Coulomb Interference in High-Energy Proton-Proton Scattering*

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The use of Coulomb interference in the phase-shift analysis of $p-p$ scattering is considered. To this end the Coulomb amplitude is calculated relativistically in the Born approximation, using a static anomalous magnetic moment. This amplitude is used to obtain formulas for the Coulomb interference contribution to the cross section and polarization. An application to the phase-shift analysis of the 200-Mev data is discussed.

I. INTRODUCTION

It has been recognized that interference between the electromagnetic and nuclear scattered amplitudes in proton-proton scattering at high energies provides additional information useful for the determination of nuclear phase shifts. In particular Breit and co-workers have given expressions for the cross section and polarization in terms of phase shifts for the case of a central Coulomb force.[1] This writer has reported an analysis of 200 Mev data in terms of s and p waves including Coulomb interference effects.[2] For this calculation an approximate relativistic formula was derived for the Coulomb scattering amplitude that takes account of the anomalous magnetic moment of the protons. While at 200 Mev the unpolarized cross section derived from this expression does not differ appreciably from that obtained using the Mott formula, the polarization differs considerably.

II. THE ELECTROMAGNETIC SCATTERING AMPLITUDE

We shall first calculate the Born approximation matrix for two Dirac particles with static anomalous magnetic moments. The protons interact with the electromagnetic field through an interaction Hamiltonian

$$H=-ei\beta\gamma_\mu A_\mu+\tfrac{1}{4}\mu_0\mu_p i\beta\sigma_{\mu\nu}F_{\mu\nu}, \tag{1}$$

where A_μ is the four-vector potential, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, $\sigma_{\mu\nu}=\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu$, $\mu_0=e\hbar/2Mc$, and μ_p is the proton's anomalous magnetic moment, i.e., $\mu_0(1+\mu_p)\boldsymbol{\sigma}$ is the total moment. The first term in Eq. (1) is the usual interaction between the charge and the electromagnetic field, and the second is the Pauli term describing the interaction of a static anomalous moment with the field. We obtain the Born approximation transition matrix by calculating the four diagrams of the type shown in Fig. 1, where one or the other of the two terms in H are used at the two vertices to emit or absorb the photon.

The resultant scattering amplitude may be written as a matrix as follows:

$$\begin{aligned}\mathcal{R}^{\mathrm{ord}}=\frac{e^2}{Mc^2}\frac{\epsilon}{Q^2}\{&(v_1{}^\dagger\gamma_\alpha u_1)(v_2{}^\dagger\gamma_\alpha u_2)+\tfrac{1}{4}\mu_p[(v_1{}^\dagger\xi_\alpha u_1)(v_2{}^\dagger\gamma_\alpha u_2)\\&-(v_1{}^\dagger\gamma_\alpha u_1)(v_2{}^\dagger\xi_\alpha u_2)]\\&-\tfrac{1}{16}\mu_p{}^2(v_1{}^\dagger\xi_\alpha u_1)(v_2{}^+\xi_\alpha u_2)\}.\end{aligned} \tag{2}$$

$\mathcal{R}^{\mathrm{ord}}$ is the amplitude for a final state in which particle $j(=1,2)$ has momentum p_j', spin m_j', and corresponding spinor v_j, when in the initial state particle j had momentum p_j, spin m_j, and spinor u_j; the transition being accomplished by the exchange of one photon. ϵMc^2 is the energy of each particle in the center-of-mass system, $Mc^2Q_\mu=(p_1)_\mu-(p_1')_\mu$, and $\xi_\alpha=iQ_\mu\sigma_{\mu\alpha}$. The superscript "ord" indicates that the formula is not antisymmetrized. The first term in Eq. (2) is simply the Møller formula.[3]

In order to calculate interference effects, $\mathcal{R}$ must be expressed in terms of the combined spin space of the two particles. To do this we take as basis states the positive energy spinors ψ_{m_j}, $m_j=\pm\frac{1}{2}$ (this in effect defines the "spin"), where the large components of $\psi_{\frac{1}{2}}$ and $\psi_{-\frac{1}{2}}$ are $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ respectively. The four components of the $\mathcal{R}$ matrix are thus designated

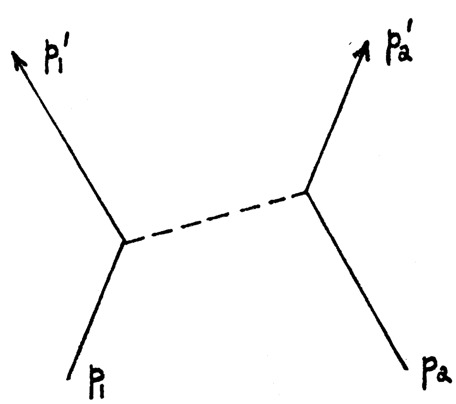

FIG. 1. Born approximation diagram.

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[1] J. Bengston and R. M. Thaler, Phys. Rev. 94, 679 (1954); G. Breit and J. B. Ehrman, Phys. Rev. 96, 805 (1954); and M. H. Hull, Jr., and A. M. Saperstein, Phys. Rev. 96, 806 (1954).

[2] A. Garren, Phys. Rev. 96, 1709 (1954).

[3] C. Møller, Ann. Physik 14, 531 (1932).

