be an isotopic spin forbidden $E1$ from the $3-$ state; the 1.72-Mev line would similarly be an isotopic spin forbidden $E1$ from the $0+$ state.[19] However, the $0+$ member would be formed via a first forbidden transition from N^{16} while the $3-$ would result from an allowed transition. Hence, the 1.72-Mev to 2.75-Mev intensity ratio would be greatly decreased following N^{16} decay as compared with $F+p$ if the doublet assumption were valid. This is contrary to observation. The unlikelihood that our new state is a single $3-$ (alpha-particle model) state has already been discussed.

We should now like to make a general comment on the situation as regards the comparison between the alpha-particle model of O^{16} and experiment. So far, attention has centered on the agreement in *energy* between the theoretical and experimental levels; but if the model is to be acceptable it must also give an adequate account of the *dynamical* properties of the states. In particular, we should ask what the model has to say about the reduced alpha-particle widths. These show experimentally a very wide range from 0.15 percent of the Wigner limit for the $2+$ state at 9.84 Mev to 85 percent of the Wigner limit for the $1-$ state at 9.58 Mev. Although one could not hope that the model would give a detailed account of the widths, it must at least predict allowed transitions for those states that show experimentally large reduced widths and forbidden transitions for those with very small widths. Unless such qualitative correspondence emerges between theory and experiment for the dynamical properties of the states the present agreement in energy must be regarded as largely fortuitous.

It would also be very valuable to know the predictions of the alpha-particle model in relation to radiative widths. It appears probable[20] that the predicted lifetime of the alpha-particle model against pair emission from the $0+$ state at 6.06 Mev is too short by a considerable factor. Again the emission of $E1$ radiation from the $1-$ state at 7.12 Mev with a width of at least 0.02 percent of the single-particle value[21] constitutes a violation of the strict alpha-particle model (as it does of the more general isotopic spin rule).

[19] The need to investigate this alternative explanation has been particularly stressed by Professor H. T. Richards.

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Differential Elastic Scattering of 14-Mev Neutrons in Bi, Ta, In, Fe, and S†

J. O. Elliot
Naval Research Laboratory, Washington, D. C., and University of Maryland, College Park, Maryland
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The differential elastic scattering cross sections of Bi, Ta, In, Fe, and S for 14-Mev neutrons were measured at scattering angles between 5° and 55° with an angular resolution varying from ±1° to ±3° using a cylindrical geometry and a biased scintillation detector with a threshold of 12 Mev. Multiple-scattering corrections were made using an approximate theoretical method. The experimental results were compared to calculated cross sections using a phase-shift analysis based on the complex square-well model of the nuclear interaction.

SEVERAL investigators[1–4] have recently reported measurements of differential elastic scattering cross sections for medium energy neutrons and have compared their results with the theoretical approach of Feshbach, Porter, and Weisskopf.[5] These experiments have been confined to the energy region 1–5 Mev. In order to examine the applicability of this theory to higher energy neutrons, the present study of the differential elastic scattering cross sections of selected medium and heavy weight elements has been undertaken for incident 14-Mev neutrons. This higher energy presents an advantage when dealing with the "gross structure" approximation of the theory in that compound elastic scattering can be essentially neglected, and comparison of the experimental results with the theoretical predictions becomes simpler. Also there is only a limited amount of experimental information available concerning 14-Mev neutron differential elastic scattering in medium and heavy weight elements. Early results of Wakatuki and Kikuchi[6,7] at this energy showed that the scattering was predominantly in the forward direction. Later Amaldi *et al.*[8] were successful

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in demonstrating in the case of lead the typical diffraction pattern associated with this scattering phenomenon and obtained qualitative agreement between their results and the Placzek-Bethe optical diffraction model of the scattering.[9] More recently preliminary results of the differential elastic scattering of 14-Mev neutrons in elements of widely varying mass numbers have been reported by various experimenters.[10–12]

EXPERIMENTAL DETAILS

The $H^3(d,n)He^4$ reaction was used as a source of 14-Mev neutrons. Targets of the thick Zr-T type[13] were bombarded with an unanalyzed beam of 180-kev deuterons in a Cockcroft-Walton accelerator.[14] During the experiment the rate of production of neutrons from the reaction was held constant and was continuously monitored by counting the companion alpha-particle rate with a KI scintillation detector.

