Inelastic Scattering of 31-Mev Protons from Beryllium*

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The range spectrum of charged particles resulting from the bombardment of a thin beryllium foil by 31.3-Mev protons has been measured at several angles. In addition to previously reported energy levels in Be^9 at 2.4, 6.8, and 11.3 Mev, evidence for new levels at 5.0, 7.9, 19.9, and 21.7 Mev was obtained. The angular distribution of each of the proton groups was interpreted in the light of the Austern-Butler-McManus peripheral scattering theory.

Deuteron groups corresponding to the Be^8 ground state and first excited states were also identified. The angular distribution of the ground state deuteron group agrees well with the prediction of a modified Butler theory for the (p,d) reaction.

INTRODUCTION

An energy level of Be^9 was first reported by Davis and Hafner in 1948[1] at an excitation of 2.41 Mev, observed by inelastic scattering of 7.1 Mev protons into photographic plates at one angle (37°). This was verified by a number of other observers[2,3] and remained the only information on the energy level structure of Be^9 until 1952, when Davis[4] published an angular distribution for this level obtained with the above experimental means, and Britten[5] reported the results of his scintillation spectrometer work at the Berkeley proton linear accelerator. Britten observed the inelastic spectrum of protons from beryllium at laboratory angles 90°, 125°, and 160° and reported seeing, in addition to the first level, new levels at 6.8 and 11.6 Mev. By this time it had become increasingly clear that a thorough study of inelastic scattering included, in addition to the location of energy levels, a measurement of the angular distribution of the particle groups. Beryllium was selected to initiate this study because (1) relatively few levels were known, and (2) the levels in this light nucleus were expected to be sufficiently separated to yield easily resolvable proton groups.

The essential features of the experimental method have been described before.[6] A beryllium target was bombarded by 31.3-Mev protons in a remotely controlled 24-inch diameter scattering chamber.[7] Scattered particles were detected in a triple-proportional counter differential range spectrometer.

RESULTS AND CONCLUSIONS

Range Spectra

Complete range spectra (Fig. 1) from the elastic peak down to about 40 to 50 mg/cm² Al, where alpha particles make a large contribution, were obtained at laboratory angles 15°, 30°, 45°, $52\frac{1}{2}$°, 60°, 75°, 90°, and 135°. Interesting regions of the spectrum measured in more detail appear in Figs. 2 and 3.

On the 30° spectrum the peaks have been identified as follows: (1) the elastic peak, (2) the 2.45-Mev level, (3) the 5.0-Mev level, (4) the 6.8-Mev level, (6) the ground state of Be^8 (deuterons), (7) the 3.0-Mev level of Be^8, (8) the 11.3-Mev level of Be^9 (9) a group of levels in Be^8 in the vicinity of 17 Mev, (10) the 19.9-Mev level of Be^9 and (11) the 21.7-Mev level in Be^9. Group (5), which represents the Be^9 nucleus left in its 7.9-Mev level, appears more prominently at backward angles (Fig. 3).

Oxygen is known[8] to have levels at about 6 and 7 Mev. To determine whether this element occurred in appreciable concentration in the beryllium foil, the range spectrum at 60° was investigated above the Be^9 elastic peak for peaks due to elastic scattering from heavier nuclei. The two small peaks found (see Fig. 4) were associated with target nuclei of mass 16 (oxygen) and 23 (sodium). A gaseous oxygen target was then bombarded with 32-Mev protons and the range spectrum of particles scattered through 60° was observed. The ratio of the cross sections of the 6- and 7-Mev levels to that of the elastic peak was obtained, and, using the area of the oxygen impurity peak, the contribution of the oxygen levels to the beryllium spectrum could be estimated. A maximum contribution of about one-fifth that of the 5.0-Mev Be^9 level was obtained for both O^{16} levels together.

Other Particles

An attempt to identify a monoenergetic group of tritons from the reaction $Be^9(p,t)Be^7$ was unsuccessful.

He^3 and He^4 particles, although difficult to distinguish from each other, are easily separable from protons since their rate of energy loss, and thus their pulse height in the proportional counters, is about four times as large. The range spectrum at 30° of particles of charge two (or greater) appears in Fig. 5. No structure

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] K. E. Davis and E. M. Hafner, Phys. Rev. **73**, 1473 (1948).
[2] E. H. Rhoderick, Proc. Roy. Soc. (London) **A201**, 348 (1950).
[3] Arthur, Allen, Bender, Hausman, and McDole, Phys. Rev. **88**, 1291 (1952).
[4] K. E. Davis, Phys. Rev. **88**, 1433 (1952).
[5] Roy Britten, Phys. Rev. **88**, 283 (1952).
[6] J. Benveniste and B. Cork, Phys. Rev. **89**, 422 (1953).
[7] R. M. Eisberg and G. Igo, Phys. Rev. **93**, 1039 (1954).
[8] F. Ajzenberg and T. Lauritzen, Revs. Modern Phys. **27**, 77 (1955).

