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First 0^+ Level in C^{12} and O^{16}*

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A model is proposed to explain the electric monopole ($0^+ \rightarrow 0^+$) matrix element for the transition between the ground and 7.68-Mev state in C^{12} and between the ground and 6.06-Mev state in O^{16}. In this model the first excited 0^+ state in C^{12} and O^{16} is described by an independent-particle wave function which is identical to the ground-state wave function except for the fact that one of the $1s$ nucleons in the "alpha-particle core" is replaced by a $2s$ nucleon. The calculation is performed using harmonic-oscillator wave functions and the results are found to be in excellent agreement with the present experimental information.

I. INTRODUCTION

IN a recent paper by Schiff[1] the electric monopole ($0^+ \rightarrow 0^+$) transitions between the ground and 7.68-Mev level in C^{12} and the ground and 6.06-Mev level in O^{16} were discussed. These transitions require that the matrix element of the electric monopole operator $\sum_p r_p^2$ between initial and final 0^+ states be approximately 3.8×10^{-26} cm^2 for both nuclei, where r_p is the distance of a proton from the center of the nucleus. He shows that calculations based on collective models, the alpha-particle model and *A*. Bohr's liquid drop model yields values for this matrix element which are too large by a factor of three to five. In addition, a calculation for C^{12} is performed based on the j-j coupling independent-particle model in which the excited state differs with respect to two p-particles from the ground state. In order to get a nonzero result for the matrix element it is necessary to consider the effects of the internucleon forces. Nevertheless, the calculation yields a value too small by a factor of approximately six. There is every reason to believe that a similar calculation for O^{16} would also yield too small a value.

II. DISCUSSION

These results are not surprising. We shall see that it is possible to get excellent agreement with the experimental results in terms of a transition involving a single particle. Collective models involving the correlated motion of several nucleons would be expected to give a value larger than the single-particle value, as they do. Also, any calculation based on the independent-particle model which describes the excited state by a configuration which differs from the ground-state configuration by two or more particles, can give a nonzero value for the matrix element only by virtue of the fact that the nucleon-nucleon interaction mixes the ground state with states which differ from the excited state by at most one nucleon and mixes the excited state with states which differ from the ground state by at most one nucleon.[2] If the first-order perturbation upon which such a calculation is based has any validity, these admixtures must be small and therefore the matrix element must be smaller than the single-particle value. The fact that Schiff's calculation yields a result which is too low confirms the expectation.

Another argument against describing the excited state of C^{12} and O^{16} in terms of multiple excitation of the p-shell can be advanced. Experimentally the matrix element of $\sum_p r_p^2$ is approximately the same in both nuclei. However, the zero-order configurations that might be chosen and the configurations that are likely to be mixed in by the nucleon-nucleon forces are very different in the two cases. It would be very surprising if the two calculations gave approximately the same results.

We are thereby led to postulate the following configurations for the first excited 0^+ states of C^{12} and O^{16} respectively: $(1s)^3 2s(1p)^8$, $(1s)^3(2s)p^{12}$. The excited 0^+ state is imagined to be identical in every way with the ground state except that one of the nucleons in the "alpha-particle core," $(1s)^4$, of the ground state is replaced by a $2s$ particle. We shall try to show that this configuration is reasonable for O^{16} where the experimental evidence is most reliable. In C^{12}, this configuration does not seem as "natural" as in O^{16} but it will be postulated because of the failure of the more straightforward configuration assignments to explain the experimental data.

If the central potential for the shell model is chosen to be a harmonic-oscillator well, the configurations $(1s)^3(2s)(1p)^{12}$, $(1s)^4(1p)^{10}(1d)^2$, $(1s)^4(1p)^{10}(2s)^2$, and $(1s)^4(1p)^{10}(1d)(2s)$ are all degenerate. The first configuration is the one we have chosen and the last three are the lowest configurations involving two-particle excitation which can give a 0^+ state. The degeneracy between these configurations is removed by the interparticle forces. In the state we have chosen the symmetry of the excited state is identical to the ground state symmetry and the p-shell particles are completely undisturbed. The "pseudo-alpha particle" $(1s)^3 2s$ might also be expected to make effective use of the interparticle force since it is so similar to the true alpha

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[1] L. I. Schiff, Phys. Rev. **98**, 1281 (1955). This article contains references to the experimental evidence determining the required value of the matrix element.

