be $(1060\pm 200)m_e$ by momentum-range measurements. These values are quite consistent with the precisely measured mean mass of $(958\pm 5)m_e$ for K^+ mesons[3] and we shall use this (latter) value in the discussion to follow.

We define a four vector $\tilde{P}_0$ with energy $E_0=E_++E_-=95\pm 10$ Mev, and momentum $\mathbf{P}_0=\mathbf{P}_++\mathbf{P}_-$, whose magnitude is (94.92 ± 10) Mev/c; we can consider this as the energy-momentum four vector of a fictituous particle of rest mass $m_0=[(\tilde{P}_++\tilde{P}_-)^2]^{\frac{1}{2}}=3.7$ Mev and velocity $\beta_0=P_0/E_0=0.9991$ emitted at an angle to the μ^+ meson whose cosine is -0.245. If we first assume that the decay is $K_{\mu 3}{}^+\rightarrow e^++e^-+\mu^++x$, application of the conservation laws determines the mass of x, $M_x=[(M_K-E_0-E_\mu)^2-(\mathbf{P}_0+\mathbf{P}_\mu)^2]^{\frac{1}{2}}$; we find $236<M_x$ (Mev) <265. Since no known particle of this mass exists, we conclude that the decay must involve at least two neutral particles; $K_{\mu 3}{}^+\rightarrow e^++e^-+\mu^++y+z$. If we do not wish to invent a new interaction, the most plausible assumption (and indeed the only one that need concern us if we exclude the materialization of a real γ ray) for the source of an electron pair of this energy in this type of event is that it arises from the alternate decay of the $\pi^0\rightarrow e^++e^-+\gamma$. We thus have $K_{\mu 3}{}^+\rightarrow\pi^0(\rightarrow e^++e^-+\gamma)+\mu^++z$. We can now set limits on the mass of z.

To obtain limits on M_z, we must first obtain limits on E_{π^0} and its angle of emission, θ, with respect to the resultant momentum $\mathbf{P}_0$ of the electron pair; $\cos\theta=\mathbf{P}_0\cdot\mathbf{P}_{\pi^0}/P_0P_{\pi^0}$. By squaring the equation $\tilde{P}_\gamma=\tilde{P}_{\pi^0}-(\tilde{P}_++\tilde{P}_-)$ (which represents the alternate π^0 decay) we obtain $\gamma(1-\beta\beta_0\cos\theta)=[(\mu_{\pi^0})^2+m_0{}^2]/(2\mu_{\pi^0}E_0)=A$, where $E_{\pi^0}=\gamma\mu_{\pi^0}$, $\beta=P_{\pi^0}/E_{\pi^0}$ and $\cos\theta$ is defined above. For the case $A<1$, which is of interest here, one can show by solving the above equation for β that $\beta\cos\theta\geqslant(1-A^2)^{\frac{1}{2}}$. We then obtain $\beta_{\max}=(1-A^2)^{\frac{1}{2}}$ for $\cos\theta=(1-A^2)^{\frac{1}{2}}/\beta_0$ and $\beta_{\min}=[\beta_0-A(A^2+\beta_0{}^2-1)^{\frac{1}{2}}]/(A^2+\beta_0{}^2)$ at $\cos\theta=1$. For our case, $0.645<A<0.796$ and we obtain $E_{\pi^0}(\min)=(\mu_{\pi^0}+3.5)$ Mev at $\cos\theta=1$ and $E_{\pi^0}(\max)=(\mu_{\pi^0}+74.5)$ Mev at $\cos\theta=0.763$. These limiting values of energy and angle are then used to obtain limits on M_z via the relation $M_z=[(M_K-E_\mu-E_{\pi^0})^2-(\mathbf{P}_\mu+\mathbf{P}_{\pi^0})^2]^{\frac{1}{2}}$. We find $0<M_z$ (Mev) <217. Thus it is possible for the missing neutral particle to be either a π^0 meson, a γ ray, or a neutrino.

The $K_{\mu 3}{}^+$ decay is thus $K_{\mu 3}{}^+\rightarrow\mu^++\pi^0+(\pi^0$ or γ or $\nu)$ if we restrict ourselves to three-body decays.[4] The existence of the π^0 meson among the decay products rules out the hypothesis[5] of a universal Fermi interaction to explain the $K_{\mu 3}$ (and K_{e3}) decay scheme. If the $K_{\mu 3}{}^+$ meson is a boson, as has been established for three of the four phenomenological decay modes, $K_{\pi 2}{}^+$ (θ^+), $K_{\pi 3}{}^+$ (τ^+), and $K_{\mu 2}{}^+$, then the unknown neutral particle must be a neutrino and the decay is $K_{\mu 3}{}^+\rightarrow\mu^++\pi^0+\nu$.