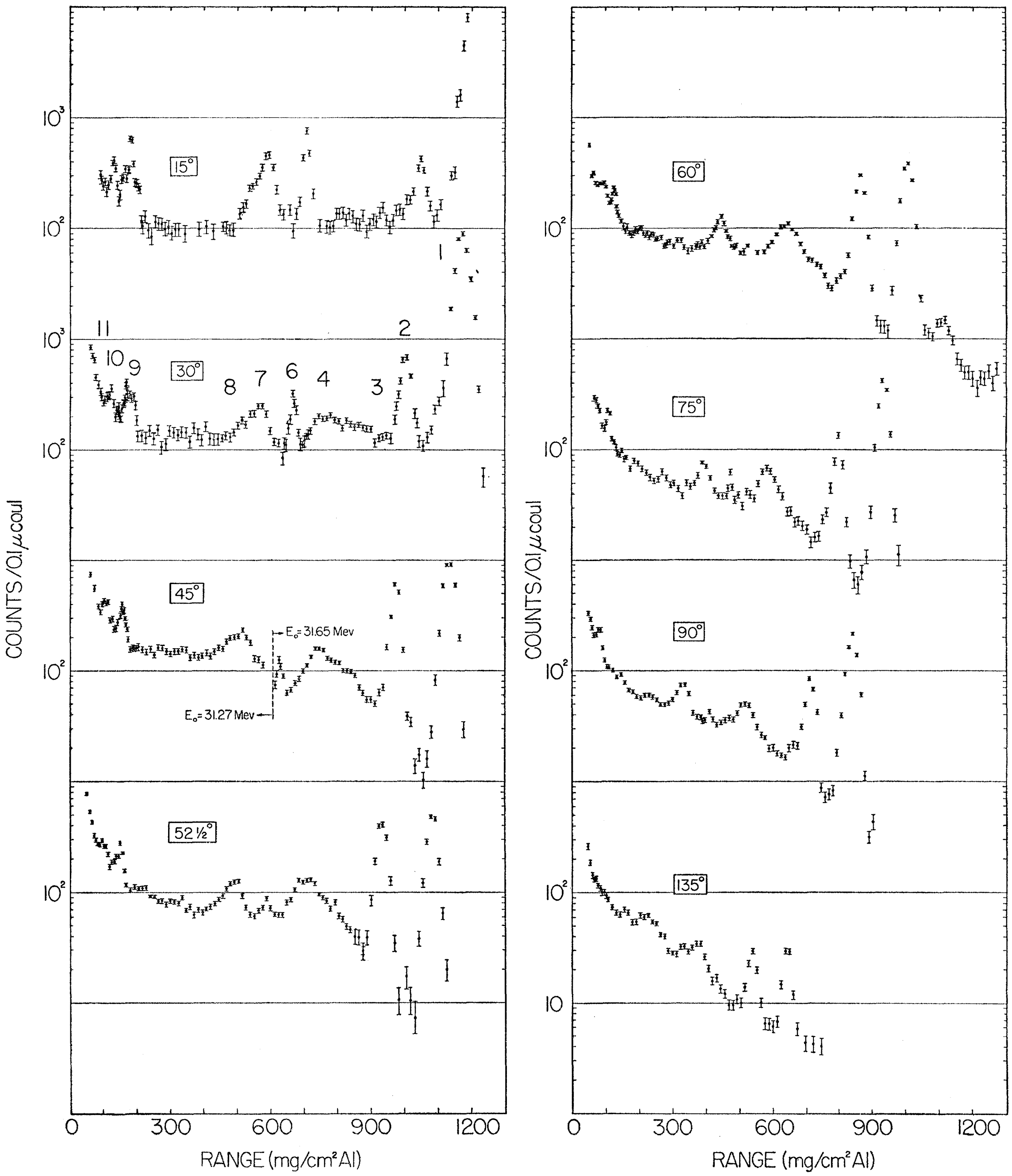

FIG. 1. Composite of range spectra of charged particles from bombardment of Be^9 by ~31.3-Mev protons, observed with the differential-range proportional counter telescope. (Ordinate is in arbitrary units.)

is evident that would identify the ground states of Li^6 or Li^7.

Energy Levels

The bombardment energy is determined from a measurement of the range of the elastically scattered protons and an application of the range-energy relation[9] for protons in aluminum. A check on this determination may be made by a measurement of the energy of the deuteron group which has left Be^8 in its ground state, since the Q of the reaction $Be^9(p,d)Be^8$ is well known. Once the bombardment energy is determined the kinematics of the (p,p') and (p,d) reactions yields expressions relating the energy of the proton or deuteron groups and the laboratory scattering angle.

[9] J. H. Smith, Phys. Rev. 71, 32 (1947).

Comparison of the observed and kinematical relations serves to identify the groups as protons or deuterons.

A tabulation of the excitation energies of Be^9 and Be^8 as measured at the various angles appears in Tables I and II. The reported excitation energy is usually taken as the average of the several determinations. In the case of the 5.0-Mev level, however, which is observed at only a few angles, greater weight was given to an observation of a small portion of the 45° spectrum (Fig. 2) obtained with good statistics.

At angles less than 60°, Group 4 (Fig. 1) appears to leave Be^9 excited by energies decreasing with angle to 6.2 Mev. At angles larger than 90°, the excitation energy appears to be constant at 6.8 Mev. This shift in energy could be accounted for by the existence of two levels with different angular dependences for the inelastically scattered proton. Attempts to establish this conjecture more firmly by improving statistics and looking for an especially wide peak at intermediate angles were not fruitful. However, the possibility of the existence of two levels has not been ruled out.