The scattering experiment was performed at 90° to the direction of the deuteron beam and is shown schematically in Fig. 1. The scatterer was fabricated as a circular cylindrical shell and was placed midway between the neutron source and the neutron detector; the axis of the scatterer was aligned with the accelerator target and the neutron detector. A tungsten shadow shield of suitable cross sectional area and about six mean free paths in length was positioned along the axis of symmetry midway between the neutron source and detector to keep the primary beam of neutrons from striking the detector while measuring the effect of the scatterer.

The scattering cross section was deduced from an experimentally determined scattering ratio defined as the intensity of the elastically scattered beam divided by the intensity of the direct beam from the neutron source, both intensities being measured with the detector kept at the same spatial position. This ratio was obtained from the results of the following three runs: (1) a "source" run made with the scatterer and the shielding bar in place; (2) a "background" run made with the scatterer removed but with the shielding bar in place; and (3) a "direct" run made with both the scatterer and shielding bar removed. For a given nominal scattering angle θ_0, the scattering ratio $g(\theta_0)$ is then given by

$$g(\theta_0)=\frac{\text{"source"}-\text{"background"}}{\text{"direct"}-\text{"background"}}. \tag{1}$$

θ_0 was varied either by changing the radius of the scatterer, or by changing the distance between the source and detector, or by both of these means. $g(\theta_0)$ was

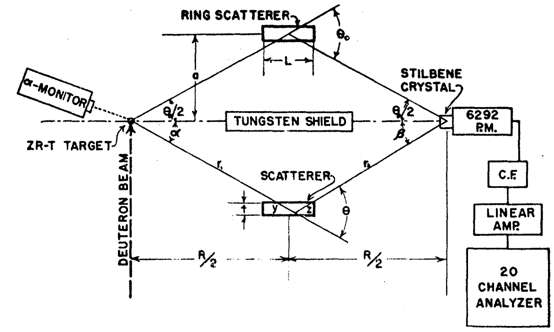

FIG. 1. Schematic diagram of the experimental arrangement.

measured at 5° intervals for θ_0 lying between 5° and 55° for scattering rings of bismuth, tantalum, indium, iron, and sulfur; the angular resolution varied from $\pm 1°$ at small angles to $\pm 3°$ at the larger angles. The tantalum and indium cylinders were built up of layers of foil, and the others were cast to size; in all cases the wall thickness of the cylinder was designed to scatter approximately 10% of the incident beam elastically. The fabricated cylinders were investigated spectroscopically for impurities and, except for iron, exhibited none greater than 0.1%; in the case of iron less than 1% of manganese was detected.

A scintillation counter consisting of a cylindrical stilbene crystal 1 cm in diameter and 1 cm in length and a Dumont 6292 photomultiplier was used as the neutron detector. Since this detector was sensitive to inelastically scattered neutrons and gamma rays as well as to elastically scattered neutrons, some criterion had to be established to discriminate against unwanted events; in this experiment pulse heights corresponding to a recoil proton energy of 12 Mev or greater were accepted for analysis. Such a criterion completely eliminated the possibility of confusing pulses from gamma rays produced by inelastic scattering for this small stilbene crystal. It eliminated almost all inelastically scattered neutrons also, for data published on the energy spectra of 14-Mev neutrons undergoing inelastic scattering[15] show that the great majority of these scattered neutrons have energies considerably below 12 Mev.

The energy and angular sensitivities of the detector and the energy and intensity variations of the primary neutrons have to be considered due to the geometry chosen for the experiment. The energy variation of the neutron source with the angle of emission is available from the kinetics of the reaction.[16] The intensity variation of the source with the angle of emission can be determined from the work of Allan and Poole.[17] The angular sensitivity of the detector was determined

[9] G. Placzek and H. A. Bethe, Phys. Rev. **57**, 1075(A) (1940).
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experimentally using just the detector and the neutron source; the detector was rotated around a vertical axis passing through it and the change in the counting rate of primary neutrons was noted. The energy sensitivity of the detector was obtained experimentally by moving the detector in the horizontal plane about a vertical axis through the accelerator target keeping the face of the detector at a constant distance from the target and normal to the flux. The resulting count rate variation with angle was converted to sensitivity *vs* angle of emission by dividing out the angular distribution of the source; the corresponding sensitivity *vs* neutron energy curve was obtained from the kinetics of the reaction.