We are indebted to the many members of the University of California Radiation Laboratory for their assistance in making the exposure and to the Weitzmann Institute for the loan of the emulsions. We also wish to thank Dr. J. Blum and Mr. J. Klarmann for their assistance in the preparation of the emulsions and Mrs. J. Milks and B. Sherwood for their aid in scanning and tracing.

* This research was supported in part by the joint program of the U. S. Atomic Energy Commission and the Office of Scientific Research, U. S. Air Force.

† On leave from the Weitzmann Institute of Science, Rehovoth, Israel.

[1] In this way we rule out the possibility that one of the scatterings is a nuclear interaction of a π^+ meson from which a proton is ejected.

[2] This probability is less than $0.15\times 5\times 10^{-5}$. The factor 0.15 is the probability of not seeing a high-energy electron in our plates and the factor 5×10^{-5} is the upper limit on the relative probability of the decay $\pi^+\rightarrow e^++\nu$ as reported by S. Lokanathan and J. Steinberger, Phys. Rev. **98**, 240(A) (1955).

[3] D. M. Ritson (private communication). This value is relative to the τ-meson mass of $963m_e$; within experimental errors all K^+ mesons are found to possess the same mass.

[4] The designation $K_{\mu 3}$ is of course a phenomenological one, and there exists the possibility that those K particles classified as $K_{\mu 3}$ in reality represent more than one distinct decay mode. If they represent one distinct decay mode, the observation of a μ meson of kinetic energy greater than 75.7 Mev but less than 152 Mev (corresponding to the μ meson from the $K_{\mu 2}$ decay) would require the additional neutral particles (i.e., other than the π^0) to be massless.

[5] Kaplon, Klarmann, and Yekutieli, Phys. Rev. **99**, 1528 (1955).

Energy Dependence of the Optical Model Parameters*

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(Received November 4, 1955)

WE report some results of an analysis of proton elastic scattering at 5.25 and 31.5 Mev based on the diffuse surface optical model.[1] Together with results obtained previously at 17 Mev,[2] these provide some idea of the energy dependence of the optical model parameters. In this model, it is recalled, the nuclear part of the interaction between the incoming proton and the target nucleus is taken to be

$$-V(r)=\frac{V+iW}{1+\exp[(r-R_0)/a]},\tag{1}$$

and the Coulomb part of the interaction is chosen to be that appropriate to a uniform distribution of the nuclear charge over a sphere of radius R_0. We shall not present detailed comparisons between experimental and calculated cross sections at this time; we merely list the values of the optical model parameters which appear to give the best agreement with experiment.[3]

We first remark that R_0 and a, which describe the space dependence of the interaction, do not vary significantly with energy. Except for the lighter elements, we have found that R_0 is accurately given by $1.33A^{\frac{1}{3}}\times10^{-13}$ cm for all elements and for all energies considered while a is about 0.5×10^{-13} cm. Actually a does fluctuate somewhat but no systematic behavior has yet been discerned.

With respect to the remaining parameters V and W, our results are summarized in Table I.[4] In the table, V, W, and the incident energy E are given in Mev. The values at 17 and 31.5 Mev represent "averages" over the periodic table, neglecting elements much lighter than, say, Fe, since these behave erratically. The spread in the listed values simply indicates the variability of the parameters for the eight or ten elements studied, which ranged from Fe to Pb. On the other hand, the values at 5.25 Mev were obtained from an analysis of the scattering from Ni alone and from a preliminary look at neighboring elements. Consequently, these values may not be representative; our belief that they are is based only on our experience at the higher energies. Because of our lack of information, we have not attempted to indicate any variability at this energy.

First we remark on W, the imaginary part of the potential, which is seen to increase rapidly with energy, as expected. Indeed, as shown in Fig. 1, the results are

TABLE I. Energy dependence of the diffuse-surface optical-model parameters.