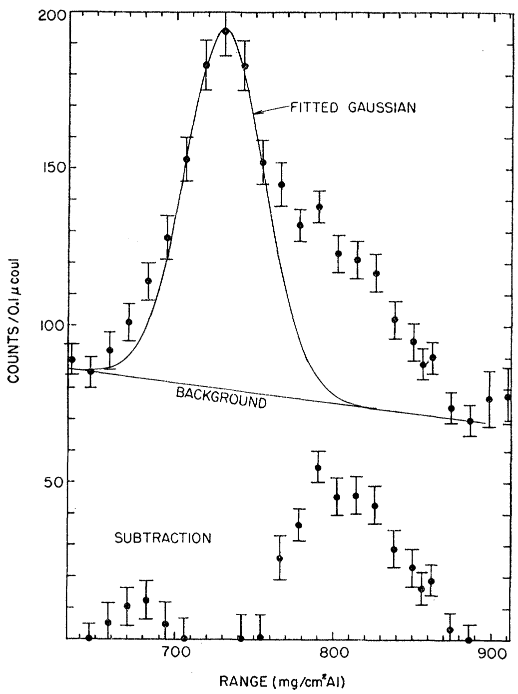

FIG. 2. Portion of 45° range spectrum in vicinity of 6.76-Mev level (solid curve), showing evidence for existence of a 5.0-Mev Be^9 excited state (subtraction curve).

The existence of the 11.3-Mev level in Be^9 would have been rather hard to establish with the range method alone, since at a bombarding energy of 31 Mev, the deuterons from the 3-Mev Be^8 level splash across its position in range at nearly all angles. Only at small angles does the proton group appear as a small bump on the side of the deuteron peak (Group 8, Fig. 1).

An attempt was made to identify the sharp peak,

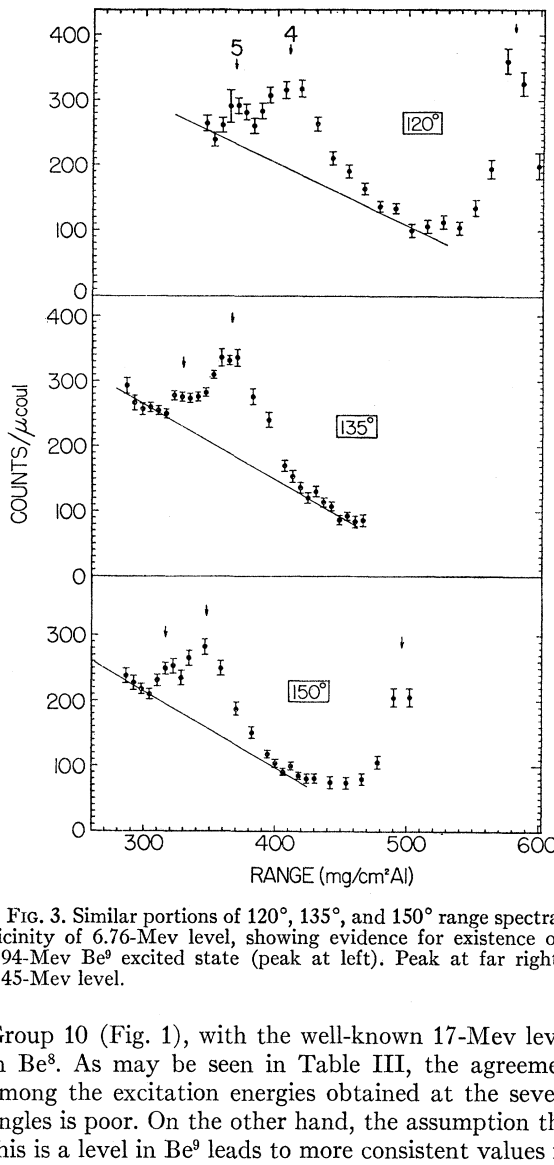

FIG. 3. Similar portions of 120°, 135°, and 150° range spectra in vicinity of 6.76-Mev level, showing evidence for existence of a 7.94-Mev Be^9 excited state (peak at left). Peak at far right is 2.45-Mev level.

Group 10 (Fig. 1), with the well-known 17-Mev level[8] in Be^8. As may be seen in Table III, the agreement among the excitation energies obtained at the several angles is poor. On the other hand, the assumption that this is a level in Be^9 leads to more consistent values for

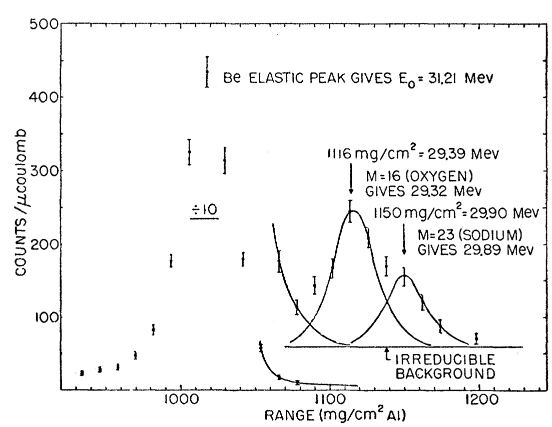

FIG. 4. Beryllium elastic peak and spectrum of longer-range protons elastically scattered from impurities at 60° (lab).

