Proton Cross Sections of Bi^{209}†

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The cross sections for the reactions $Bi^{209}(p,2n)Po^{208}$, $Bi^{209}(p,n)Po^{209}$, and $Bi^{209}(p,\gamma)Po^{210}$ have been measured for proton energies up to 10.65 Mev. The threshold for the $(p,2n)$ reaction is 9.65 Mev (c.m.) which leads to a neutron binding energy for Bi^{210} of $\geqslant 4.59\pm0.14$ Mev. The total cross section as a function of energy agrees with the theoretical calculations for a radius $R=1.5A^{1/3}\times10^{-13}$ cm. The energy dependence of the ratio of $\sigma_{(p,2n)}/[\sigma_{(p,n)}+\sigma_{(p,2n)}]$ corresponds to a nuclear temperature of 4 Mev for Po^{209}.

I. INTRODUCTION

The neutron binding energy of Bi^{210} measured by the $Bi^{209}(n,\gamma)Bi^{210}$ reaction[1] and the $Bi^{209}(d,p)Bi^{210}$ reaction[2] is about 0.5 Mev lower than the value calculated from closed energy cycles.[3] Pryce[4] has pointed out that the probability of producing the ground state of Bi^{210} in the above reactions is small, and therefore the possibility exists that the measured values of the Bi^{210} neutron binding energy are too small. A threshold value of the $Bi^{209}(p,2n)Po^{208}$ reaction, in conjunction with existing measured neutron binding energies and several known disintegration energies, enables one to calculate the Bi^{210} neutron binding energy.

The determination of the relative cross sections for primary and secondary reactions can furnish information about the level spacing for intermediate and heavy nuclei. We have studied the reactions $Bi^{209}(p,n)Po^{209}$ and $Bi^{209}(p,2n)Po^{208}$ as a function of proton energy. The energy distribution of the primary neutrons (assuming we can treat this case as that of a compound nucleus) is given by

$$I(\epsilon)d\epsilon=\text{const}\,\epsilon\sigma_c(\epsilon)\omega(\epsilon_{\max}-\epsilon)d\epsilon,$$

where $\omega(\epsilon_{\max}-\epsilon)$ is the level density.[5] A determination of the level density directly from the measured neutron spectra can be made, but several difficulties are encountered in such a determination, one of which is to estimate the secondary neutron emission. The level density is often expressed in terms of a nuclear temtemperature, θ, where θ is the temperature of the "Maxwell" energy distribution of particles evaporated from the nucleus. Assuming a Maxwellian energy distribution of the evaporated neutrons, we can determine the nuclear temperature from the cross sections of the $Bi^{209}(p,n)Po^{209}$ and $Bi^{209}(p,2n)Po^{208}$ reactions. If we assume that the second neutron is evaporated whenever it becomes energetically possible, then

$$\frac{\sigma_{(p,2n)}}{\sigma_{(p,n)}+\sigma_{(p,2n)}}=\frac{\int_0^{\Delta\epsilon}I(\epsilon)d\epsilon}{\int_0^{\epsilon_{\max}}I(\epsilon)d\epsilon},$$

where $\Delta\epsilon=\epsilon_p-T_{2n}$ and $\epsilon_{\max}=\Delta\epsilon+T_{2n}-T_n$. T_n and T_{2n} are the thresholds for the (p,n) and $(p,2n)$ reactions, respectively. Taking $\sigma_c(\epsilon)=\text{const}$ and integrating, we get

$$R=\frac{\sigma_{(p,2n)}}{\sigma_{(p,n)}+\sigma_{(p,2n)}}=\frac{1-(1+\Delta\epsilon/\theta)e^{-\Delta\epsilon/\theta}}{1-(1+\epsilon_{\max}/\theta)e^{-\epsilon_{\max}/\theta}}. \quad (1)$$

An evaluation of this expression at a series of values of ϵ_p gives both the $(p,2n)$ threshold and the nuclear temperature.

II. EXPERIMENTAL PROCEDURE

The stacked foil technique was used to obtain the range of proton energies necessary to determine the the threshold of the $Bi^{209}(p,2n)Po^{208}$ reaction. Aluminum foils of 0.75-in. diameter and 0.0005-in. thickness provided energy increments of about $\frac{1}{8}$ Mev from the maximum beam energy of 10.65 ± 0.08 Mev. Bismuth was evaporated on to the aluminum foils to a thickness of about 100 $\mu g/cm^2$ over an area of 1.62 cm^2.

The all quartz apparatus in which bismuth metal was evaporated on to the aluminum discs is shown in Fig. 1. The aluminum foils and their holders were cooled by liquid air in a Styrofoam "Dewar" which was sealed to the quartz tubing with water. The rest of the apparatus is described in the drawing. In operation, the iron cylinder was heated inductively to a previously determined temperature at which bismuth vapor condensed on the plates at the rate of 1 to 2 $\mu g\ cm^{-2}\ sec^{-1}$. No shutter was employed; rather each target was exposed for the given time and then ejected, exposing the next plate. Twenty-five plates were usually prepared at one time.