ANALYSIS

The single scattering ratio g can be related to the differential elastic scattering cross section $\sigma(\theta)$ by

$$g=\int_V R^2 r_1^{-2} r_2^{-2} M(\alpha,\phi)\epsilon(E)f(\beta)\sigma(\theta) \times\{\exp[-\mu(y+z)]\}ndV, \quad (2)$$

where $M(\alpha,\phi)$ is the neutron intensity per unit solid angle per unit monitor flux normalized along the cylindrical axis of symmetry, ϕ is the angle measured about the axis of symmetry in the plane normal to the axis ($\phi=0$ being in the forward direction of the deuteron beam), $\epsilon(E)$ is the energy sensitivity of the detector normalized for the neutron energy along the symmetry axis, $f(\beta)$ is the angular sensitivity of the detector normalized along the symmetry axis, n is the number of scatterers per unit volume, μ is the absorption coefficient based on the total cross section, and the meaning of the remaining symbols is evident from Fig. 1.

An approximate solution, which will be referred to as $\sigma_I(\theta_0)$, is obtained for Eq. (2) if the measured scattering ratio $g(\theta_0)$ is substituted for g and if all the scatterings are imagined to occur midway between the source and detector through the angle θ_0. Such an approximate solution requires corrections for two effects introduced by the finite size of the scatterer—namely, multiple scattering and angular resolution.

Some of the neutrons reaching the detector and included in the measured scattering ratio have undergone more than one scattering. The contributions from such events must be removed from the final cross-section values. This multiple-scattering effect was investigated theoretically, and for simplicity third and higher orders of scattering were neglected in the calculations. Three types of second order scattering events were studied—scattering from the shielding bar to the cylindrical sample and thence to the detector, scattering from the cylindrical sample to the shielding bar and thence to the detector, and double scattering in the sample itself. It can be argued on physical grounds that the first two types of double scattering are less important than the third because much larger scattering angles (and correspondingly smaller cross sections) are involved. To investigate this point more concretely, numerical integrations were carried out for double scattering from the bar to the cylinder and the reverse event and compared to single scattering in the cylinder. Estimates of the magnitude of these contributions for the sample case of bismuth were 1% or less throughout the angular range utilized in the experiment and were therefore neglected in all of the multiple scattering corrections.

For calculating the double scattering in the sample itself, an approximate semianalytic method was chosen which compared the ratio of double scattering to single scattering for an infinite plane of thickness equal to the wall thickness of the cylindrical scatterer. The neutrons in the calculation were required to be incident upon the plane at the same angle as they were upon the cylinder

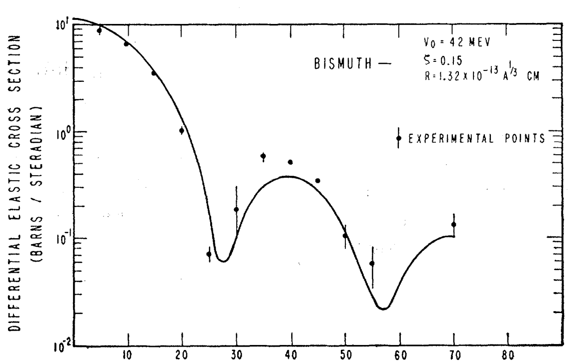

FIG. 2. Differential elastic scattering of 14-Mev neutrons in bismuth.

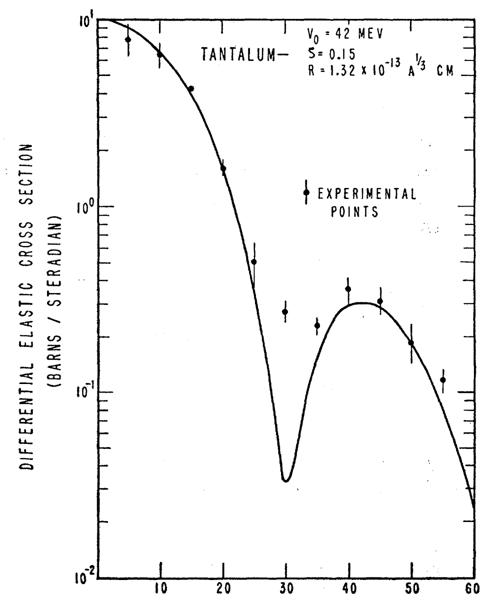

FIG. 3. Differential elastic scattering of 14-Mev neutrons in tantalum.