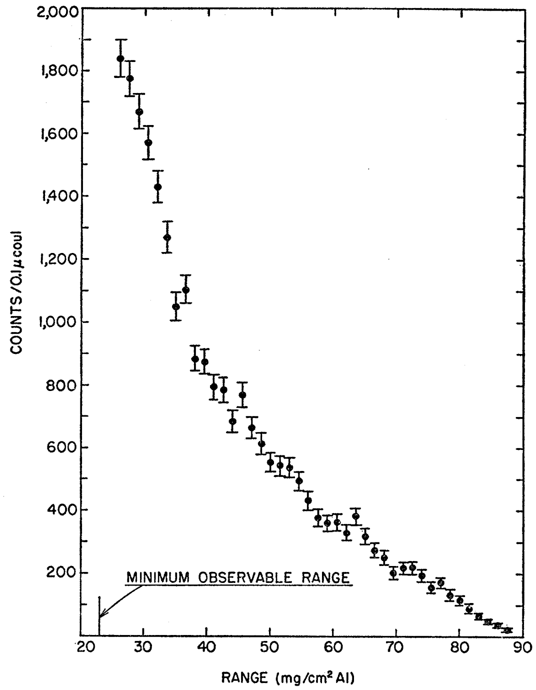

FIG. 5. Range spectrum of He (and heavier) ions from p(31.3 Mev)+Be^9 at 30° (lab).

the excitation energy, 19.9 Mev. In the same manner Group 11 (Fig. 1) has been identified as a proton group which leaves Be^9 excited to 21.7 Mev. It should be kept in mind, however, that this mode of identification is rather insensitive for high excitation energies so that for the two foregoing assignments, some reservations might be held.

There can be seen just to the left of the 3.0-Mev Be^8 level in the 75°, 90°, and 135° spectra in Fig. 1 two small peaks. Taking the observations individually, the peaks are no larger than the statistical fluctuations one might expect in the data. Taken together, however, the several observations are rather suggestive because they yield consistent values for the excitation energies of levels at 14.5 and 17.5 Mev in Be^9.

The levels in Be^9 at 1.8 and 3.1 Mev reported by Moak *et al.*[10] were not observed, even though a cross section as small as 10^{-29} cm^2/sterad could have been detected. It would be interesting to understand why these levels are rather strongly excited in some nuclear interactions and not in others.

The only excited states observed in Be^8 were the 3-Mev level and a group of levels in the vicinity of 17 Mev.

TABLE I. Excitation energies of levels in Be^9 as determined at various angles.

Proton groups as identified in Fig. 1	1	2	3	4	5	8	10	11
θ_{lab}	Bombarding energy (Mev)	Energies of levels in Be^9 (Mev)						
15	31.1	2.52	4.38	6.20		11.2	19.9	21.6
30	31.1	2.57		6.33		11.2	19.9	21.6
45	31.7	2.51	5.49	6.45				
45	31.3	2.42		6.49		11.4	19.9	21.7
45	31.4		5.06	6.44				
$52\frac{1}{2}$	31.5	2.42	5.34				19.8	21.8
60	31.0	2.40		6.53		11.5	19.8	
75	31.3	2.44		6.69	7.94		20.1	21.7
90	31.5	2.40		6.72	7.87			
90	31.3	2.47		6.68	7.93			
120	31.4			6.80	7.96			
135	31.5	2.45		6.78	8.09			
135	31.2			6.83	7.98			
150	31.4			6.80	7.84			
Average of all measurements		2.46	5.0	6.76[a]	7.94	11.3	19.9	21.7
Std. deviation		0.05	0.3	0.06	0.08	0.2	0.1	0.1

[a] Average of measurements beyond 60°.

Angular Distribution of Proton-Groups

The inelastic scattering process may be pictured in at least two separate ways: (1) The compound nucleus theory[11,12] assumes that the incoming proton is captured by the target nucleus Z^A forming a compound nucleus $(Z+1)^{A+1}$ in an excited state. There are two cases which may be simply analyzed. The first is the case in which the compound nucleus is formed in a single excited state. The transition from this state to a discrete final state means that the wave function of the emitted proton has a definite parity. Thus one expects the angular distribution of these protons to be symmetrical about 90°. The second is the case in which many states of the compound nucleus are excited because the level

TABLE II. Excitation energies of levels in Be^8 as determined at various angles.

θ_{lab}	Group 6 bombarding energy (Mev)	Group 7 energies of levels (Mev)
15	31.1	3.2
30	31.1	2.77
45	31.2	2.88
$52\frac{1}{2}$	31.5	2.92
60	31.0	2.84
90	31.4	3.10
90	31.5	3.25
Average of all measurements		2.99 ± 0.17

TABLE III. Comparison of calculated excitation energies for group 10, 11.

θ_{lab}	Group 10 proton excit. of Be^9 (Mev)	Group 10 deut. excit. of Be^8 (Mev)	Group 11 proton excit. of Be^9 (Mev)	Group 11 deut. excit. of Be^8 (Mev)
15	19.9	17.5	21.6	19.9
30	19.9	17.2	21.6	19.8
45	19.9	16.9	21.7	19.5
52.5	19.8	16.7	21.8	19.5
60	19.8	16.8		
75	20.1	16.7	21.7	19.1

[10] Moak, Good, and Kunz, Phys. Rev. **96**, 1363 (1954).
[11] V. F. Weisskopf, Helv. Phys. Acta **23**, 187 (1950).
[12] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 535.

