequalities connecting the boundary values of the $F^{(n)}$. Given a set of $F^{(n)}$ satisfying the conditions listed, one can reconstruct a theory of a neutral scalar field.

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Radiative Corrections to Decay Processes

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Radiative corrections associated with the electromagnetic field have been determined for the decay of a fermion of arbitrary mass into a lighter one with the emission of a single boson or of two other fermions; no special assumptions have been made about the nature of the interaction responsible for the instability. The particular example of the muon-electron decay has been worked through in detail. Sufficiently accurate experimental determination of the muon spectrum would permit the observation of a Lamb term without vacuum polarization. Modified formulas for the Michel parameter ρ are given.

INTRODUCTION

ALL instabilities of the elementary particles are somewhat modified by fluctuations of their electromagnetic fields.[1,2] These fluctuations are responsible first for the emission of real photons, simultaneous with the decay and independent of the surrounding matter (inner bremsstrahlung) and second for damping effects associated with the unradiated field. This damping may be described in terms of virtual photons and is exactly similar to the processes responsible for the Lamb-Retherford shift. The total probability of decay with and without inner bremsstrahlung would of course exceed the probability of unperturbed decay, were it not for the damping effect of the virtual photons; however both effects are of the same order and must be considered together.

We have considered the decay of an arbitrary charged fermion into a lighter one with the emission of a single boson or of two other fermions, without making any

[1] S. Hanawa and T. Miyazima, Progr. Theoret. Phys. (Japan) 5, 459 (1950).
[2] T. Nakano *et al.*, Progr. Theoret. Phys. (Japan) 5, 1014 (1950).

special assumptions about the nature of the interaction responsible for the decay; and for this general situation have calculated perturbations associated with both real and virtual photons. The simplest reactions covered by these general results might appear to be

$$S_1 \rightarrow S_2 + \mathcal{B}, \qquad \text{(a)}$$

$$S_1 \rightarrow S_2 + 2\nu. \qquad \text{(b)}$$

Reaction (a), when the single boson is a photon, as well as the corrections in question both belong entirely to quantum electrodynamics; the discussion of (a) might in this case appear entirely unambiguous. However, the fact is that no example in which $\mathcal{B}=\gamma$ has been experimentally established. When $\mathcal{B}$ is a strongly-coupled boson, the present calculations are not adequate. Therefore, our general results will not be discussed for case (a), but they will be illustrated in detail for (b). A well known example of the latter is the muon-electron decay; this is the case of main interest. Here one is relatively safe in applying quantum electrodynamics, since the muon as well as the electron is very weakly coupled to other fields. As we shall see, however, some questions of principle do arise in connection with the beta interaction; fortunately the corresponding numerical uncertainties appear to be small.

If the loss of rest mass is large, the order of magnitude of the effect (measured by the fractional change in the probability of decay) is

$$\Delta\mathcal{P}/\mathcal{P} \cong (1/137)[\ln(m_1/m_2)]^2.$$

In the muon-electron decay $(\ln 207)^2=28.4$ so that the effect is large for an electromagnetic correction. In addition, $\Delta\mathcal{P}/\mathcal{P}$ is energy-dependent.

Our main reason for starting this calculation is related to this last point and has to do with the procedure for inferring the beta interaction from the shape of the electron (positron) spectrum. The interpretation of this spectrum is usually based on Michel's one-parameter formula; that is, it has become customary to express the experimental results by giving the value of ρ. However, since Michel's formula ignores radiative corrections, it seemed possible that the observed spectrum might differ from his in a significant way, especially in the interval near the endpoint, which is most important for fixing ρ and where the electron has an energy ~ 100 rest masses. In that case, one would be misled about the value of ρ and therefore about the nature of the primary beta interaction. It turns out that although one would not get into serious error in this way about the general structure of the beta interaction, nevertheless these corrections, of the order of 5 percent, ought to be taken into account, especially in discussing the possibility of a universal Fermi interaction.

FORMULATION

We consider the decay of a spinor particle (rest energy m_1) into a lighter one (rest energy m_2) as the result of an arbitrary tensor interaction (Γ). If $m_1=m_2$ and $\Gamma=\gamma_\mu$, one has the usual examples of quantum electrodynamics.

An ordinary scattering of a particle without change in rest mass may be described as a Lorentz rotation of its momentum vector (p_i); a scattering with change in rest mass may also be regarded as a rotation in suitable variables.[3] Let the familiar angle of the Lorentz rotation be θ and let the new angle correlated with the change in mass be ω. They may be defined by the relations

$$\cosh\theta = p_1\cdot p_2/m_1m_2, \qquad \text{(1a)}$$

$$\omega = \ln(m_1/m_2). \qquad \text{(1b)}$$

We shall express all results in terms of these angles. For example, Michel's formula for the muon decay with no radiative corrections is[4]

$$\begin{aligned}\mathcal{P}d^3p = [m_1^2m_2^2/2(2\pi)^4\hbar^7E_1E_2c^3][&K_1(\cosh\theta)\\ &\times(\cosh\omega-\cosh\theta)+\tfrac{2}{3}K_2\sinh^2\theta\\ &+K_3(\cosh\omega-\cosh\theta)]d^3p, \qquad (2)\end{aligned}$$

$$\begin{aligned}K_1&=g_0^2+2(g_1^2+g_2^2+g_3^2)+g_4^2,\\ K_2&=g_1^2+2g_2^2+g_3^2, \qquad (3)\\ K_3&=g_0^2-2g_1^2+2g_3^2-g_4^2.\end{aligned}$$

The interaction Lagrangian suitable for studying radiative corrections may be written as follows[5]:

$$L^{\text{int}} = g\sum_{\sigma,\rho} a_\sigma\varphi_\rho{}^\sigma(\bar\psi_1\Gamma_\rho{}^\sigma\psi_2)+e\sum_{k=1}^{2}\sum_\mu A_\mu(\bar\psi_k\gamma_\mu\psi_k). \qquad (4)$$

Here the electromagnetic interaction is written in its usual form. The first term is some arbitrary linear combination of invariants mixed in the proportions $g_\sigma=ga_\sigma$, where $\sigma=0\cdots 4$, and constructed from the tensor fields $\varphi_\rho{}^\sigma$ which are responsible for the decay [for an example, see Eq. (14)]. The index ρ runs over all components of each tensor. Equation (4) will be abbreviated

$$L^{\text{int}} = g\bar\psi_1\mathbf{\Phi}\psi_2+e\sum_{k=1}^{2}\bar\psi_k\mathbf{A}\psi_k, \qquad (5)$$

where

$$\mathbf{\Phi}=\textstyle\sum_{\sigma\rho} a_\sigma\varphi_\rho{}^\sigma\Gamma_\rho{}^\sigma, \qquad \text{(5a)}$$

$$\mathbf{A}=\textstyle\sum_\mu A_\mu\gamma_\mu. \qquad \text{(5b)}$$

RADIATIVE CORRECTIONS (VIRTUAL PHOTONS)

Denote the three diagrams of order e^2 by a, b, and c; and let a be the vertex modification. The correction to

[3] These new variables (P_μ) may be expressed in terms of the usual four components (p_i) as follows: $\rho P_i=p_i$, $\rho P_5=(m^2-\sum p_i^2)/2m$, $\rho P_6=(m^2+\sum p_i^2)/2im$.