It was not possible to determine the amount of bismuth on each target by a vapor pressure–geometry calculation; so each target was "weighed" colorimetri-

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[1] Kinsey, Bartholomew, and Walker, Phys. Rev. **82**, 380 (1951).
[2] J. A. Harvey, Phys. Rev. **81**, 353 (1951).
[3] Huizenga, Magnusson, Simpson, and Winslow, Phys. Rev. **79**, 908 (1950).
[4] M. H. L. Pryce, Proc. Phys. Soc. (London) **A65**, 773 (1952).
[5] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. 8.

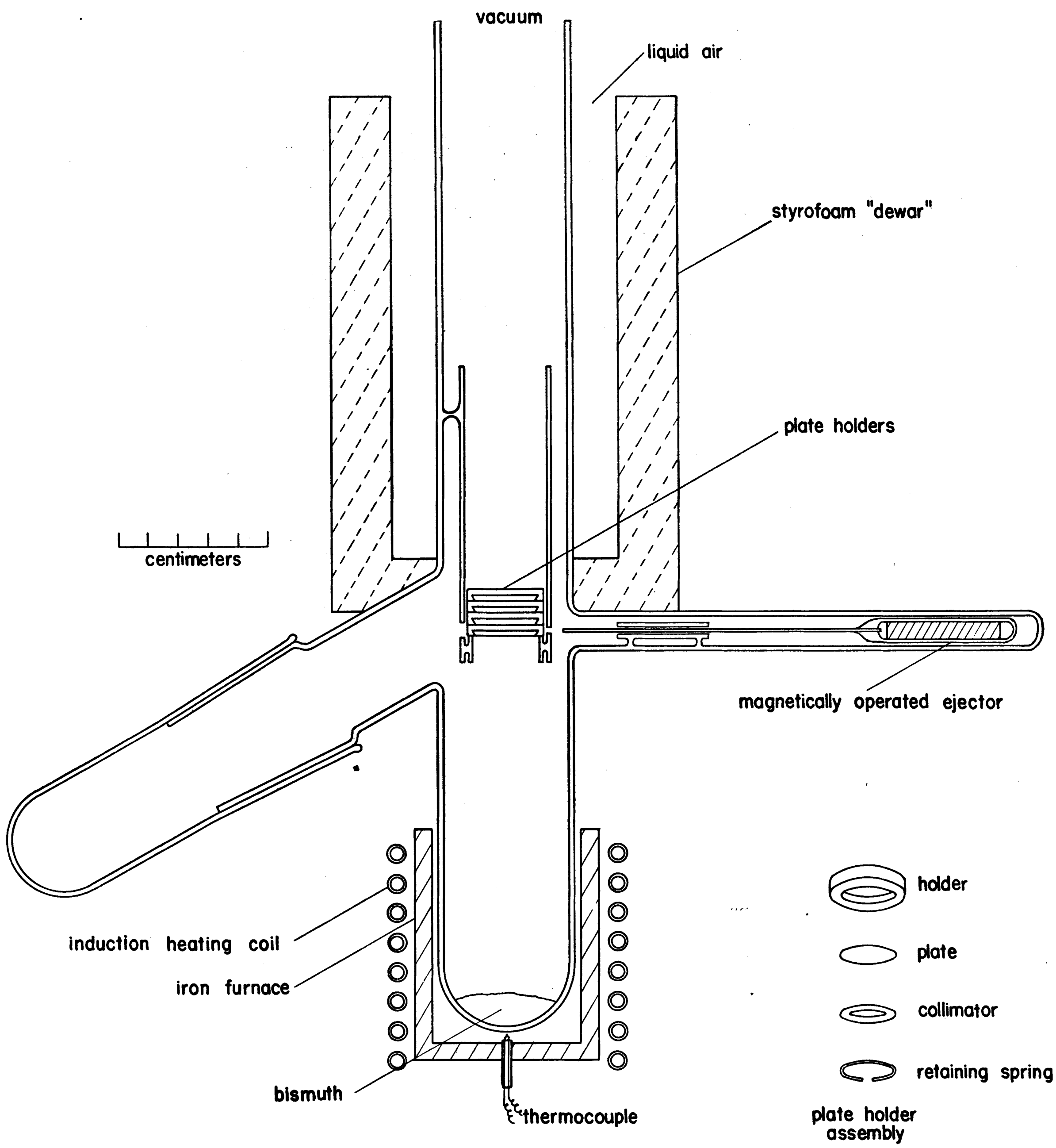

FIG. 1. The evaporation apparatus.

cally after cyclotron bombardment and counting. The bismuth was dissolved off the aluminum backing plate with 1 ml of HNO_3. To the nitric acid solution were added 5 ml of 12% KI solution and two drops of saturated SO_2 solution, and the whole was diluted to 50 ml. The transmission was measured with a Beckman spectrophotometer at 4600 A in 5-cm cells. The slope of the log transmission–weight curve was determined from standard solutions of $Bi(NO_3)_3$. The estimated uncertainty in each weight was about $\pm 2\ \mu g$.