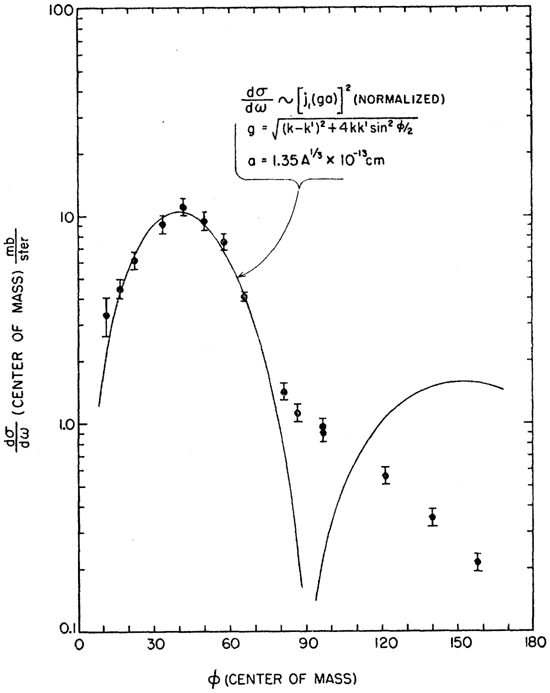

FIG. 6. Angular distribution of the differential cross section for the reaction p(31.3 Mev)$+Be^9 \rightarrow p'+Be^{9*}$ (2.45 Mev) and the Austern-Butler-McManus curve for $l=1$ and $r_0=1.35$. (Curve not corrected for finite angular resolution of 1.6°.)

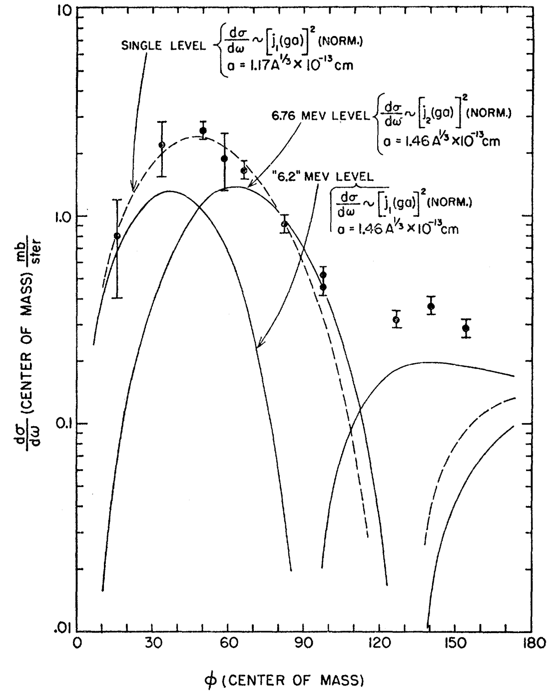

FIG. 7. Angular distribution of the differential cross section for the reaction p(31.3 Mev)$+Be^9 \rightarrow p'+Be^{9*}$ (6.76 Mev and possible "6.2 Mev" unresolved) and the Austern-Butler-McManus curves for the two cases—single level, $l=1$ and $r_0=1.17$ (dotted), and two unresolved levels with different l (2 and 1) and same $r_0=1.46$ (solid curve).

density is high. The analysis of this case by means of a statistical theory also leads to angular distributions symmetrical about 90°. (2) The second picture of inelastic scattering[13] views the collision as a proton-nucleon interaction instead of a proton-nucleus interaction. In this theory the proton does not penetrate very deeply into the nucleus nor does it stay in the vicinity longer than a time of the order of the nuclear diameter divided by the proton's velocity. Since the proton's mean free path in nuclear matter increases with energy it is more likely that at higher energies a proton will interact with individual nucleons rather than form a compound nucleus. Thus one would except that the mechanism responsible for inelastic scattering would be mostly compound nucleus formation at low energies (e.g., 7.1 Mev),[4] a mixture of compound nucleus and peripheral scattering at a higher energy (e.g., 10 Mev),[14] and mostly peripheral scattering at a still higher energy (e.g., 31 Mev). Even at this energy, however, compound nucleus formation might be expected to play an important part when the emergent proton has low energy.

For the case of a peripheral collision, Austern, Butler, and McManus give for the angular distribution of the scattered nucleon

$$d\sigma/d\omega \approx \sum_l C_l\{j_l(|\mathbf{k}_i - \mathbf{k}_f|a)\}^2,$$

[13] Austern, Butler, and McManus, Phys. Rev. **92**, 350 (1953).
[14] Gerhard E. Fischer, University of California Radiation Laboratory Report UCRL-2546 (unpublished).

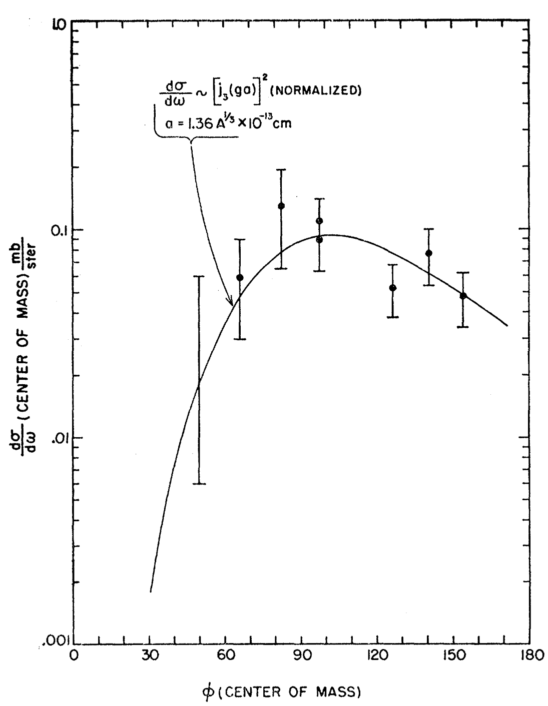

FIG. 8. Angular distribution of the differential cross section for the reaction p(31.3 Mev)$+Be^9 \rightarrow p'+Be^{9*}$ (7.94 Mev) and the Austern-Butler-McManus curve for $l=3$ and $r_0=1.36$.