Proton incident energy, E (Mev)	Real part of nuclear potential, V (Mev)	Imaginary part of nuclear potential, W (Mev)
5.25	52.5	0.9
17	47 ± 1	8.5 ± 0.5
31.5	36 ± 1	15.5 ± 0.5

in remarkable qualitative agreement with the behavior predicted on the simple model of Lane and Wandel.[5] We note, but offer no explanation for, the rather small value of W we obtain at low energies compared to the value reported on the basis of a square well analysis of neutron total cross sections.[6] We also note that Sternheimer's analysis of proton polarization experiments yields values of W over the energy region from 50 to 130 Mev which connect reasonably well with our results.[7]

Finally, we remark on the behavior of V, the real part of the potential. As shown in Fig. 2, the values given here are such that they extrapolate to about 55 Mev at zero energy, a value somewhat larger than that deduced from analysis of neutron scattering data.[6,8] So far, it is not clear whether this is due entirely to the different radii and well shapes assumed in the different calculations or whether it might perhaps imply a deeper average potential for protons than for neutrons in heavy nuclei. However, our result appears reasonable on the basis of the following rough estimate of the potential felt by the most loosely bound nucleon in a

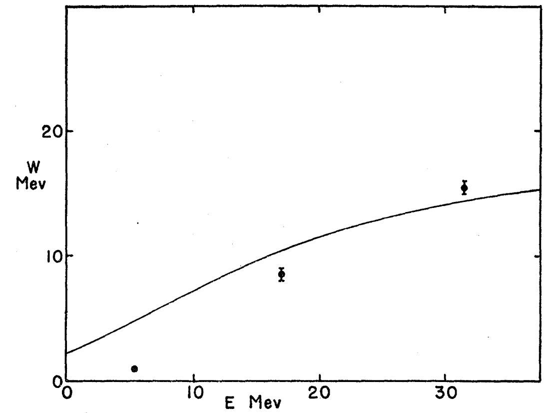

FIG. 1. Energy dependence of the imaginary part of the nuclear potential, W. The curve was calculated as in reference 5 for a square well radius $1.33A^{\frac{1}{3}}\times10^{-13}$ cm, while the points are those of Table I.

stable nucleus. The removal of this nucleon from the nucleus may be regarded as proceeding in two stages[9]; first the nucleon is removed and then the remaining nucleons are readjusted so that they occupy a slightly smaller volume corresponding to the original density. Using reasonable estimates of the nuclear compressibility,[10] it is found that the rearrangement energy is about 10 Mev. Thus the energy of the most loosely bound nucleon is not minus 8 Mev but more nearly minus 20 Mev. With the nuclear radii used here, this requires a well depth of about 50–55 Mev at an energy of minus 8 Mev. (If the rearrangement energy had been neglected, as is often done, the calculated well depth would be only 40–45 Mev.)

The empirical decrease of V with increasing energy exhibited in the table and in Fig. 2 seems consistent with other evidence[11] and is qualitatively reasonable on the basis of the saturation requirement that the interaction decrease for two-nucleon states of high relative energy. It is possible to interpret the observed energy dependence as roughly equivalent to a smaller

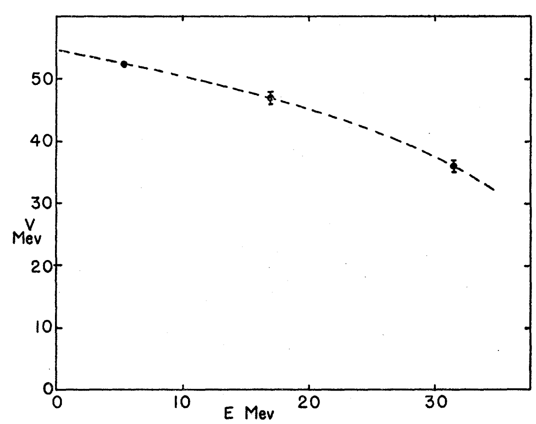

FIG. 2. Energy dependence of the real part of the nuclear potential, V.

effective mass of nucleons in nuclear matter. Our data imply an effective mass of about one-half the free nucleon mass, a value which has been previously mentioned in attempts to explain nuclear saturation and N/Z ratios in heavy nuclei.[9,12]

* It is a pleasure to acknowledge support from the National Science Foundation, the Office of Naval Research, and the Office of Ordnance Research.