[2] This can be most clearly seen by referring to Eq. (8) in Schiff's paper.

particle which has the highest binding energy per bond of any nucleus. Thus it may not be unreasonable to expect this state to be lower in energy than the other candidates.

III. CALCULATION AND RESULTS

The electric monopole operator is

$$\Omega=\sum_i r_i^2(\tfrac{1}{2}-\tau_{zi}), \tag{1}$$

where τ_{zi} is the z-component of isotopic spin for the ith nucleon. Since the initial and final states have $T=0$, the isotopic spin part of this operator gives no contribution. The calculation of the matrix element is straightforward, and we find

$$\langle\Omega\rangle=(4)^{\frac{1}{2}}\Omega_{2s,\,1s}$$
$$=r_{2s,\,1s}^2. \tag{2}$$

The factor of $(4)^{\frac{1}{2}}$ arises essentially[3] because the $2s$ particle can change into any one of the four $1s$ particles present in the ground state. We shall evaluate this using harmonic oscillator wave functions whose range is chosen so as to give a mean squared radius for the nucleus which is the same as for a uniform distribution of nucleons up to a nuclear radius $R=r_0A^{\frac{1}{3}}$.

The following easily derived relationships valid for harmonic oscillator wave functions will be useful.

$$r_{2s,\,1s}^2=(2/3)^{\frac{1}{2}}r_{1s,\,1s}^2 r_{1p,\,1p}^2=(5/3)r_{1s,\,1s}^2. \tag{3, 4}$$

The mean squared radius for a nucleus of mass A in the p-shell, using the above assumption, is

$$\langle r^2\rangle A=[4r_{1s,\,1s}^2+(A-4)r_{1p,\,1p}^2]/A=(3/5)r_0^2A^{\frac{2}{3}}. \tag{5}$$

Using Eqs. (3), (4), (5) we can solve for r_0 in terms of the observed value of the matrix element Ω_{if}. Thus

$$r_0^2=(5/9)(3/2)^{\frac{1}{2}}A^{-5/3}(5A-8)\Omega_{if}. \tag{6}$$

Substituting $\Omega_{if}=3.8\times10^{-26}$ cm^2 and $A=12$ and 16 respectively, we find for C^{12}, $r_0=1.45\times10^{-13}$ cm; and for O^{16}, $r_0=1.35\times10^{-13}$ cm. These results are perfectly reasonable values for r_0. It should also be noted that within the framework of this calculation the lifetime is very sensitive to r_0 ($T\alpha r_0^{-4}$), and the experimental uncertainty of 10% in the lifetime of O^{16} restricts r_0 to $1.35\pm0.04\times10^{-13}$ cm.

[3] The possibility that the independent particle model can lead to transition rates substantially larger than one particle values when the initial and final state are similar is discussed in by A. M. Lane and D. H. Wilkinson, Phys. Rev. **97**, 1199 (1955).

IV. CONCLUSION

The success of the model in explaining the lifetime of the 6.06-Mev level in O^{16} and its consistency with the much less reliable data on the 7.68-Mev level in C^{12} suggests its application to other nuclei in the p-shell. Taken literally the model suggests that, for all low-lying states which can be described by a configuration which does not involve any $1s$-particle excitation, there should exist analogous states with the same spin, parity and isotopic spin. From the nature of the model, the energy separation of the analogous states might be expected to vary smoothly with mass number. A change in the energy separation may be expected since the difference in the interaction energy of the $1s$ and $2s$ particles with the p particles would vary with the number and state of the p particles. In addition, there might be a variation in the size of the alpha particle and the pseudo-alpha particle cores. Other effects which would have to be considered in trying to predict the energy separations, such as interconfigurational mixing, would tend to make the energy separation change from nucleus to nucleus and from state to state.

Recent work[4] suggests that the 6.89-Mev level in C^{14} has positive parity. Other evidence[5] that its spin is 0 suggests that this is the analog of the ground state of C^{14} and is the isotopic spin counterpart of the 8.62-Mev state in N^{14} which is known to be 0^+. It is interesting to note that the separation of analog states in C^{14}, 6.89 Mev, is almost exactly the arithmetic mean of the corresponding excitations in O^{16} and C^{12} (6.06 and 7.89 Mev, respectively).

Attempts have been made to identify other analogous pairs but unfortunately there is not sufficient evidence in the regions of interest to test the model.

[4] McGruer, Warburton, and Bemder (unpublished).
[5] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).