In the new variables, the change in length of a four vector appears as a rotation in the 5,6-plane; i.e., if $p_i'=\lambda p_i$, then $P_5'=P_5\cosh\omega-iP_6\sinh\omega$, $P_6'=-P_5\sinh\omega+iP_6\cosh\omega$, where $\omega=\ln\lambda$.

[4] L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1949).

[5] We define the $\Gamma_\rho{}^\sigma$ as follows: $\Gamma^0=1$; $\Gamma_\rho{}^1=\gamma_\rho$; $\sqrt{2}\Gamma_{\mu\nu}{}^2=\frac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$; $\Gamma_\rho{}^3=-i\gamma_\rho\gamma_5$; $\Gamma^4=-i\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. We use the γ's as defined in reference 6.

the matrix element associated with this diagram is

$$\Delta_a M(\mathbf{\Phi})=(ge^2/4\pi^3 i)\int \gamma_\mu(\mathbf{p}_2-\mathbf{k}-m_2)^{-1}$$
$$\times\mathbf{\Phi}(\mathbf{p}_1-\mathbf{k}-m_1)^{-1}\gamma_\mu k^{-2}C(k^2)d^4k, \quad (6)$$

where[6]

$$C(k^2)=-\lambda^2k^2(k^2-\lambda^2)^{-1}(k^2-\lambda_{\min}{}^2)^{-1},$$
$$M(\mathbf{\Phi})=\sum a_\sigma\varphi_\rho{}^\sigma M(\Gamma_\rho{}^\sigma).$$

By a standard reduction, one obtains

$$\Delta_a M(\mathbf{\Phi})=(-ge^2/8\pi)\{\gamma_\mu(\mathbf{p}_2+m_2)\mathbf{\Phi}(\mathbf{p}_1+m_1)\gamma_\mu J_1 - [\gamma_\mu\gamma_\sigma\mathbf{\Phi}(\mathbf{p}_1+m_1)\gamma_\mu+\gamma_\mu(\mathbf{p}_2+m_2)\mathbf{\Phi}\gamma_\sigma\gamma_\mu]J_{2\sigma} + \gamma_\mu\gamma_\sigma\mathbf{\Phi}\gamma_\tau\gamma_\mu J_{3\sigma\tau}\}, \quad (7)$$

where

$$J_1=\int_0^1 (dy/p_y{}^2)\ln(p_y{}^2/\lambda_{\min}{}^2), \quad (7a)$$

$$J_{2\sigma}=2\int_0^1 (dy/p_y{}^2)p_{y\sigma}, \quad (7b)$$

$$J_{3\sigma\tau}=\int_0^1 dy[p_{y\sigma}p_{y\tau}/p_y{}^2+\tfrac{1}{4}\delta_{\sigma\tau}-\tfrac{1}{2}\delta_{\sigma\tau}\ln(\lambda^2/p_y{}^2)], \quad (7c)$$

$$p_y=yp_1+(1-y)p_2.$$

A method of evaluating these integrals is indicated in the Appendix. The results are:

$$J_1=[F_1+\theta(\omega-2\omega_<)]/m_1m_2\sinh\theta, \quad (8a)$$

$$J_{2\sigma}=[(F_2-F_3)p_{2\sigma}+F_3p_{1\sigma}]/m_1m_2, \quad (8b)$$

$$J_{3\sigma\tau}=(q_\sigma q_\tau/q^2)F_4-[(p_{2\sigma}q_\tau+q_\sigma p_{2\tau})/2m_1m_2]F_3 + (p_{2\sigma}p_{2\tau}/2m_1m_2)F_2+F_5\delta_{\sigma\tau}, \quad (8c)$$

where q denotes the momentum transfer in the decay:

$$q=p_2-p_1. \quad (8d)$$

The angles θ and ω are defined in Eq. (1) and $\omega_<$ is the minimum cut-off angle:

$$\omega_<=\ln(\lambda_{\min}/m_2). \quad (8e)$$

The functions F_k are defined as follows:

$$F_1(\theta,\omega)=L\left(\frac{2\sinh\theta}{e^\omega-e^{-\theta}}\right)-L\left(\frac{2\sinh\theta}{e^\theta-e^{-\omega}}\right) + (\omega-\theta)\ln\left(\frac{\sinh\frac{1}{2}(\omega-\theta)}{\sinh\frac{1}{2}(\omega+\theta)}\right), \quad (9a)$$

$$F_2(\theta)=2\theta/\sinh\theta, \quad (9b)$$

$$F_3(\theta,\omega)=\frac{1}{\sinh\theta}\left(\theta+\frac{\omega\sinh\theta-\theta\sinh\omega}{\cosh\omega-\cosh\theta}\right), \quad (9c)$$

$$F_4(\theta,\omega)=1+\omega+\frac{\theta\sinh\theta-\omega\sinh\omega}{\cosh\omega-\cosh\theta}+\frac{\theta e^{-\omega}}{\sinh\theta}, \quad (9d)$$

$$F_5(\theta,\omega)=\frac{\omega\sinh\omega-\theta\sinh\theta}{2(\cosh\omega-\cosh\theta)}+\tfrac{1}{2}(\omega-2\omega_>)-\tfrac{3}{4}. \quad (9e)$$