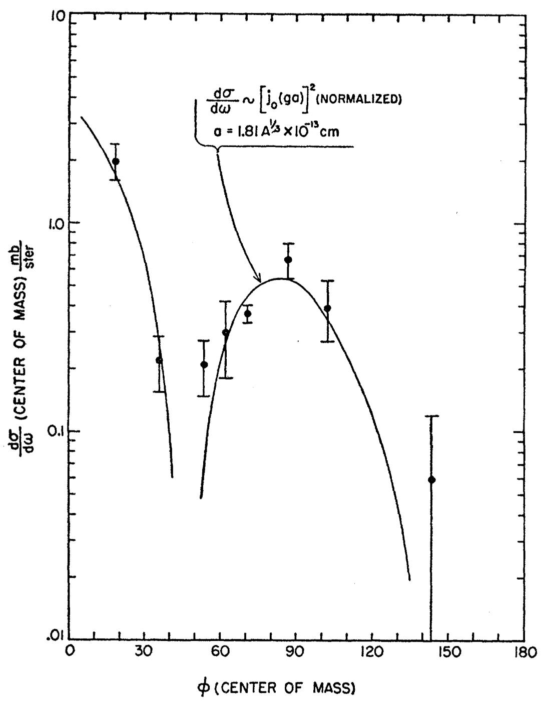

FIG. 9. Angular distribution of the differential cross section for the reaction p(31.3 Mev)$+Be^9 \rightarrow p'+Be^{9*}$ (19.91 Mev) and the Austern-Butler-McManus curve for $l=0$ and $r_0=1.81$.

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the wave numbers of the incident and scattered nucleons, a is a measure of the radius of the nuclear shell in which the inelastic collision takes place, the C_l are constants, and the j_l, regular spherical Bessel functions of order l; l is an index which characterizes the reaction. Conservation of angular momentum restricts the range of l values to

$$J_x+J_y+1 \geqslant l \geqslant |\mathbf{J}_x+\mathbf{J}_y+\mathbf{S}|_{\min},$$

where J_x and J_y are the spins of the initial and final states of the target nucleus and $\mathbf{S}$ is a vector of unit magnitude. Conservation of parity places the further restriction that l may assume either even or odd values in this range. From the properties of the spherical Bessel functions of order l, the position of the most forward peak will serve to determine $l_{\min}$. Thus the total angular momentum change suffered by the target nucleus will be restricted to the values obtained from

$$l_{\min}=\Delta J-1, \quad \text{or} \quad l_{\min}=\Delta J.$$

The parity will change or not depending on whether $l_{\min}$ is odd or even.

ANGULAR DISTRIBUTION OF PROTON GROUPS

The cross section for the 2.45-Mev level is well determined at most angles. Figure 6 illustrates that j_1 yields a very good fit for $r_0=1.35$, where $a=r_0A^{1/3} \times 10^{-13}$ cm. A poorer fit is found for $r_0=1.30$ or 1.41. Since the spin of the ground state[8] of Be^9 is $J_x=3/2^-$, the spin of the 2.45-Mev level is given to be $J_y=\frac{1}{2}$, 5/2 or 7/2, all even parity ($l=1$).

In the case of the 6.76-Mev level, where there is some evidence for the existence of two levels, a single level assumption leads to a good fit with j_1 but with an extremely small nuclear radius, $r_0=1.17$. To fit the data well using a larger nuclear radius ($r_0=1.46$) requires the use of two values of l. The amount of admixture of the two angular distributions was chosen to fit the observed angular dependence of the excitation energy (Table I). Since we now have three parameters to fit the observed angular distribution, it is not surprising that the composite curve fits so well. (See Fig. 7.) For a single level, $J_y=\frac{1}{2}$, 5/2 or 7/2 with even parity. For two levels, the "6.2"-Mev level has this angular momentum and parity, and the 6.76-Mev level has $J_y=\frac{1}{2}$, $\frac{3}{2}$, 7/2 or 9/2 and odd parity.

The cross section for exciting the 7.94-Mev level is very low ($\sim$0.1 mb/sterad maximum) and the estimated error is rather high. From the indication that the cross section becomes smaller at forward angles, one concludes that a high value of l is required. A fit is obtained with j_3 and r_0 between 1.36 and 1.46 (Fig. 8). For $l=3$, $J_y=\frac{3}{2}$, 5/2, 9/2 or 11/2 with even parity.

The proton group from the excitation of the 11.3-Mev level, is almost completely obscured at intermediate angles by a deuteron group so no attempt to analyze its angular distribution was made.

