

be $(1060 \pm 200)m_e$ by momentum-range measurements. These values are quite consistent with the precisely measured mean mass of $(958 \pm 5)m_e$ for K^+ mesons³ and we shall use this (latter) value in the discussion to follow.

We define a four vector \bar{P}_0 with energy $E_0 = E_+ + E_- = 95 \pm 10$ Mev, and momentum $\mathbf{P}_0 = \mathbf{P}_+ + \mathbf{P}_-$, whose magnitude is (94.92 ± 10) Mev/c; we can consider this as the energy-momentum four vector of a fictitious particle of rest mass $m_0 = [(\bar{P}_+ + \bar{P}_-)^2]^{\frac{1}{2}} = 3.7$ Mev and velocity $\beta_0 = P_0/E_0 = 0.9991$ emitted at an angle to the μ^+ meson whose cosine is -0.245 . If we first assume that the decay is $K_{\mu 3}^+ \rightarrow e^+ + e^- + \mu^+ + x$, application of the conservation laws determines the mass of x , $M_x = [(M_K - E_0 - E_\mu)^2 - (\mathbf{P}_0 + \mathbf{P}_\mu)^2]^{\frac{1}{2}}$; we find $236 < M_x$ (Mev) < 265 . Since no known particle of this mass exists, we conclude that the decay must involve at least two neutral particles; $K_{\mu 3}^+ \rightarrow e^+ + e^- + \mu^+ + y + z$. If we do not wish to invent a new interaction, the most plausible assumption (and indeed the only one that need concern us if we exclude the materialization of a real γ ray) for the source of an electron pair of this energy in this type of event is that it arises from the alternate decay of the $\pi^0 \rightarrow e^+ + e^- + \gamma$. We thus have $K_{\mu 3}^+ \rightarrow \pi^0 (\rightarrow e^+ + e^- + \gamma) + \mu^+ + z$. We can now set limits on the mass of z .

To obtain limits on M_z , we must first obtain limits on E_{π^0} and its angle of emission, θ , with respect to the resultant momentum \mathbf{P}_0 of the electron pair; $\cos\theta = \mathbf{P}_0 \cdot \mathbf{P}_{\pi^0} / P_0 P_{\pi^0}$. By squaring the equation $\bar{P}_\gamma = \bar{P}_{\pi^0} - (\bar{P}_+ + \bar{P}_-)$ (which represents the alternate π^0 decay) we obtain $\gamma(1 - \beta\beta_0 \cos\theta) = [(\mu_{\pi^0})^2 + m_0^2] / (2\mu_{\pi^0} E_0) = A$, where $E_{\pi^0} = \gamma\mu_{\pi^0}$, $\beta = P_{\pi^0}/E_{\pi^0}$ and $\cos\theta$ is defined above. For the case $A < 1$, which is of interest here, one can show by solving the above equation for β that $\beta \cos\theta \geq (1 - A^2)^{\frac{1}{2}}$. We then obtain $\beta_{\max} = (1 - A^2)^{\frac{1}{2}}$ for $\cos\theta = (1 - A^2)^{\frac{1}{2}}/\beta_0$ and $\beta_{\min} = [\beta_0 - A(A^2 + \beta_0^2 - 1)^{\frac{1}{2}}] / (A^2 + \beta_0^2)$ at $\cos\theta = 1$. For our case, $0.645 < A < 0.796$ and we obtain $E_{\pi^0}(\min) = (\mu_{\pi^0} + 3.5)$ Mev at $\cos\theta = 1$ and $E_{\pi^0}(\max) = (\mu_{\pi^0} + 74.5)$ Mev at $\cos\theta = 0.763$. These limiting values of energy and angle are then used to obtain limits on M_z via the relation $M_z = [(M_K - E_\mu - E_{\pi^0})^2 - (\mathbf{P}_\mu + \mathbf{P}_{\pi^0})^2]^{\frac{1}{2}}$. We find $0 < M_z$ (Mev) < 217 . Thus it is possible for the missing neutral particle to be either a π^0 meson, a γ ray, or a neutrino.

The $K_{\mu 3}^+$ decay is thus $K_{\mu 3}^+ \rightarrow \mu^+ + \pi^0 + (\pi^0 \text{ or } \gamma \text{ or } \nu)$ if we restrict ourselves to three-body decays.⁴ The existence of the π^0 meson among the decay products rules out the hypothesis⁵ of a universal Fermi interaction to explain the $K_{\mu 3}$ (and $K_{e 3}$) decay scheme. If the $K_{\mu 3}^+$ meson is a boson, as has been established for three of the four phenomenological decay modes, $K_{\pi 2}^+$ (θ^+), $K_{\tau 3}^+$ (τ^+), and $K_{\mu 2}^+$, then the unknown neutral particle must be a neutrino and the decay is $K_{\mu 3}^+ \rightarrow \mu^+ + \pi^0 + \nu$.

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¹ In this way we rule out the possibility that one of the scatterings is a nuclear interaction of a π^+ meson from which a proton is ejected.

² This probability is less than $0.15 \times 5 \times 10^{-6}$. The factor 0.15 is the probability of not seeing a high-energy electron in our plates and the factor 5×10^{-6} is the upper limit on the relative probability of the decay $\pi^+ \rightarrow e^+ + \nu$ as reported by S. Lokanathan and J. Steinberger, Phys. Rev. **98**, 240(A) (1955).

³ D. M. Ritson (private communication). This value is relative to the τ -meson mass of $963m_e$; within experimental errors all K^+ mesons are found to possess the same mass.

⁴ The designation $K_{\mu 3}$ is of course a phenomenological one, and there exists the possibility that those K particles classified as $K_{\mu 3}$ in reality represent more than one distinct decay mode. If they represent one distinct decay mode, the observation of a μ meson of kinetic energy greater than 75.7 Mev but less than 152 Mev (corresponding to the μ meson from the $K_{\mu 2}$ decay) would require the additional neutral particles (i.e., other than the π^0) to be massless.

⁵ Kaplon, Klarmann, and Yekutieli, Phys. Rev. **99**, 1528 (1955).

Energy Dependence of the Optical Model Parameters*

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WE report some results of an analysis of proton elastic scattering at 5.25 and 31.5 Mev based on the diffuse surface optical model.¹ Together with results obtained previously at 17 Mev,² these provide some idea of the energy dependence of the optical model parameters. In this model, it is recalled, the nuclear part of the interaction between the incoming proton and the target nucleus is taken to be

$$-V(r) = \frac{V + iW}{1 + \exp[(r - R_0)/a]}, \quad (1)$$

and the Coulomb part of the interaction is chosen to be that appropriate to a uniform distribution of the nuclear charge over a sphere of radius R_0 . We shall not present detailed comparisons between experimental and calculated cross sections at this time; we merely list the values of the optical model parameters which appear to give the best agreement with experiment.³

We first remark that R_0 and a , which describe the space dependence of the interaction, do not vary significantly with energy. Except for the lighter ele-