In F_5 appears the upper cut-off angle $\omega_>$:

$$\omega_>=\ln(\lambda/m_2). \quad (9f)$$

Here $I(x)$ is one of the Spence functions and is defined by[7]

$$L(x)=\int_0^x \ln(1-t)(dt/t). \quad (9g)$$

The matrix element (7) simplifies slightly when taken between the initial and final plane wave states. Then

$$\Delta_a M(\mathbf{\Phi})=(-ge^2/2\pi)\{(p_1\cdot p_2)J_1\mathbf{\Phi} - \tfrac{1}{2}[\mathbf{p}_1\mathbf{J}_2\mathbf{\Phi}+\mathbf{\Phi}\mathbf{J}_2\mathbf{p}_2]+\tfrac{1}{4}\gamma_\mu\gamma_\sigma\mathbf{\Phi}\gamma_\tau\gamma_\mu J_{3\sigma\tau}\}. \quad (10)$$

After mass and wave function renormalization the contribution of the other two diagrams (the self-energy diagrams b and c) is

$$-(ge^2/2\pi)(R_b+R_c)\mathbf{\Phi}, \quad (11)$$

where

$$R_b=\tfrac{1}{2}\ln(\lambda/m_2)-\ln(m_2/\lambda_{\min})+9/8=\tfrac{1}{2}\omega_>+\omega_<+9/8. \quad (11a)$$

To get R_c, substitute m_1 for m_2. Then

$$R=R_b+R_c=\omega_>+2\omega_<-\tfrac{3}{2}\omega+9/4. \quad (11b)$$

To order e^2, the total radiative correction to the matrix element arising from the virtual photons is therefore

$$\Delta M(\mathbf{\Phi})=(-ge^2/2\pi)\{[(p_1\cdot p_2)J_1+R]\mathbf{\Phi} - \tfrac{1}{2}[\mathbf{p}_1\mathbf{J}_2\mathbf{\Phi}+\mathbf{\Phi}\mathbf{J}_2\mathbf{p}_2]+\tfrac{1}{4}\gamma_\mu\gamma_\sigma\mathbf{\Phi}\gamma_\tau\gamma_\mu J_{3\sigma\tau}\}. \quad (12)$$

After elimination of the Dirac matrices, as far as this

[6] R. P. Feynman, Phys. Rev. **76**, 769 (1949). We follow the notation of that paper, except for the definition of θ [Eq. (1a)] and for $d^4K=dK_1dK_2dK_3dK_4$. In particular, the scalar product of two four-vectors a and b is defined by $(a\cdot b)=a_0b_0-a_1b_1-a_2b_2-a_3b_3$. $C(k^2)$ is a convergence factor depending on the upper limit λ and the infrared limit $\lambda_{\min}$.

[7] The following relations were found useful in our calculations:

$$L(x)=-\sum_{n=1}^{\infty} x^n/n^2, \quad x\leq 1;$$

$$L(x)=-\tfrac{1}{3}\pi^2-L(1/x)+\tfrac{1}{2}(\ln x)^2+\ln(-1)(\ln x), \quad x>1;$$

$$L(x)=\ln(1-x)\ln(x)-\tfrac{1}{6}\pi^2-L(1-x), \quad 0\leq x\leq 1.$$

A detailed study and table of this and related functions is given by K. Mitchell, Phil. Mag. **40**, 351 (1949).

is possible, Eq. (12) may be written as follows[5,8]:

$$\Delta M(\Gamma^{(0,4)})=(ge^2/2\pi)\{S+T\mp\theta/\sinh\theta+r_0\}\Gamma^{(0,4)}, \quad (13a)$$

$$\Delta M(\Gamma_\mu{}^{(1,3)})=(ge^2/2\pi)\{(S-\tfrac{1}{2}T\pm\theta/\sinh\theta+r_1)\Gamma_\mu{}^{(1,3)}+A_\mu\Gamma^{(0,4)}\}, \quad (13b)$$

$$\Delta M(\Gamma_{\rho\lambda}{}^{(2)})=(ge^2/2\pi)\{(S-T-\theta/\sinh\theta+r_2)\Gamma_{\rho\lambda}{}^{(2)}+\delta_{\rho\lambda}{}^{\nu\mu}B_\nu\Gamma_\mu{}^{(1)}\}, \quad (13c)$$

where

$$S=(\theta-F_1)\coth\theta+(1-\theta\coth\theta)(\omega-2\omega_<), \quad (13d)$$

$$T=(\theta\sinh\theta-\omega\sinh\omega)/(\cosh\omega-\cosh\theta), \quad (13e)$$

$$A_\mu=[(F_3-F_2)/2m_1]p_{1\mu}\mp(F_3/2m_2)p_{2\mu}+[(m_2\mp m_1)(F_4/q^2)+(F_2-2F_3)/2m_1]q_\mu, \quad (13f)$$

$$\sqrt{2}B_\mu=i[(F_2-F_3)/m_1]p_{1\mu}-i(F_3/m_2)p_{2\mu}, \quad (13g)$$

$$r_0=3\omega_>-\tfrac{3}{2}\omega-\tfrac{1}{4}, \quad (13h)$$

$$r_1=-2, \quad (13i)$$

$$r_2=-\omega_>+\tfrac{1}{2}\omega-9/4. \quad (13j)$$

$\delta_{\rho\lambda}{}^{\nu\mu}$ is the completely antisymmetric tensor of absolute value unity. The result (13) is the lowest order radiative correction to a transition associated with an an arbitrary tensor interaction and connecting two spinor states of different rest mass. It is applicable to both reactions (a) and (b). It is seen that the transition amplitude for the vector interaction (13b) reduces in the limiting case where $m_1=m_2$ (or $\omega=0$) to the usual electromagnetic result.[6,9]

The infrared divergence appears in exactly the same form in all five interactions. This contribution will be cancelled by a similar term arising from inner bremsstrahlung, i.e., from the real photons.

On the other hand the cutoff for short wavelengths is not cancelled except in the vector and pseudovector cases. In addition, if there is no transition, i.e., if $\omega=\theta=0$, then ΔM does not vanish except in the vector case.

A field theory containing a beta interaction is not renormalizable, i.e., it is not known how sensible results may be extracted from the infinite terms arising in such a theory. In the present calculations, which are made to the lowest order, these divergent terms are contained in r_0 and r_2. Fortunately, however, they depend only logarithmically on the upper cutoff λ, and if λ is chosen to correspond to the Compton wavelength of the proton, the λ-dependent terms amount to only about 20 percent of the total calculated correction. We shall interpret our results by assuming that the calculation is meaningful except for these uncertainties in r.