The angular distribution for the 19.9-Mev level is j_0 with $r_0=1.81$. The fit is quite good and the experi-

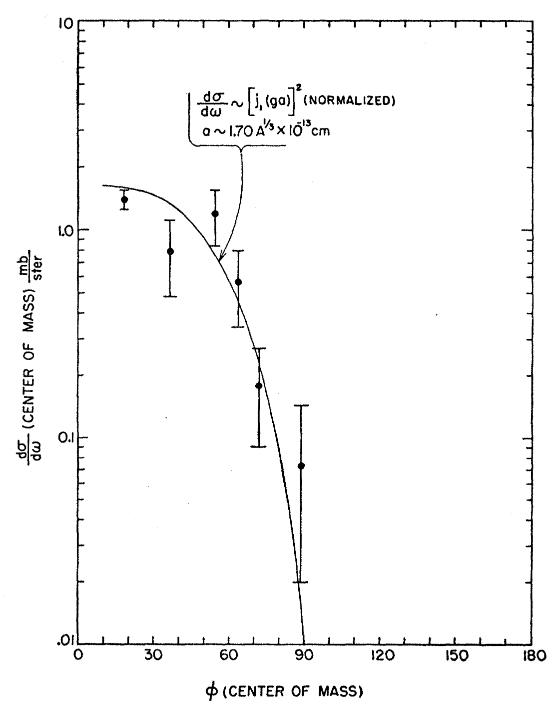

FIG. 10. Angular distribution of the differential cross section for the reaction p(31.3 Mev)$+Be^9 \rightarrow p'+Be^{9*}$ (21.7 Mev) and the Austern-Butler-McManus curve for $l=1$ and $r_0=1.70$.

mental points are fairly well determined (Fig. 9). No significance is attached to the large variations observed in r_0. For the 19.9-Mev level, $J_y=\frac{1}{2}$, $\frac{3}{2}$ or 5/2 with odd parity.

The angular distribution for the 21.7-Mev level (Fig. 10), because of large errors, is difficult to match unambiguously. It is possible with reasonable choice of r_0 to fit the curve with j_0 or j_1. This suggests that for this level $J_y=\frac{1}{2}$, $\frac{3}{2}$, 5/2 (odd) or $J_y=\frac{1}{2}$, 5/2, 7/2 (even).

Angular Distribution of Deuteron Groups

The differential cross section for observation of the reactions $Be^9(p,d)Be^8$ and $Be^9(p,d')Be^{8*}$ is plotted in Figs. 11 and 12. Attempts were made to fit the ground state angular distribution with Butler theory predictions for $r=1.4A^{\frac{1}{3}}\times10^{-13}$ cm and $r=1.4(A^{\frac{1}{3}}+1)\times10^{-13}$ cm and $l=0$, 1, and 2 in each case. The best fit (and not a good one) is obtained with $r=1.4(A^{\frac{1}{3}}+1)\times10^{-13}$ cm and $l=1$ in good agreement with the known change in J from $\frac{3}{2}^-(Be^9)$ to $0^+(Be^8)$. No real choice could be made between the two radii. It should be kept in mind that the Butler theory assumes that the interaction between the nucleons takes place at the periphery of the nucleus. If, however, it is assumed the proton may pick up the neutron anywhere throughout the nuclear volume,[15] it may be shown that the angular distribution is multiplied by a factor with a singularity which may wipe out one of the minima and give a distribution such as we find.

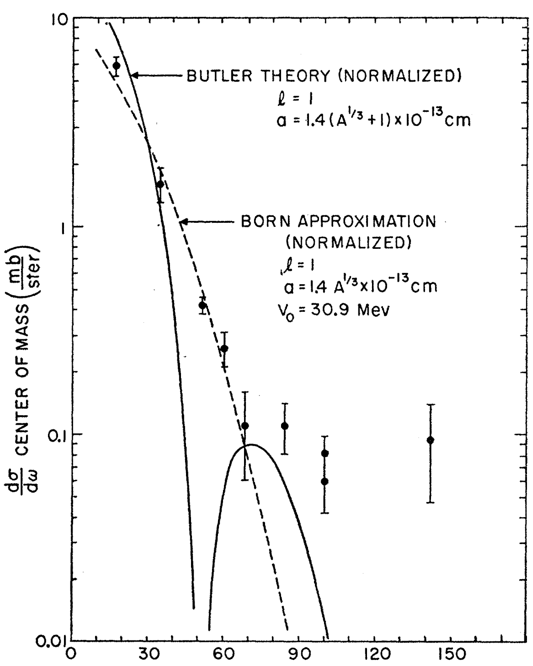

FIG. 11. Angular distribution of the differential cross section for the reaction p(31.3 Mev)$+Be^9\rightarrow d+Be^8$, the Butler theory prediction (solid curve), and the Born approximation (Daitch and French) curve (dotted).

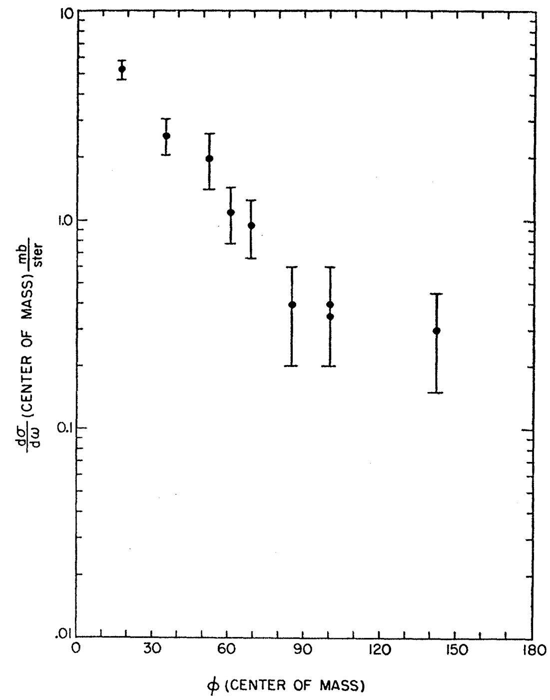

FIG. 12. Angular distribution for the differential cross section for the reaction p(31.3 Mev)$+Be^9\rightarrow d'+Be^{8*}$ (3.0-Mev level).