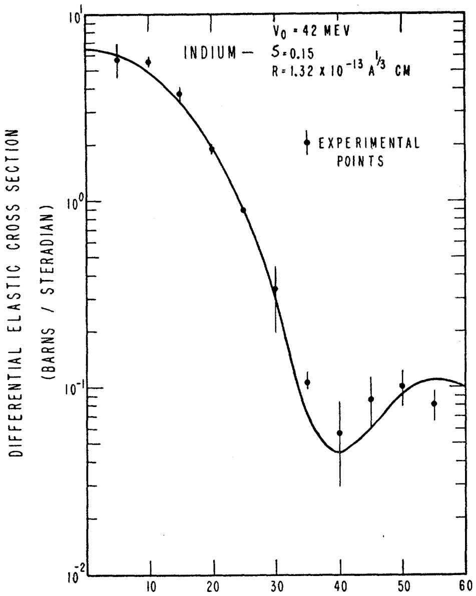

FIG. 4. Differential elastic scattering of 14-Mev neutrons in indium.

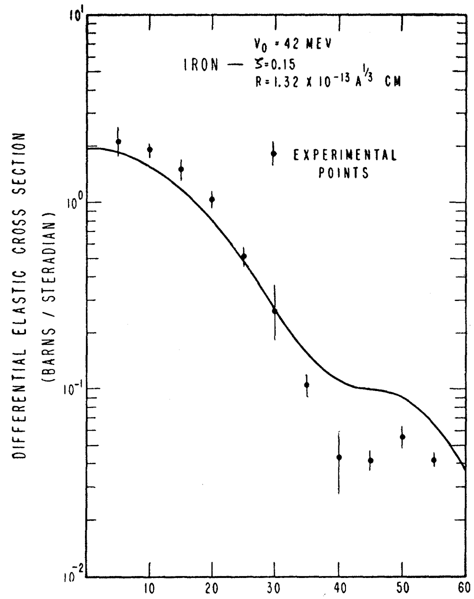

FIG. 5. Differential elastic scattering of 14-Mev neutrons in iron.

in the experiment, and, of the neutrons leaving the plane, only those were retained in the calculation which departed at the correct angle to have been detected in the experiment. The scattering process was treated in a manner similar to that of Vineyard[18] in that a set of transport equations was written expressing the fact that scattering of order $n-1$ provides a source, while scattering and absorption of neutrons of order n provide a sink, for the current of scattering of order n at a given depth in the plane. From this set of equations the current of singly scattered neutrons $S_1(\theta_0)$ headed toward the detector and the current of doubly scattered neutrons $S_2(\theta_0)$ headed toward the detector were compared.[19] In particular, $S_2(\theta_0)$ is an integral expression containing the product of two differential cross sections under the integration sign.

The procedure then is to plot $\sigma_{\mathrm{I}}(\theta_0)$ as a smooth curve. These cross-section values are known to be too large because they include contributions from higher order scatterings. If one had a curve of the corrected differential cross section $\sigma_{\mathrm{II}}(\theta_0)$, a point by point comparison with $\sigma_{\mathrm{I}}(\theta_0)$ could be obtained by multiplying $\sigma_{\mathrm{II}}(\theta_0)$ by the ratio of the sum of the first and higher order scattering contributions to the first order contribution alone. Ignoring third and higher order contributions and utilizing the plane scatterer approximation, this comparison would be written

$$\sigma_{\mathrm{II}}(\theta_0)[1+S_2(\theta_0)/S_1(\theta_0)]=\sigma_{\mathrm{I}}(\theta_0). \tag{3}$$

[18] G. H. Vineyard, Phys. Rev. **96**, 93 (1954).
[19] J. O. Elliot, Ph.D. thesis, University of Maryland, 1955 (unpublished).

This represents a nonlinear integral equation for the cross section; its solution was obtained numerically by successive approximations. As a first approximation the values of the differential cross section needed in $S_2(\theta_0)$ were chosen from the smooth curve of $\sigma_{\mathrm{I}}(\theta_0)$ *vs* θ_0. The resulting improved cross-section values were used in the next approximation. The method effectively converged after at most three iterations.

The approximate cross section, now suitably corrected for multiple scattering, should be corrected for the angular spread introduced in the measurement of $g(\theta_0)$ due to the finite dimensions of the scatterer. To accomplish this the integrand of Eq. (2) was expanded in a Taylor series about θ_0 retaining terms of second order,[19] and the series was integrated term by term yielding a differential equation in $\sigma(\theta_0)$. The solution to this equation was obtained in terms of $\sigma_{\mathrm{II}}(\theta_0)$, its first derivative, and known functions of θ_0.