TRANSITION PROBABILITIES

In the two-neutrino decay:

$$\varphi_\rho{}^\sigma=\bar{\varphi}_2\Gamma_\rho{}^\sigma\varphi_1, \quad (14)$$

where the tensor $\varphi_\rho{}^\sigma$ is formed from the wave functions φ_1 and φ_2 of the two neutrinos.

The exact expression for the transition probability may be written

$$\mathcal{P}d^3p=(d^3p/2(2\pi)^5E_1E_2)\sum_{\sigma\sigma'\rho\rho'}a_\sigma a_{\sigma'}(I_{\rho\rho'}{}^{\sigma\sigma'}/4)\times\mathrm{Tr}\{(\mathbf{p}_2+m_2)M_\rho{}^\sigma(\mathbf{p}_1+m_1)\bar{M}_{\rho'}{}^{\sigma'}\}, \quad (15)$$

where E_1 and E_2 are the energies of the initial and final charged particles and p is the momentum of the latter.[10] $M_\rho{}^\sigma(=M^{(0)}{}_\rho{}^\sigma+\Delta M_\rho{}^\sigma)$ is the complete matrix element where $\Delta M_\rho{}^\sigma$ are the virtual photon corrections given in (13). Finally,

$$I_{\rho\rho'}{}^{\sigma\sigma'}=\tfrac{1}{4}\,\mathrm{Tr}[\gamma_\mu\Gamma_\rho{}^\sigma\gamma_\nu\bar{\Gamma}_{\rho'}{}^{\sigma'}]\times\int\int d^3k\,d^3k'\,\delta(p_1-p_2-k-k')k_\mu k_\nu'/KK', \quad (15a)$$

where the k and K represent the momentum and energy of the two neutrinos. After the sum and integration over neutrino spin and momenta indicated in Eq. (15a) have been performed, then Eq. (15) gives the spectrum of the decay product. The coefficients defined in (15a) have the following simple properties, which we take from Lenard[11]:

$$I^{\sigma\sigma'}=0, \quad \sigma\neq\sigma', \quad (15b)$$

$$I^{00}=I^{44}=\pi q^2, \quad (15c)$$

$$I_{\nu\lambda}{}^{11}=I_{\nu\lambda}{}^{33}=(2\pi/3)[q_\nu q_\lambda-q^2\delta_{\nu\lambda}], \quad (15d)$$

$$I_{\lambda\rho;\lambda'\rho'}{}^{22}=(\pi/12)(1-P_{\lambda\rho})(1-P_{\lambda'\rho'})\times(q^2\delta_{\rho\rho'}-4q_\rho q_{\rho'})\delta_{\lambda\lambda'}, \quad (15e)$$

where $P_{\lambda\rho}$ is a transposition operator. In virtue of (15b), the double sum reduces to a single one

$$\mathcal{P}=[1/2(2\pi)^5E_1E_2]\sum_\sigma a_\sigma{}^2\sum_{\rho\rho'}(I_{\rho\rho'}{}^{\sigma\sigma}/4)\times\mathrm{Tr}[(\mathbf{p}_2+m_2)M^{(0)}{}_\rho{}^\sigma(\mathbf{p}_1+m_1)(\bar{M}^{(0)}{}_{\rho'}{}^\sigma+2\Delta\bar{M}_{\rho'}{}^\sigma)]$$

---

[8] This notation means the following: for the scalar case ($\sigma=0$), we pick up the upper signs and the left index in $\Gamma^{(0,4)}$; for the pseudoscalar case ($\sigma=4$) we pick up the lower signs and the right index in $\Gamma^{(0,4)}$. An identical convention is understood for the vector and pseudovector cases: always the upper and lower signs correspond to the left and right indices in $\Gamma^{(i,j)}$, respectively. Equations (13b) and (13c) for the matrix elements are dependent on the representation of Γ_ρ. Of course, this does not affect the transition probabilities.

[9] It is easily seen that the term proportional to q_μ in Eq. (13f), vector case, vanishes identically in the case of electrodynamics ($\omega=0$). On the other hand, it turns out that, for arbitrary ω, this term does not contribute to the transition probability for the two neutrino decay, discussed in the next section.

[10] Hereafter $\mathbf{p}$ and p refer to the three-vector momentum of particle 2 while p_2, as before, represents its four-vector momentum.

[11] A. Lenard, Phys. Rev. **90**, 968 (1953). Equation (15b) is an immediate consequence of the simple relation

$$\mathrm{Tr}[\gamma_\mu\Gamma_\sigma\gamma_\nu\Gamma_{\sigma'}]=-\mathrm{Tr}[\gamma_\nu\Gamma_\sigma\gamma_\mu\Gamma_{\sigma'}], \quad \sigma\neq\sigma'.$$

to order e^2. With the aid of (15), (15c), (15d), and (15e), the calculation may now be completed. In the tensor case, there is a simplification because $M^{(0)}{}_{\lambda\rho}$ and $\Delta M_{\lambda'\rho'}$ are antisymmetric in $\lambda\rho$ and $\lambda'\rho'$, respectively. Hence, I^{22} may be replaced by

$$I^{22}=(\pi/3)\delta_{\lambda\lambda'}(q^2\delta_{\rho\rho'}-4q_\rho q_{\rho'}). \tag{15f}$$

The final result may be expressed in the form

$$\mathcal{P}=\sum_{\sigma=0}^{4}\left(1+\frac{e^2}{2\pi}\alpha_\sigma\right). \tag{16}$$

Here the $\mathcal{P}_\sigma^{(0)}$ are the unperturbed transition probabilities and the α_σ are

$$\tfrac{1}{2}\alpha_{0,4}=S\mp\frac{\theta}{\sinh\theta}+T+r_0, \tag{16a}$$

$$\tfrac{1}{2}\alpha_{1,3}=S\pm\frac{\theta}{\sinh\theta}+\frac{\frac{1}{2}\omega\sinh\omega-\frac{1}{3}\theta\sinh\theta}{\cosh\omega-\cosh\theta+\frac{1}{3}(\cosh\theta\pm1)}+r_1, \tag{16b}$$

$$\tfrac{1}{2}\alpha_2=S+\frac{(\omega\sinh\omega-\frac{1}{3}\theta\sinh\theta)\cosh\theta-(\cosh\omega-\cosh\theta)(\theta/\sinh\theta)}{(\cosh\theta)(\cosh\omega-\cosh\theta)+\frac{2}{3}\sinh^2\theta}, \tag{16c}$$

where S, T, and r_i are given in Eq. (13d), ff.