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[3] The calculations were performed on the SWAC, Numerical Analysis Research Project, Department of Mathematics, University of California, Los Angeles, using the code prepared by Roger D. Woods.
[4] The experimental cross sections from which these results were determined were obtained by Bromley, Hashimoto, and Wall (Rochester) at 5.25 Mev, by Dayton and Schrank (Princeton) at 17 Mev and by Wright, Kinsey, and Leahy (Berkeley) at 31.5 Mev. We are indebted to each of these authors for providing us with his data prior to publication.
[5] A. M. Lane and C. F. Wandel, Phys. Rev. **98**, 1524 (1955). In this paper the nucleus is treated as a Fermi gas filling a sphere of radius R. We have extended their calculation to the case $R=1.33A^{\frac{1}{3}}\times10^{-13}$ cm and are presently examining the effects of rounding the nuclear surface.
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[7] R. Sternheimer, Phys. Rev. **100**, 886 (1955). In this paper a Thomas-type spin-orbit term is added to the potential of Eq. (1). Thus direct comparisons with our results may be misleading, particularly with respect to the real part of the potential. However, since absorption takes place over a large part of the nuclear volume and since the spin-orbit term is negligible except near the surface, it seems reasonable to expect the imaginary part of his potential to behave similarly to ours. We are indebted to Dr. Sternheimer for sending us his results prior to publication.
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Effect of Hard Core on the Binding Energies of H^3 and He^3

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(Received October 5, 1955)

THE nuclear two-body problems have been extensively studied in order to obtain some knowledge about nuclear forces. The next important sources of information about nuclear forces are the nuclei H^3 and He^3. Using two-body forces only, many authors have analyzed these three-body systems; the most reliable computation was that carried out by Pease and Feshbach,[1] assuming Yukawa potentials both for the central and tensor forces. Their results are as follows: The binding energy of H^3 can be reproduced by using reasonable force parameters as required by the two-body data, if we take a suitable tensor force range; however, the calculated energy difference between H^3 and He^3, which is regarded as mostly due to the Coulomb energy is too large by about 36%.

TABLE I. Binding energies of H^3 and energy differences between H^3 and He^3 computed with variational method.

Hard core radius in 10^{-13} cm	$r_{0s}=2.7\times10^{-13}$ cm B.E.(H^3) in Mev	Coulomb energy in Mev	$r_{0s}=2.4\times10^{-13}$ cm B.E.(H^3) in Mev	Coulomb energy in Mev
0.0	10.26	0.986	11.38	1.037
0.2	7.86	0.810	8.88	0.845
0.4	6.49	0.729	7.65	0.777
0.6	4.78	0.676	6.19	0.723
Experimental value	8.49	0.764	8.49	0.764

Recent investigations concerning nucleon-nucleon scattering[2] and nuclear saturation[3] have given evidence in favor of a hard-core interaction. However, it seems difficult to draw definite conclusions about the saturation character of the nucleus, owing to the great complexity of the many-body problem.

Calculation of the binding energy of H^3 with a hard-core potential has been carried out for the central part of the Lévy potential by Feshbach and Rubinow.[4] The purpose of the present note is to point out (1) the effect of the hard core on the Coulomb energy difference between H^3 and He^3 and (2) the dependence of the binding energy of H^3 on the radius of the hard core, matters which were not treated by these authors.[4]

The main purpose of the present note is to suggest that the hard-core interaction reduces the binding energy of H^3, and pushes the wave function out so that the Coulomb energy decreases to the experimental value. We expect that this investigation will also serve for the explanation of nuclear saturation.

We choose charge-independent central potentials of exponential type outside the hard cores, whose radii D are equal for both the singlet and triplet spin states. Tensor forces are not taken into account, because they require very laborious calculations. However, we believe that the effect of the hard core will be understood even if tensor forces are neglected. The potential parameters are so chosen as to agree with the binding energy of deuteron and the scattering lengths of the singlet and triplet spin states of the n-p system. As the effective range in the singlet state, the following two values are taken:

$$r_{0s}=2.7\times10^{-13}\text{ cm}\quad\text{and}\quad 2.4\times10^{-13}\text{ cm}.$$

The binding energy of H^3 is computed by the usual variational method assuming the following trial function. The notation is the same as reference 1, except that ρ is written here as r_3.)

$$\psi(r_1,r_2,r_3)=\begin{cases}\prod_{i=1}^{3}(e^{-\mu(r_i-D)}-e^{-\nu(r_i-D)})\cdot\chi & \text{for}\quad D\leqslant r_i\\ 0 & \text{otherwise,}\end{cases}$$