The total alpha activity of each foil was measured after the proton bombardment, and then the foils were pulse analyzed to determine the relative amounts of the polonium isotopes produced. Since the alpha energies of Po^{208}, Po^{209}, and Po^{210} are 5.11 Mev, 4.88 Mev, and 5.30 Mev, respectively, it was possible with our thin layers of bismuth to detect the activity of a given isotope to less than 1% of the total activity. The coincidence that the half-lives of these isotopes are roughly in the same proportion as the cross sections for protons on bismuth producing these isotopes, facilitated the cross-section measurements.

The 10.65-Mev protons used in this experiment were obtained by accelerating molecular hydrogen (H_2^+) at the deuteron resonant frequency of the Argonne cyclotron. This is a constant frequency, 11.1 Mc/sec, sixty-inch accelerator. The deflected beam was collimated by a 3-mm-diameter hole in a water-cooled copper plate, which transmitted about 1% of the deflected beam. A 3.4-mg/cm² aluminum foil was mounted on the target side of this plate to strip the H_2^+ ions. The proton beam then entered the evacuated target chamber in which were located both the range determination foils of aluminum and the stacked bismuth foil targetry. The proton current was about 0.5 μa. The current incident on the insulated target was amplified and recorded by a Brown recording millivoltmeter. The total flux of 279.5 ± 3.5 μa-min was obtained by planimetering the area under the trace.

The beam energy was determined at regular intervals during the bombardment by measuring the proton absorption in aluminum. The transmitted and absorbed currents were measured for each thickness of an eight-step wedge. Each step consisted of two 82 mg/cm² 2*S* aluminum plates, one on each side of an appropriate amount of aluminum foil. From the fraction of total beam current transmitted for the various thicknesses of aluminum absorber, the mean range was determined, and by using the table of Smith[6] the energy was found. The mean range was 186 ± 2.5 mg/cm² corresponding to an energy of 10.65 ± 0.08 Mev. Figure 2 shows a typical energy calibration curve. The straggling $S=(R_{\text{mean}}-R_{\text{extreme}})/R_{\text{mean}}$ was 1.6%, which compares favorably with the theoretical value of 1.5%. The variations of the mean range due to changes in deflector voltage, beam angle, and absorber temperature or due to secondary emission, air ionization, and stray currents (voltaic) were investigated in detail.

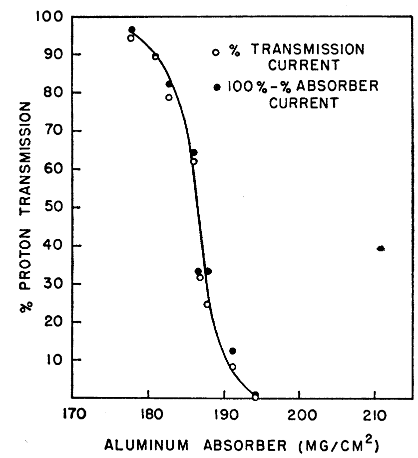

FIG. 2. Energy calibration curve.

It was necessary to have a proton beam free of any deuterium contamination, as even a few parts of deuterium per 10 000 parts of hydrogen would give rise to resultant polonium alpha activities comparable to that obtained from some proton reactions. This was accomplished by having the stripping foil located far enough in front of the target so that the fringing field of the cyclotron would act as a momentum selector. The lateral separation at the target of the deuteron beam from the proton beam was about one cm. Figure 3 is a schematic diagram of the target chamber.

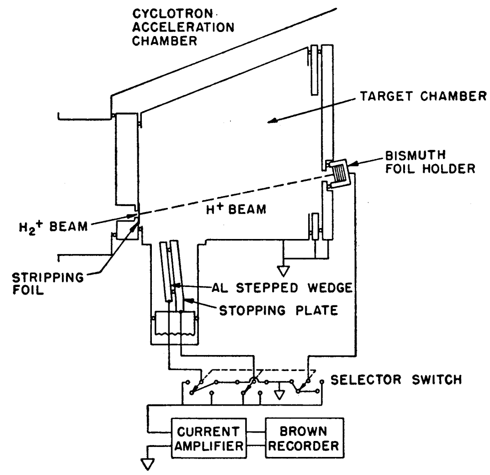

FIG. 3. A schematic diagram of the target and the aluminum absorber wedge.