INNER BREMSSTRAHLUNG

The process of inner bremsstrahlung also contributes corrections of order $e^2/2\pi$ to the spectrum of particle 2.

The differential transition probability for this process has been worked out by Lenard.[11] However, Lenard used the approximation of neglecting m_2 in comparison with the momenta and energies involved. Following the pattern of that calculation, we obtain the following differential transition probability for the case of arbitrary masses m_1 and m_2:

$$\Delta\mathcal{P}_\gamma d^3p d^3\kappa=[d^3pd^3\kappa e^2/8(2\pi)^6E_1E_2\epsilon]\textstyle\sum_\sigma g_\sigma^2N_\sigma, \tag{17}$$

$$N_{0;4}=[(m_1\pm m_2)^2-G^2]G^2\Omega+2(\kappa\cdot G)^2G^2/(p_1\cdot\kappa)(p_2\cdot\kappa), \tag{17a}$$

$$\tfrac{3}{4}N_{1;3}=\{G^2[\tfrac{1}{2}(m_1\mp m_2)^2\mp2m_1m_2-G^2]+\tfrac{1}{2}(m_1^2-m_2^2)\}\Omega+4G^2+(\kappa\cdot G)^2\times[2G^2+(m_1\mp m_2)^2]/(p_1\cdot\kappa)(p_2\cdot\kappa), \tag{17b}$$

$$\begin{aligned}\tfrac{3}{4}N_2=&[(m_1^2-m_2^2)^2-4(p_1\cdot\kappa)(p_2\cdot\kappa)-\tfrac{1}{2}G^2\\&-\tfrac{1}{2}(m_1^2+m_2^2)G^2]\Omega-(\kappa\cdot G)^2\\&\times[G^2+4(p_1\cdot p_2)]/(p_1\cdot\kappa)(p_2\cdot\kappa)-4G^2\\&+4[(p_1\cdot\kappa)(p_2\cdot\kappa)^{-1}-(p_2\cdot\kappa)(p_1\cdot\kappa)^{-1}]\\&\times[p_1\cdot G+p_2\cdot G].\end{aligned} \tag{17c}$$

Here κ and ϵ are the momentum and energy of the real photon; G and Ω are defined by

$$G=p_1-p_2-\kappa, \tag{17d}$$

$$\Omega=\sum_{i=1,2}[(p_2\cdot e_i)/(p_2\cdot\kappa)-(p_1\cdot e_i)/(p_1\cdot\kappa)]^2, \tag{17e}$$

where e_i are the polarization vectors of the photon. Setting $m_2=0$ in Eq. (17), we obtain Lenard's results. In that case, the pseudoscalar case coincides with the scalar and the pseudovector with the vector.

We are interested in the spectrum of particle 2 rather than in the differential transition probability. Thus, the next step is to integrate over the photon momenta. This leads to an invariant integral, which is most conveniently calculated in the rest system of particle 1. As a rule, we integrate first over the photon energy and, afterwards, over the angle. As a consequence of the conservation laws, the maximum energy of the photon depends on its direction. Explicitly, in the rest system of particle 1:

$$\omega_0=q^2/2(m_1-E_2+p\cos\delta), \tag{18}$$

where δ is the angle between the photon and particle 2. When rewritten in covariant notation, these integrations yield

$$\mathcal{P}_\gamma d^3p=d^3p(e^2/2\pi)\textstyle\sum_\sigma b_\sigma\mathcal{P}^{(0)}{}_\sigma, \tag{19}$$

$$b_{0;4}=2D+(\cosh\theta\pm1)^{-1}(\cosh\omega-\cosh\theta)W$$

$$b_{1;3}=2D+[Q+(\cosh\omega-\cosh\theta)^2Y]\times[3(\cosh\omega-\cosh\theta)(\cosh\theta\mp1)+\sinh^2\theta]^{-1},$$

$$b_2=2D+[2Q+2(\cosh\omega-\cosh\theta)^2Z]\times[3(\cosh\omega-\cosh\theta)\cosh\theta+2\sinh^2\theta]^{-1},$$

where

$$\begin{aligned}D=&2(\theta\coth\theta-1)(\omega-\omega_<-1-\ln2)+[L(e^{-\theta-\omega})\\&-L(e^{\theta-\omega})](\coth\theta)-1-\theta e^{-\omega}/\sinh\theta\\&+[2\theta\coth\theta+(\sinh\omega)/(\sinh\theta)-1]\ln(1-e^{-\theta-\omega})\\&+[2\theta\coth\theta-(\sinh\omega)/(\sinh\theta)-1]\ln(1-e^{\theta-\omega}),\end{aligned}$$

$$Q=4(\theta\coth\theta-1)\sinh^2\omega,$$

$$W=(5/3)\theta\coth\theta+\theta(\cosh\omega)/3\sinh\theta-2,$$

$$Y=(10/3)(\theta\coth\theta-1)+[(5/3)\cosh\omega\mp1](\theta/\sinh\theta),$$

$$Z=-\frac{1}{6}\frac{\theta\cosh\omega}{\sinh\theta}+\frac{5}{6}\theta\coth\theta-\frac{5}{3}-\frac{\sinh\omega}{\sinh\theta}\ln\frac{\sinh\frac{1}{2}(\omega-\theta)}{\sinh\frac{1}{2}(\omega+\theta)}.$$

It is important to observe that in the expression for D there is a factor [namely, $\ln(1-e^{\theta-\omega})$] that is logarithmically divergent at the end of the spectrum ($\theta=\omega$). Now, let us suppose we ask for the probability of finding particle 2 between $p_2-\Delta p_2$ and p_2, where Δp_2 represents the experimental interval (more explicitly, Δp_2 is a four vector of components ΔE_2 and $\Delta\mathbf{p}_2$,[10] the last two quantities representing the energy and momentum intervals, respectively). In order to answer that question, we imagine that we integrate Eq. (19) with respect to d^3p between $\mathbf{p}-\Delta\mathbf{p}$ and $\mathbf{p}$. As $\Delta\mathbf{p}$ is assumed to be very small in comparison with the energies and momenta involved, all the well behaved functions of Eq. (19) may be regarded as constants and taken out from the integral sign. The divergent function $\ln(1-e^{\theta-\omega})$, however, requires a more careful consideration. To an excellent approximation, an elementary calculation yields

$$\frac{1}{\Delta^3 p}\int_{\mathbf{p}-\Delta\mathbf{p}}^{\mathbf{p}} \ln(1-e^{\theta-\omega})d^3p$$
$$=\ln(1-e^{\theta-\omega}+e^{\theta-1}(p_1\cdot\Delta p_2)/m_1^2\sinh\theta). \quad (20)$$

The final result of the bremsstrahlung calculation is then given by Eq. (19), ff., where in the expression for D, we must replace the factor $\ln(1-e^{\theta-\omega})$ by the right-hand side of Eq. (20). Of course, we imagine that in Eq. (19), d^3p is replaced by Δ^3p, the experimental momentum interval.