EXPERIMENTAL RESULTS

The differential elastic scattering cross sections so developed are presented as the plotted points of Figs. 2–6 for bismuth, tantalum, indium, iron, and sulfur, respectively. The vertical bars indicate the probable error of each cross-section measurement. The greatest error entering into the cross-section values is the statistical probable error of the measured scattering ratio $g(\theta_0)$. This error is due to the relatively small net counting rate obtained between the "source" and "background" runs of Eq. (1); in the experiment the "background" counting rate varied from 50% to 95% of the "source" counting rate, the higher percentages usually occurring where the differential cross section was

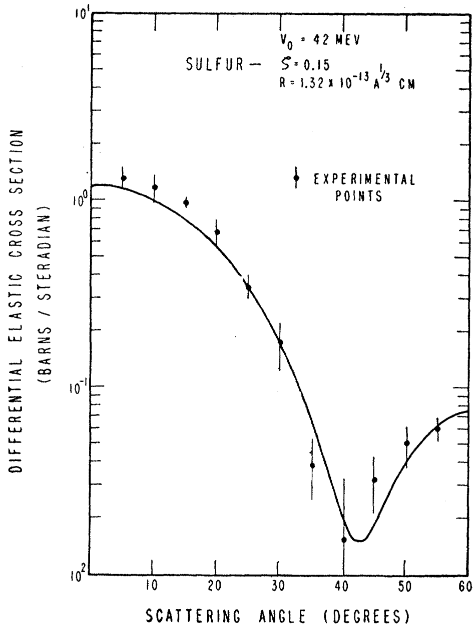

FIG. 6. Differential elastic scattering of 14-Mev neutrons in sulfur.

smaller. The procedure for determining the multiple scattering correction introduces some uncertainty. The iterations at each point were carried out until successive approximations would reproduce themselves within two or three percent; so, within the validity of the method, the calculated corrections are sufficiently accurate. In general, the corrected cross-section values fell within the experimental error of $\sigma_{\mathrm{I}}(\theta_0)$ except near the minima of Figs. 2–6 where the corrections were larger. Any errors associated with the angular resolution correction were completely negligible since the correction itself was very small.

THEORETICAL COMPARISON

Calculated values of the differential elastic cross section were compared to the experimentally determined ones utilizing the approach of Feshbach, Porter, and Weisskopf.[5] Although other investigators have recently introduced modifications into the nuclear potential such as adding a tail to the square well or including spin-orbit terms,[20,21] it was felt worthwhile to explore the simple square-well model insofar as feasible with extensive calculations; therefore, using the potential

$$\begin{aligned} V &= -V_0(1+i\zeta) \quad \text{for} \quad r<R, \\ V &= 0 \qquad\qquad\quad\ \ \text{for} \quad r>R, \end{aligned} \tag{4}$$

as a model of the nuclear interaction, the differential elastic scattering cross sections were calculated as a function of the scattering angle by a partial wave analysis for specified values of the nuclear radius R, the well depth V_0, and the absorption parameter ζ using the SEAC electronic computer of the National Bureau of Standards.

In order to limit the number of cases given to the SEAC, it was decided to restrict V_0 approximately to values suggested by previous work of other investigators —namely, 19 Mev,[1,2,5] 32–36 Mev,[22] and 42 Mev.[1,3–5] To determine which well depth would be of the greatest value in describing the 14-Mev scattering for all the elements investigated, calculations were made letting the radius R range from 1.0 to $1.5\times10^{-13}\,A^{\frac{1}{3}}$ cm, where A is the mass of the nucleus, and letting ζ range from 0.03 to 0.30. It was immediately evident that a good fit to the bismuth data could be obtained by using the 19-Mev well,[23] but for the lighter elements this well depth gave rather poor agreement. The best over-all agreement was obtained with the 42-Mev well; for this V_0, $R=1.32\times10^{-13}\,A^{\frac{1}{3}}$ cm and $\zeta=0.15$ were chosen for the values of these parameters giving the best general agreement for all the elements studied experimentally. The extent of the agreement for this set of parameters is shown by the curves of Figs. 2, 3, 4, 5, and 6 for bismuth, tantalum, indium, iron, and sulfur, respectively. The fact that one is able to reproduce the measured distributions as well as has been done with such a simple model of the nuclear interaction and with only three parameters is certainly remarkable. It is especially interesting to note that the theory is able to predict the relatively flat distribution for iron in the region of the second maximum of Fig. 5 and yet predict the pronounced maxima and minima of the bismuth distribution of Fig. 2.

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