III. RESULTS AND DISCUSSION

The threshold of the $Bi^{209}(p,2n)Po^{208}$ reaction was 9.70 ± 0.08 Mev in the laboratory system, 9.65 ± 0.08 Mev in the c.m. system, the error being largely due to uncertainty in the beam energy. The value of the thresh-

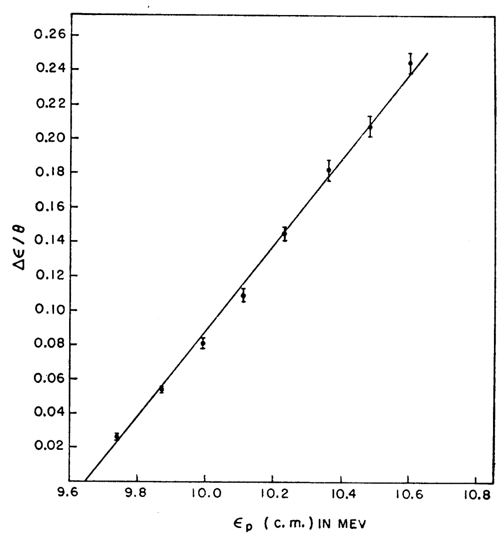

FIG. 4. Determination of $Bi^{209}(p,2n)Po^{208}$ threshold. A plot of $\Delta\epsilon/\theta$ *vs* ϵ_p. The errors indicated are the rms sum of standard deviations.

[6] J. H. Smith, Phys. Rev. **71**, 32 (1947).

TABLE I. Disintegration energies (c.m.) and neutron binding energies (in Mev).

$E_\alpha(Po^{208})=5.21\pm(0.005)$[a]	$E_n(Pb^{206})=8.10\pm0.10$[g]
$E_\alpha(Po^{210})=5.40\pm0.005$[b]	$E_n(Pb^{207})=6.72\pm0.01$[g, h]
$E_\beta(Bi^{210})=1.17\pm0.01$[c]	$E_n(Pb^{208})=7.38\pm0.01$[g, h]
$E_\beta(Pb^{209})=0.64\pm0.01$[d]	$E_n(Pb^{209})=3.87\pm0.05$[g]
$E_\beta(Tl^{204})=0.77\pm0.01$[e]	$E_n(Tl^{205})=7.58\pm0.11$[g, i]
$E_\beta(Tl^{206})=1.51\pm0.01$[f]	$E_n(Tl^{206})=6.23\pm0.05$[h]

[a] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).
[b] W. J. Sturm and V. Johnson, Phys. Rev. **83**, 542 (1951).
[c] L. M. Langer and M. D. Whitaker, Phys. Rev. **51**, 713 (1937); G. J. Neary, Proc. Roy. Soc. (London) **175A**, 71 (1940); L. M. Langer and H. C. Price, Jr., Phys. Rev. **76**, 641 (1949).
[d] A. H. Wapstra, Ph. D. thesis, University of Amsterdam (unpublished); Wagner, Freedman, Engelkemeir, and Magnusson, Phys. Rev. **88**, 171 (1952).
[e] Lidofsky, Macklin, and Wu, Phys. Rev. **87**, 391 (1952); D. Saxon and J. Richards, Phys. Rev. **76**, 928 (1949).
[f] D. E. Alburger and G. Friedlander, Phys. Rev. **82**, 977 (1951).
[g] See reference 2.
[h] See reference 1.
[i] Sher, Halpern, and Mann, Phys. Rev. **84**, 387 (1951); Hanson, Duffield, Knight, Diven, and Palevsky, Phys. Rev. **76**, 578 (1949).

old for the reaction was obtained by plotting $\Delta\epsilon/\theta$ *vs* ϵ_p as determined by Eq. (1) and then fitting a straight line to the data by the method of least squares. The intersection of this line with the abscissa determined the threshold energy (see Fig. 4). The error in the least squares determination was 0.01 Mev. An evaluation of Eq. (1) for $\Delta\epsilon/\theta$ at the various proton energies required a knowledge of the nuclear temperature, θ. However, the value for the threshold energy was insensitive to changes in θ; so only an approximate estimate was needed. Once the threshold energy was obtained, ϵ_{max} was expressed in terms of the threshold energy T_{2n} plus the energy increment, and the nuclear temperature was determined.