The divergence at $\theta=\omega$ is now removed. Furthermore, Eq. (20) shows that in the vicinity of the end point, the shape of the spectrum depends logarithmically on the experimental energy interval.

TOTAL CORRECTED TRANSITION PROBABILITIES

Combining Eqs. (16) and (19) and taking into account Eq. (2), our final result may be written as follows:

$$\mathcal{P}d^3p=(m_1^2m_2^2d^3p/2(2\pi)^4\hbar^7E_1E_2c^3)$$
$$\times[\bar{K}_1(\cosh\theta)(\cosh\omega-\cosh\theta)+\tfrac{2}{3}\bar{K}_2\sinh^2\theta$$
$$+\bar{K}_3(\cosh\omega-\cosh\theta)], \quad (21)$$

where

$$\bar{K}_1=\bar{g}_0^2+2(\bar{g}_1^2+\bar{g}_2^2+\bar{g}_3^2)+\bar{g}_4^2, \quad (21a)$$

$$\bar{K}_2=\bar{g}_1^2+2\bar{g}_2^2+\bar{g}_3^2, \quad (21b)$$

$$\bar{K}_3=\bar{g}_0^2-2\bar{g}_1^2+2\bar{g}_3^2-\bar{g}_4^2, \quad (21c)$$

$$\bar{g}_\sigma^2=g_\sigma^2[1+(e^2/2\pi)(\alpha_\sigma+b_\sigma)]. \quad (21d)$$

Thus, the influence of the virtual photon corrections and inner bremsstrahlung may be regarded as a perturbation of the interaction constants g_σ. Of course, this perturbation depends on the momenta p_1 and p_2 through the angle θ.

LIMIT CASE OF SMALL MASS

When m_2 is negligible in comparison with the energy E_2, our final expressions for the radiative corrections can be greatly simplified. Of course, this approximation is applicable to the study of the muon decay, provided that we do not consider the low-energy part of the spectrum. In this section, we shall retain terms of order m_2/E_0 or $e^2/2\pi$ and neglect terms of higher order (e.g., $e^2m_2/2\pi E_0$ or $(m_2/E_0)^2$) whenever this approximation does not introduce divergences in the high-energy part of the spectrum; moreover, we shall express our results in the rest frame of particle 1. In this approximation the scalar and pseudoscalar, as well as the vector and pseudovector, radiative corrections coincide.

Let us define

$$E_0=\tfrac{1}{2}m_1, \quad \eta=E_2/E_0. \quad (22)$$

Thus, E_0 is the maximum energy attainable by particle 2. Then, our results may be expressed as follows:

$$\mathcal{P}(\eta)d\eta=A[3\bar{K}_1\eta(1-\eta)+2\bar{K}_2\eta^2$$
$$+3\bar{K}_3(m_2/E_0)(1-\eta)]\eta d\eta, \quad (23)$$
$$A=m_1E_0^4/3(2\pi)^3\hbar^7c^6,$$

where the K's are defined in Eqs. (21a), (21b), (21c), and (21d). The α's now reduce to

$$\alpha_0=\alpha_4=2U+2(1-\eta)^{-1}\ln\eta+6(\omega_>-\omega)-\tfrac{1}{2}, \quad (24a)$$

$$\alpha_1=\alpha_3=2U+6(3-2\eta)^{-1}(1-\eta)\ln\eta-4, \quad (24b)$$

$$\alpha_2=2U+2(3-2\eta)(3-\eta)^{-1}\ln\eta-2(\omega_>-\omega)-(9/2), \quad (24c)$$

where

$$U=\sum_{m=1}^{\infty}\eta^m/m^2-\tfrac{1}{6}\pi^2+(\ln\eta)[\ln(\eta^{-1}-1)-2\omega]$$
$$+(5/2)\omega-\omega^2+2\omega_<[\ln\eta+\omega-1], \quad (24d)$$

$$b_0=b_4=2V+\tfrac{1}{3}\eta^{-1}[\eta^{-1}+4-17\eta]$$
$$\times[\ln\eta+\omega]+6-2\eta^{-1}, \quad (25a)$$

$$b_1=b_3=2V+\tfrac{1}{3}[(1-\eta)\eta^{-1}(3-2\eta)^{-1}]$$
$$\times\{[5\eta^{-1}+17-34\eta][\ln\eta+\omega]+34\eta-22\}, \quad (25b)$$

$$b_2=2V+\tfrac{1}{3}[(1-\eta)\eta^{-1}](3-\eta)^{-1}]\{[7\eta^{-1}+22-17\eta]$$
$$\times[\ln\eta+\omega]+22\eta-34-6(\eta^{-1}-1)\ln(1-\eta)\}, \quad (25c)$$

where

$$V=\sum_{m=1}^{\infty}\eta^m/m^2-1+2(\omega-\omega_<-\ln 2)(\ln\eta+\omega-1)$$
$$+(2\ln\eta+2\omega-1-\eta^{-1})\ln[1-\eta+(2/m_1e)\Delta E_2]. \quad (25d)$$

We observe again that the infrared divergences arising from the virtual photons are compensated by similar terms originating in the inner bremsstrahlung calculation. The divergent terms associated with high photon energies in the radiative corrections again cancel for the vector case, but in the other cases we find uncompensated logarithmically divergent terms [see discussion following Eq. (13)]. Moreover, in the vicinity of the end point of the spectrum ($\eta=1$), these results depend logarithmically on the experimental energy interval ΔE_2 (the last term in the expression for V).