The 3.0-Mev level angular distribution decreases with increasing angle much more slowly than any Butler theory expression using reasonable radii and $l=0$, 1, 2, or 3. This may not be significant since for all except the smallest angles there is an unknown admixture of protons from the 11.3-Mev level.

DISCUSSION

The positions of known energy levels in Be^9 may be calculated reasonably well using an intermediate coupling model of the nucleus.[16] This model also predicts the total angular momentum of these states. These predictions are consistent with the assignments for total angular momentum of the Be^9 states obtained by application of the peripheral scattering theory.

It was seen in a previous section that the Austern-Butler-McManus theory gave for the 2.45-Mev state in Be^9 the assignment $J_y=\frac{1}{2}$, 5/2, or 7/2, even parity. This same level was observed in a $B^{10}(n,d)Be^{9*}$ reaction by Ribe and Seagrave.[17] Application of the Butler theory for (n,d) reactions lead them to the assignment $J_y=\frac{3}{2}$, 5/2, 7/2, or 9/2, *odd parity*, which is consistent with the prediction of the alpha-particle model, $J=5/2$, *odd parity*. The intermediate coupling model also yields $J=5/2$. If one takes seriously the value $J=5/2$, there remains a disagreement in the parity assigned to this

[15] P. B. Daitch and J. B. French, Phys. Rev. **85**, 695 (1952).

[16] D. R. Inglis, Revs Modern Phys. **27**, 76 (1955); **25**, 390 (1953).

[17] F. L. Ribe and J. D. Seagrave, Phys. Rev. **94**, 934 (1954).

state. In order to get an assignment $J=5/2$, odd parity, for this state using the Austern-Butler-McManus theory, it is necessary that $l=0$ or 4. $l=0$ is ruled out because the angular distribution is clearly not peaked forward, while the first lobe of $j_4(ka)$ with a reasonable choice of nuclear radius, occurs at much too large an angle.

If the restriction $J_y=5/2$ is removed, then one can make a parity and angular momentum assignment consistent with that of Ribe and Seagrave by choosing $l=2$. This yields $J_y=\frac{1}{2}, \frac{3}{2}, 7/2, 9/2$ *odd parity*.

It is clear that $j_2(ka)$ will not fit the data as well as $j_1(ka)$; however, it is not certain that the present status of the theory allows one to make a clear distinction.

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Quantum Calculation of Coulomb Excitation. II. Quadrupole Excitation: Numerical Results*

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The formal methods for the quantum-mechanical treatment of Coulomb excitation discussed earlier are here applied to obtain numerical results for the electric quadrupole case. Tables of the excitation function and the directional correlation parameters are presented and discussed for a wide range of their arguments.

INTRODUCTION

THE electric quadrupole ($E2$) transition is usually the most strongly favored transition in the Coulomb excitation process[1] and at present one of the most interesting from an experimental point of view. Analysis of the experimental data[2] calls for an accurate treatment of the quadrupole excitation, for both the total cross section and the directional correlation parameters. The semiclassical approximation[3] does not lead to sufficiently accurate results over the entire experimental region.[4] In this paper numerical results for the quantum-mechanical treatment of the electric quadrupole excitation are presented. These results are obtained through the application of the formalism and mathematical techniques for the general multipole in Coulomb excitation, discussed earlier by Biedenharn, McHale, and Thaler.[5] Subsequent work will treat other pure multipoles as well as the cases of mixed transitions.

Results are given for the excitation function and the correlation parameters a_2 and a_4 for values of the arguments in the range $0.1\leqslant\eta\leqslant 15$, $1\leqslant\rho\leqslant 1.4$, where $\eta\equiv Z_1Z_2e^2/\hbar v_{\text{initial}}$ and $\rho\equiv k_{\text{initial}}/k_{\text{final}}$. This range covers energy losses of up to 50% for all energies of experimental interest. Numerical values for computing the total cross section may be taken from Table I. Values for the particle parameters a_2 and a_4 are presented in Tables II and III. These values are plotted in various ways in order to exhibit the general behavior in any region of interest. Such plots should prove helpful in suggesting interpolation procedures.

The limitations of the present treatment have been discussed in I. Of these, the neglect of center of mass corrections is probably the most serious. The results presented below are accordingly most accurate for medium and heavy target nuclei. Neglect of retardation is better justified for quadrupole than other multipole

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[3] L. Landau, Physik Z. U.S.S.R. **1**, 88 (1932).
[4] The relevant references to the literature are given in reference 5. The following references should be added to the bibliography given there: K. Mayr [Math. Z. **39**, 597 (1935)]—integrals of Laguerre polynomials in terms of F_2; A. Erdelyi [Math. Z. **40**, 693 (1936)]—confluent hypergeometric integrals evaluated in terms of F_2; J. C. Jaeger [J. London Math. Soc. **13**, 254 (1938)]—an analytic continuation of F_2. In addition, see the following recent papers: Benedict, Daitch, and Breit, Phys. Rev. **101**, 171 (1956); F. D. Benedict, Phys. Rev. **101**, 178 (1956); Breit, Ebel, and Benedict, Phys. Rev. **100**, 428 (1956); Gluckstern, Lazarus, and Breit, Phys. Rev. **101**, 175 (1956).
[5] Biedenharn, McHale, and Thaler, Phys. Rev. **100**, 375 (1955). This paper is referred to as I in the text.