From a knowledge of the threshold energy of the $Bi^{209}(p,2n)Po^{208}$ reaction, the neutron binding energy of Bi^{210} was obtained as follows: $E_n(Bi^{210}) =(n-p)+E_n(Pb^{205})+E_n(Pb^{206})+E_\alpha(Po^{208})-E_\alpha(Po^{210}) -E_\beta(Bi^{210})-T_{2n}Bi^{209}(p,2n)Po^{208}$. This relation results from a consideration of the three closed cycles indicated by the dashed lines in Fig. 5. The experimental values of the disintegration energies and the binding energies are given in Table I. All the quantities have been measured except $E_n(Pb^{205})$. It can be calculated from the thallium binding energies by using the dotted cycle in Fig. 5. This gives a value 6.45 ± 0.12 Mev. From a study of the systematics of the binding energies of heavy nuclei one would expect this value to be greater than 6.72 Mev, the value for $E_n(Pb^{207})$. Thus a discrepancy in either one or both of the thallium binding energies appears to exist. If we take $E_n(Pb^{205})\geqslant 6.72$ and substitute the experimental values into the equation for $E_n(Bi^{210})$, we get $E_n(Bi^{210})\geqslant 4.59\pm0.14$ Mev. A calculation of $E_n(Bi^{210})$ from the closed cycle indicated by the solid line gives a value of 4.67 ± 0.11 Mev.[3] The agreement between these two values is within the experimental error.

The nuclear temperature was determined by substituting the value of the threshold we obtained into Eq. (1) and solving it for θ at several of the points. The average value of the nuclear temperature as determined in this manner was 4 Mev. This is the temperature of the nucleus emitting the second neutron where we assume that the distribution in energy of these neutrons is Maxwellian.

This value for the temperature is larger than that reported by several authors.[7–9] Most of this difference

TABLE II. The experimental values of the cross sections for the $Bi^{209}(p,2n)Po^{208}$, the $Bi^{209}(p,n)Po^{209}$, and the $Bi^{209}(p,\gamma)Po^{210}$ reactions at the various energies of the bombarding protons (c.m.). The errors indicated are the rms sum of standard deviations. (We used $T_{\frac{1}{2}}(Po^{209})$ of 103 yr; see text of paper).

Proton energy (in Mev) (c.m.)	$\sigma_{(p,2n)}$ (in millibarns) Run I	Run II	$\sigma_{(p,n)}$ (in millibarns) Run I	Run II	$\sigma_{(p,\gamma)}$ (in millibarns) Run I	Run II	$10^3\sigma_{(p,2n)}/[\sigma_{(p,n)}+\sigma_{(p,2n)}]$ Run I	Run II
10.60		6.98±0.24		159±5		0.184±0.008		42.1±2.1
10.49							31.7±1.4	
10.48		4.75±0.16		147±5		0.176±0.007		31.3±1.6
10.38							21.0±1.3	
10.36		3.38±0.12		133±4		0.192±0.008		24.8±1.3
10.26	1.80±0.10		118±7		0.145±0.008		15.0±0.9	
10.23		2.12±0.08		127±5		0.172±0.007		16.4±0.9
10.13							10.0±0.7	
10.11		1.17±0.05		119±4		0.158±0.007		9.7±0.5
10.02	0.61±0.05		106±6		0.132±0.007		5.7±0.3	
9.99		0.62±0.03		110±4		0.147±0.007		5.6±0.4
9.89							1.8±0.4	
9.87		0.25±0.01		99±3		0.141±0.005		2.5±0.1
9.77	0.08±0.01		88±5		0.109±0.006		0.91±0.24	
9.74		0.065±0.005		103±3		0.135±0.005		0.63±0.05
9.62				84±3		0.111±0.004		
9.49				81±3		0.109±0.004		
9.37				75±2		0.105±0.004		
8.84				47±2		0.073±0.004		
7.68				15.6±0.7		0.033±0.002		
6.36				1.4±0.1		0.0038±0.0002		

[7] Bleuler, Stebbins, and Tendam, Phys. Rev. **90**, 460 (1953).
[8] L. Rosen and L. Stewart, Los Alamos Scientific Laboratory Report LA-1560, 1953 (unpublished).
[9] E. L. Kelly, Ph. D. thesis, University of California, UCRL-1044, 1950 (unpublished).

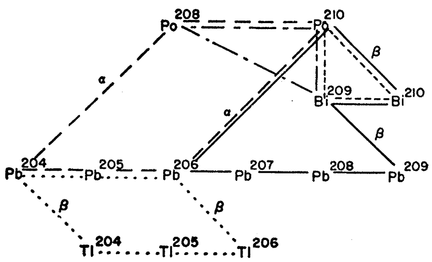

FIG. 5. The closed mass-energy cycles used in determining $E_n(Bi^{210})$.

probably can be attributed to the nearness of a closed shell. It is doubtful whether the statistical model is valid in such a region. A small change in the number of neutrons, when near a closed shell, has a great effect on the level densities as shown by Hughes[10] and others.[11] The statistical model also assumes that the excitation energy is shared by all of the nucleons. If, however, the excitation energy is divided among only a few nucleons,[12] (a direct interaction picture), one would expect higher nuclear temperatures. The value of the nuclear temperature is dependent on the half-life of Po^{209}, a shorter half-life giving a lower temperature. We have used a value of 103 yr for the Po^{209} alpha-disintegration half-life. This value is calculated relative to the Po^{208} half-life of 2.93 ± 0.03 yr[13] from the following data. A polonium sample containing an atom ratio of Po^{209}/Po^{208} equal to 0.176 ± 0.002[14] gave a ratio of $0.0050\pm0.0002_5$ for the abundance of the Po^{209} to Po^{208} alpha particles.[15] The alpha-disintegration half-life of Po^{209} is also a good measure of the total half-life since we have set an upper limit of one K-electron capture per hundred Po^{209} alpha disintegrations.