In Table I and Fig. 1, the functions $(e^2/2\pi)(\alpha_i+b_i)$ are given for the three cases (S, T, and V) for an experimental energy interval of $\Delta E=2m_2$. The "virtual" coefficients α_i are nearly independent of energy, while the b_i are positive and decrease near the end point;

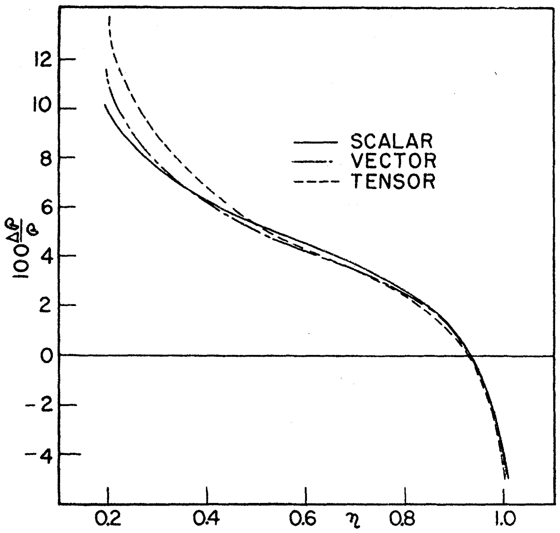

FIG. 1. Percent change ($100\,\Delta\mathcal{P}/\mathcal{P}$) in probability of decay for the various tensor cases as a function of the energy (η).

therefore the total correction α_i+b_i, also decreases there. This decrease in the b_i may be understood as follows. Over the whole spectrum most of the inner bremsstrahlung is emitted only in a narrow cone along the motion of the electron [angular opening $\sim(2m_2/E_2)^{\frac{1}{2}}$]; however, near the end point the conservation laws increasingly restrict the maximum-energy photons allowed in this cone.[12]

We also notice that the energy dependence of $(e^2/2\pi)(\alpha_i+b_i)$ is very similar for the three cases (scalar, vector, and tensor). This fact may be used to derive a simple expression which may be useful for reducing experimental results. Suppose we approximate the three curves $(e^2/2\pi)(\alpha_i+b_i)$ by a function $h(\eta,\Delta E)$.[13] Then taking into account Eqs. (3) and (21), we can write

$$\bar{K}_i=K_i[1+h(\eta)]. \qquad (27)$$

With this approximation, Eq. (23) reads

$$\mathcal{P}(\eta)d\eta=A\,(1+h(\eta;\Delta E))\eta d\eta[3K_1\eta(1-\eta) +2K_2\eta^2+(3m_2/E_0)K_3(1-\eta)]. \qquad (27a)$$

This expression has the advantage that the whole effect of the radiative corrections is concentrated in the function $h(\eta;\Delta E)$. Equation (27a) is a three parameter expression. Following Michel,[14] we can eliminate one of the parameters by integrating Eq. (27a) from $\eta=0$ to $\eta=1$ and equating the result to τ^{-1}, where τ is the mean lifetime of the muon. We obtain

$$4/\tau A=K_1(1+\Lambda_1)+2K_2(1+\Lambda_2)+(2m_2/E_0)K_3, \qquad (27b)$$

where

$$\Lambda_{1;2}(\Delta E)=4\int_0^1 h(\eta;\Delta E)[3\eta^2(1-\eta);\eta^3]d\eta. \qquad (27c)$$

Here the subscripts 1 and 2 in $\Lambda_{1;2}$ correspond to the first and second terms in [;], respectively. In Eq. (27b), we have already neglected a term of order $e^2m_2K_3/2\pi E_0$.

Solving for K_1 in Eq. (27b) and substituting in Eq. (27a), we obtain finally

$$\tau\mathcal{P}(\eta)d\eta=4\eta d\eta[1+h(\eta)-\Lambda_1][3\eta(1-\eta) +2\rho\eta\{(4/3)\eta-1+\tfrac{1}{3}\Lambda_1\eta-\Lambda_2(1-\eta)\} +(m_2/E_0)\rho'(1-\eta)(1-2\eta)]. \qquad (28)$$

Here ρ and ρ' are two parameters related to the K_i by

$$\rho=\tfrac{3}{4}K_2\tau A=3K_2[K_1(1+\Lambda_1) +2K_2(1+\Lambda_2)+2(m_2/E_0)K_3]^{-1}, \qquad (28a)$$

$$\rho'=\tfrac{3}{4}K_3\tau A. \qquad (28b)$$

We notice that for $\rho'\leq 1$, the last term in Eq. (28) is extremely small in comparison with the main terms except for the region of small energies (see footnote 13). Neglecting this term, we get

$$\tau\mathcal{P}(\eta)d\eta=4\eta^2d\eta[1+h(\eta)-\Lambda_1][3(1-\eta) +2\rho\{(4/3)\eta-1+\tfrac{1}{3}\Lambda_1\eta-\Lambda_2(1-\eta)\}]. \qquad (28c)$$

This one-parameter expression, together with Eq. (28a), represents to a good approximation the influence of radiative corrections on the muon spectrum. If we neglect the radiative corrections, i.e. if we set $h(\eta)=\Lambda_1=\Lambda_2=0$, Eq. (28c) reduces to Michel's formula.

The function $h(\eta)$ as well as Λ_1 and Λ_2 are slowly varying functions of ΔE, the energy interval used in the experiment.[15] In general, this dependence on ΔE will only affect the shape of the spectrum near the end point. As Λ_1 and Λ_2 involve integrals over the whole range of η, we can expect their dependence on ΔE to be still less than that of $h(\eta)$. (Obviously, this is especially true for Λ_1 because in that integral the region near the end point gives a very small contribution.)

We observe that the inclusion of radiative corrections modifies the interpretation of Michel's equations in two

[12] The dominant contribution to the integral of Eq. (17) may be approximated as follows

$$[\ln(q^2/2m_1\lambda_m)]\int_{1-\beta}^{1+\beta}d\rho(\beta^2-1+2\rho-\rho^2)\rho^{-2},$$

where $\rho=1-\beta\cos\varphi$. The integrand is peaked in the forward cone. As in the approximation leading to Eq. (20), q^2 may according to the conservation laws be replaced by $2m_1m_2(\cosh\omega-\cosh\theta+p_1\Delta p_2/m_1m_2e)$; therefore the logarithmic factor decreases near the end of the spectrum ($\theta\to\omega$).