The $(p,2n)$, (p,n), and (p,γ) cross sections are given in Table II and Fig. 6. The (p,γ) cross sections are a factor of 800 to 700 less than the (p,n) cross sections throughout the energy interval above 8 Mev. As we go to the two lowest energies, the ratio drops to 470 and 370, which one would expect as we approach the (p,n) threshold. The (p,n) cross section depends directly upon the Po^{209} half-life, which is taken to be 103 yr. This appears to be a good value from the comparison of our total cross section to the theoretical values. Any change in half-life would bring essentially that factor change into our total cross sections which would not give a better fit to the shape of the theoretical curve even for different values of r_0. A comparison with the total cross section of Tewes and James[16] for protons on Th^{232} also indicates good agreement. Their total cross

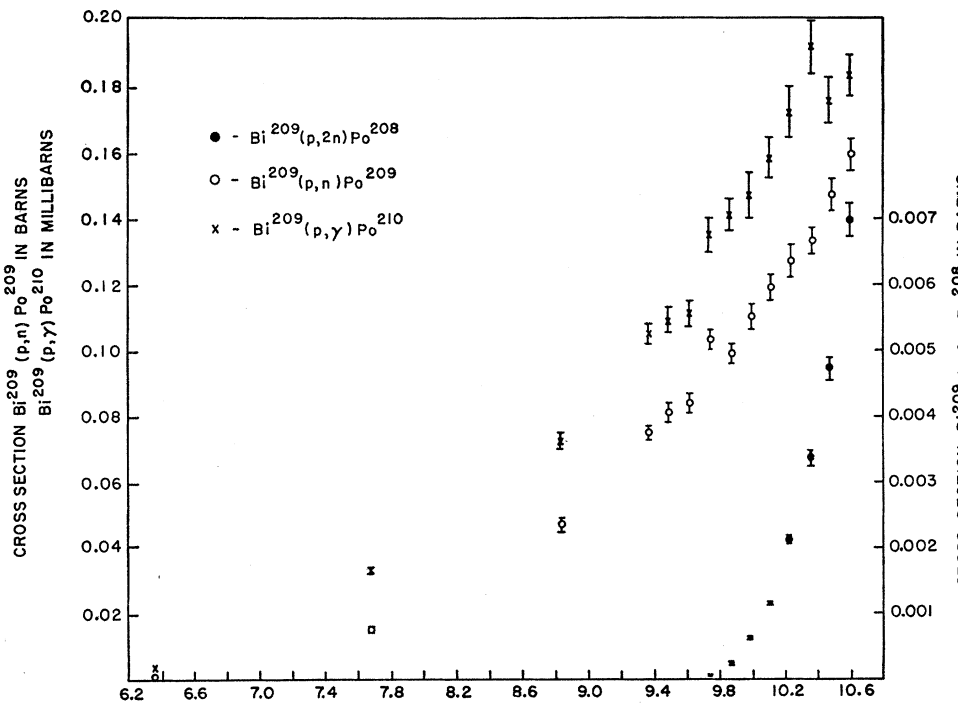

FIG. 6. Experimental values of the cross sections for the $Bi^{209}(p,2n)$-Po^{208}, the $Bi^{209}(p,n)Po^{209}$, and the $Bi^{209}(p,\gamma)Po^{210}$ reactions as a function of ϵ_p (c.m.). The errors indicated are the rms sum of standard deviations. [We used $T_{\frac{1}{2}}(Po^{209})$ equal to 103 yr, see text of paper.]

[10] Hughes, Garth, and Levin, Phys. Rev. **91**, 1423 (1953).
[11] H. Hurwitz, Jr., and H. A. Bethe, Phys. Rev. **81**, 898 (1951).
[12] D. B. Beard, Phys. Rev. **94**, 738 (1954); R. M. Eisberg, Phys. Rev. **94**, 739 (1954).
[13] D. H. Templeton, Phys. Rev. **78**, 312 (1950).
[14] This measurement was made by D. J. Hunt and G. Pish of the Mound Laboratory, Monsanto Chemical Company, and kindly communicated to us by G. R. Grove.
[15] Frank Asaro, Jr. (private communication).
[16] H. A. Tewes and R. A. James, Phys. Rev. **88**, 860 (1952).

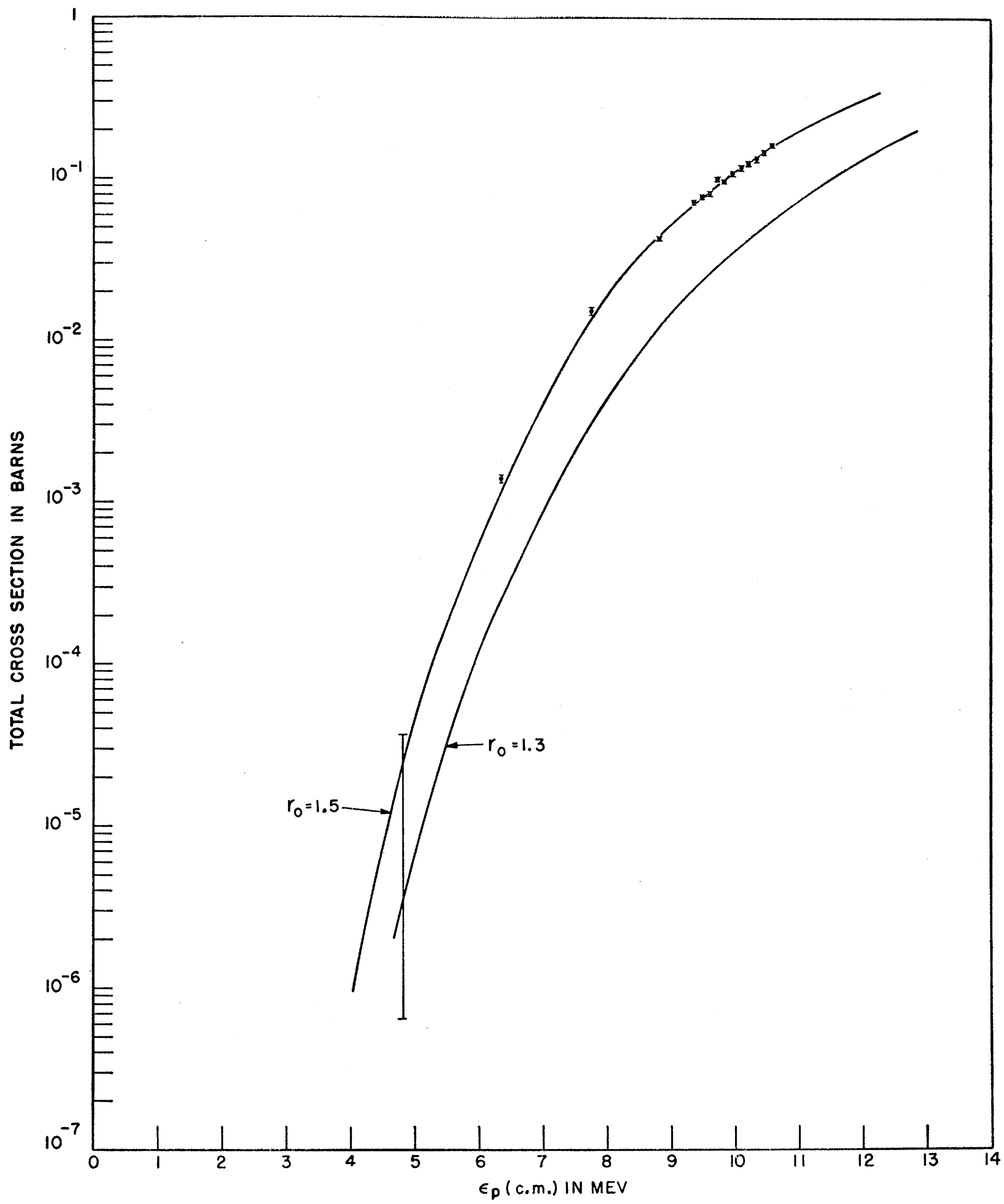

FIG. 7. The total cross section of protons on Bi^{209} compared with the theoretical values of Shapiro (see reference 14). $R=r_0A^{\frac{1}{3}}\times10^{-13}$ cm. The errors indicated are the rms sum of standard deviations. [We used $T_{\frac{1}{2}}(Po^{209})$ of 103 yr.]

section is approximately 69 millibarns at 10 Mev (c.m.), which compares with 65 millibarns of Shapiro[17] for $r_0=1.5$. Our experimental values agree closely with those of Shapiro for the same value of r_0. The total cross sections are shown in comparison with the theoretical cross-section curves of Shapiro in Fig. 7. Uncertainty in the cross section of the lowest energy point (4.8 Mev) arose from the difficulty in accurately pulse analyzing a plate with very low activity. The maximum value for the proton cross section at this point was calculated using the upper limit of the Po^{209} present from alpha pulse analysis, while the lower value was determined by assigning all the activity to Po^{210}, a product of the (p,γ) reaction. If any alpha activity were produced by deuteron contamination of the proton beam these values would be lowered. An upper limit of less than one deuteron per 10^5 protons can be calculated if we assume all of this alpha activity is due to deuteron reactions, where the cross sections of Kelly[9] at the appropriate deuteron energy were used.