[13] It is true that the three cases differ significantly in the low-energy part of the spectrum ($\eta<0.1$). However, on the one hand the intensity is very weak there and, therefore, this error will not affect appreciably the Λ_i. On the other hand, the parameter ρ is essentially determined in the region $\eta>0.2$. In our numerical calculations we have defined the function $h(\eta)$ by simply taking the average of the three $(\alpha_i+b_i)(e^2/2\pi)$ at each point but, of course, the results are not sensitive to this particular choice.

[14] L. Michel, Phys. Rev. **86**, 814 (1952).

[15] As the three functions $(\alpha_i+b_i)(e^2/2\pi)$ depend on ΔE through the same term [twice the last term of Eq. (25d)] if we want to pass from a certain ΔE to a new ΔE^* we can simply set

$$h(\eta,\Delta E^*)=h(\eta,\Delta E)+2(2\ln\eta+2\omega-1-\eta^{-1})\ln\frac{(1-\eta)m_1e+2\Delta E^*}{(1-\eta)(m_1e)+2\Delta E}.$$

ways. On the one hand, the theoretical expression for the decay is slightly modified [Eq. (28c)] and, therefore, for a given experimental curve, the ρ obtained by comparison between theory and experiment will be slightly different. On the other hand, the connection between ρ and the K_i is also modified [(28a)] and, therefore, for a given theoretical combination of the g_i (i.e., of the unperturbed interaction strengths) the theoretical ρ will also be slightly different.

In order to illustrate this last point, we have calculated the Λ_n for $\Delta E = 2m_2$ (i.e., approximately 2 percent of the spectrum range) and obtained $\Lambda_1 = 4.6\times 10^{-2}$ and $\Lambda_2 = 1.6\times 10^{-2}$. Inserting these values of Λ_i in Eq. (28a) for an interaction $S+aT+bP$ ($a=\pm 1$; $b=\pm 1$) in the charge retention order, we get $\rho = 0.727$ instead of the value $\rho = 0.750$ obtained without the consideration of radiative corrections. Similarly, for an interaction $S+aA+bP$, we get $\rho = 0.480$ instead of $\rho = \frac{1}{2}$.

With the same values for Λ_i, the form of Eq. (28c) suggests that the ρ obtained by comparison of theory and experiment will be slightly increased.

DISCUSSION

The energy-dependent radiative corrections described in Eq. (23), ff. and in the figures produce a distortion of a few percent in the Tiomno-Michel curves. If this distortion can be experimentally established, it is thereby possible to isolate the "Lamb term" for the muon, since there is no contribution from the vacuum polarization. This experiment on the fine structure of the muon spectrum therefore provides information complementary to that obtained from the mu-mesic atom where the situation is reversed: there only the vacuum polarization is observed and the Lamb term is negligible.

We have considered the muon particularly since this is the case best covered by the theory. Other particles, the neutron and one of the K-particles, for example, undergo beta decay; but at least in the case of the neutron the most important radiative correction is not electromagnetic, but mesonic.

A nonrelativistic second-order perturbation calculation with Chew's model, including the cutoff and coupling constant appropriate to the pion scattering, leads to the result that the Gamow-Teller constant is increased relative to the Fermi constant by about 30 percent.[16] The experimental evidence on nuclear beta decay implies that $g_{\mathrm{G.T.}}/g_{\mathrm{F.}}$ is greater than one, possibly 1.2. These numbers are compatible with the assumption that in the Fermi interaction itself $S=T$ and that the apparent inequality is associated with the meson cloud. Although no convincing quantitative argument can be based on meson theory, the sign and order of magnitude are correct; as with the correction to the magnetic moment, which is very similar, the prediction is qualitatively right.

[16] R. J. Finkelstein and S. A. Moszkowski, Phys. Rev. **95**, 1695 (1954).

TABLE I. Percent change ($100\,\Delta\mathcal{P}/\mathcal{P}$) in probability of decay for the various tensor cases as a function of the energy (η). The function $h(\eta)$ is the average of the three lines of this table.

η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	1
Scalar	19.3	9.8	7.4	6.1	5.2	4.4	3.6	2.5	+0.94	−0.90	−4.7
Vector	26.3	11.3	7.7	6.0	4.9	4.1	3.3	2.3	+0.99	−0.74	−4.4
Tensor	33.9	13.6	8.7	6.6	5.3	4.2	3.3	2.2	+0.66	−1.1	−5.0
$h(\eta)$	26.5	11.6	7.9	6.2	5.1	4.2	3.4	2.3	+0.86	−0.91	−4.7

According to the idea of the universal Fermi interaction, the same coupling, both in form and strength, should be responsible for the decays of the neutron and the muon. According to experiment the strengths are nearly equal. But even if the primary couplings are the same, one would expect the observed rates to be different, because for example only the neutron carries a meson cloud. The net rates will certainly be influenced by radiative corrections which are different in the two cases. We have found that the change in lifetime associated with the electromagnetic corrections to the decay of the muon amounts to about 3 to 5 percent. In order to clarify the situation about the universal interaction, it is evidently necessary to carry through a correspondingly accurate calculation for mesonic corrections to the decay of the neutron.

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APPENDIX

Consider the integral

$$\int_0^1 (dy/p_y^2)\ln(p_y^2/\lambda_m^2).$$

If we make the substitution

$$y = b\coth\alpha + a,\quad a = -(p_2\cdot q)/q^2,\quad b^2 = a^2 - m_2^2/q^2,$$
$$p_y^2 = b^2q^2\,\mathrm{csch}^2\alpha,$$

then this becomes

$$-(1/bq^2)\int_{\coth^{-1}(p_1\cdot q)/q^2}^{\coth^{-1}(p_2\cdot q)/q^2} d\alpha(-2\ln\sinh\alpha + \ln b^2q^2/\lambda_m^2).$$

The first term may be integrated by using the exponential form of $\sinh\alpha$. The result is

$$\frac{2}{m_1m_2\sinh\theta}\left[\frac{\theta}{2}\ln\left(\frac{4\sinh\frac{1}{2}(\omega-\theta)}{\sinh\frac{1}{2}(\omega+\theta)}\right) + 2L\left(\frac{\sinh\frac{1}{2}(\omega-\theta)}{\sinh\frac{1}{2}(\omega+\theta)}e^{-\theta}\right) - 2L\left(\frac{\sinh\frac{1}{2}(\omega-\theta)}{\sinh\frac{1}{2}(\omega+\theta)}e^{\theta}\right)\right].$$

Using the relations of note (7), we easily pass to the form of Eqs. (9). The other integrals are done in the same manner.