An attempt to compare our results with Kelly[9] indicates a large difference in both the $(p,2n)$ threshold and the $(p,2n)$ cross section. The (p,n) and (p,γ) cross sections are approximately the same in the 10-Mev region, but our values decrease faster as one goes to lower energies. Kelly indicates a threshold of 10.1 Mev with an appreciable $(p,2n)$ cross section down to 8.5 Mev. Evaluating his data in terms of a Maxwellian energy distribution [i.e., in terms of Eq. (1)], he obtains

[17] M. M. Shapiro, Phys. Rev. **90**, 171 (1953).

a threshold of 10.1 Mev by neglecting the lower energy points, and a nuclear temperature of 1.17 Mev. Straggling due to the reduction in energy from 31 Mev to 10 Mev probably explains part of the difference.

The data in Table II were obtained from two separate bombardments. The absolute energy of the protons was not known for the first bombardment; so these data were fit into that of the second bombardment by matching threshold energies. The absolute amount of bismuth was known only for a few plates of Run I; so only the ratio $\sigma_{(p,2n)}/[\sigma_{(p,n)}+\sigma_{(p,2n)}]$ is given at many of the energies.

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Deformation Energy of a Charged Drop. I. Qualitative Features

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The qualitative features of the deformation energy of a charged drop are discussed with special reference to the fission process. It is shown that, under conditions more general than the model of an incompressible liquid drop with a sharp surface, the threshold energy for fission should be proportional to $[(Z^2/A)_0-(Z^2/A)]^3$ near the limit of stability $(Z^2/A)_0$. Similarly, if instability against asymmetry sets in below $(Z^2/A)_c$, the degree of asymmetry of the asymmetric saddle-point shapes which appear should be proportional to $[(Z^2/A)_c-(Z^2/A)]^{\frac{1}{2}}$ and the difference between the symmetric and asymmetric thresholds should be proportional to $[(Z^2/A)_c-(Z^2/A)]^2$. Some factors governing the stability against asymmetry of a strongly deformed drop are discussed qualitatively.

ACCORDING to the liquid-drop model, the process of fission is the result of a competition between the long-range electrostatic repulsion and the attractive short-range nuclear forces, idealized in the model as a surface tension.[1] Of importance for the theory of fission is the knowledge of the potential energy of such a system as a function of deformation. The present series of papers will be concerned with this problem. In the first part, we shall consider the qualitative features of the deformation energy.

A quantity of importance for the theory of fission is the ratio of electrostatic to surface energy which, for a nucleus, is approximately proportional to $(Z^2/A^{\frac{1}{3}})\div A^{\frac{2}{3}}=Z^2/A$. A charged drop for which this quantity is less than a certain critical value $[Z^2/A<(Z^2/A)_0]$ is stable against small deformations and the potential energy is an increasing function of the deformation. For larger distortions, a maximum will occur and the energy will decrease thereafter. The least energy necessary to divide the drop (corresponding to the height of the barrier along a suitable deformation path) is of importance in the discussion of fission thresholds. The shape of the drop when in the configuration of unstable equilibrium corresponding to the top of the barrier is also of interest, especially in connection with a discussion of fission asymmetry.

A general deformation of the drop may be specified by a number of deformation coordinates (in general infinite), which, in the case of an incompressible drop with a sharp boundary, could be taken as the coefficients in the expansion of the surface in spherical harmonics. Of special interest are configurations for which the potential energy is stationary with respect to all small distortions. It is convenient to restrict our attention from the beginning to configurations for which the energy is stationary with respect to all except a limited number of deformation parameters. In the case of axially symmetric configurations one may, for example, eliminate in this way all but two coordinates, one symmetric and one asymmetric, the latter specifying deviations from reflection symmetry. The deformation energy is then explicitly a function of two coordinates only and its properties can be discussed conveniently with reference to a deformation energy surface in three dimensions. The points where the energy is stationary with respect to the remaining two coordinates specify configurations for which the energy is stationary with respect to all deformations. The choice of the two coordinates is in principle arbitrary, but in practice it is advantageous to choose them so that they are capable of describing qualitatively the division of the drop into two equal or unequal fragments, the eliminated parameters being concerned with relatively less important features of the configuration. In the case in which the deformation parameters are the coefficients α_n in an expansion of the surface of the drop in Legendre polynomials, a convenient choice is

[1] N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).