$\mathcal{R}_{m_1'm_2',m_1m_2}{}^{\mathrm{ord}}(\theta,\phi)$ for c.m. scattering angles θ, ϕ; where the z-axis is along the direction of incidence. $\mathcal{R}$ is then expressed in terms of singlet and triplet states $\psi_m{}^S$ which are the usual linear combinations of ψ_{m_1} and ψ_{m_2}, where $S=0$ refers to the antisymmetric and $S=1$ to the symmetric combination, and $m'=m_1'+m_2'$, $m=m_1+m_2$. Writing these matrix elements $\mathcal{R}_{m',m}{}^{S\ \mathrm{ord}}$ we have, for example, $\mathcal{R}_{1,-1}=\mathcal{R}_{\frac{1}{2}\frac{1}{2},-\frac{1}{2}-\frac{1}{2}}$. Finally the antisymmetrized matrix $\mathcal{R}_{m',m}{}^S$ is given by

$$\mathcal{R}_{m',m}{}^S(\theta,\phi)=\mathcal{R}_{m',m}{}^{S\ \mathrm{ord}}(\theta,\phi)+(-1)^S\mathcal{R}_{m',m}{}^{S\ \mathrm{ord}}(\pi-\theta,\ \pi+\phi). \quad (3)$$

When Eq. (2) is evaluated explicitly in this way, we obtain the following matrix elements for the p-p system. Let $\eta=e^2/\hbar V$, $V=$incident velocity in lab system, $\hbar k=$c.m. momentum of each proton, $mc^2\epsilon=$c.m. energy of each proton. Also let

$$a=\frac{2(\epsilon-1)}{2\epsilon^2-1},\quad \frac{\eta}{2k}=\frac{e^2}{4mc^2}\frac{2\epsilon^2-1}{\epsilon(\epsilon^2-1)},$$
$$\epsilon=[\tfrac{1}{2}(\gamma+1)]^{\frac{1}{2}},\quad \gamma=[1-V^2/c^2]^{-\frac{1}{2}},$$
$$s=\sin(\theta/2),\quad c=\cos(\theta/2),\quad x=\cot(\theta/2). \quad (4)$$

Then $\mathcal{R}$ may be written as follows:

$$\mathcal{R}^{\mathrm{ord}}=\frac{\eta}{2ks^2}[\Sigma+\Lambda+\mathrm{X}]. \quad (5)$$

Σ, Λ, and X are given in Eq. (5″) below, where we have written

$$\Lambda=\mu_pa(1+\epsilon)s^2L,\quad \mathrm{X}=\tfrac{1}{2}\mu_p{}^2a(1+\epsilon)s^2Y. \quad (5')$$

$$\begin{aligned}
&\Sigma_{00}{}^0=1,\\
&\Sigma_{11}{}^1=1-a[(2\epsilon+1)-\epsilon s^2]s^2,\\
&\Sigma_{00}{}^1=1-2a(\epsilon+s^2)s^2,\\
&\Sigma_{-10}{}^1=(a/\sqrt{2})e^{i\phi}s^2x[(2\epsilon+1)+2s^2],\\
&\Sigma_{01}{}^1=(a/\sqrt{2})e^{i\phi}s^2x[(2\epsilon+1)-2\epsilon s^2],\\
&\Sigma_{-11}{}^1=a\epsilon e^{2i\phi}s^2(1-s^2),\\
&L_{00}{}^0=1,\\
&L_{11}{}^1=-2\epsilon+(2\epsilon-1)s^2,\\
&L_{00}{}^1=-(4\epsilon-3)-2(3-2\epsilon)s^2,\\
&L_{-10}{}^1=\sqrt{2}xe^{i\phi}[\epsilon-(2\epsilon-3)s^2],\\
&L_{01}{}^1=\sqrt{2}xe^{i\phi}[\epsilon-(2\epsilon-1)s^2],\\
&L_{-11}{}^1=(2\epsilon-1)(1-s^2)e^{2i\phi},\\
&Y_{00}{}^0=2\epsilon^2-(\epsilon^2-1)s^2,\\
&Y_{11}{}^1=-1+(2\epsilon^2-2\epsilon+1)s^2-(\epsilon-1)^2s^4,\\
&Y_{00}{}^1=-2(\epsilon^2-1)-(3+4\epsilon-5\epsilon^2)s^2-2(\epsilon-1)^2s^4,\\
&Y_{-10}{}^1=\sqrt{2}e^{i\phi}s^2x[(2+\epsilon-2\epsilon^2)+(\epsilon-1)^2s^2]\\
&Y_{01}{}^1=-\sqrt{2}e^{i\phi}s^2x[\epsilon(2\epsilon-1)-(\epsilon-1)^2s^2],\\
&Y_{-11}{}^1=e^{2i\phi}(1-s^2)[(2\epsilon^2-1)-(\epsilon-1)^2s^2].
\end{aligned} \quad (5'')$$

Besides the matrix elements listed in Eq. (5), the following nonzero matrix elements exist:

$$\mathcal{R}_{-1-1}{}^1=\mathcal{R}_{11}{}^1,\quad \mathcal{R}_{10}{}^1=-\mathcal{R}_{10}{}^{1*},\quad \mathcal{R}_{0-1}{}^1=-\mathcal{R}_{01}{}^{1*},\quad \mathcal{R}_{1-1}{}^1=\mathcal{R}_{-11}{}^{1*}. \quad (5''')$$

A similar calculation may be carried out for the neutron-proton case. The result is as follows, where $\mu_0\mu_n\boldsymbol{\sigma}_n$ is the neutron's magnetic moment:

$$\begin{aligned}
&\mathcal{R}_{np}=-\frac{\eta}{2ks^2}[\Lambda'+\mathrm{X}']\\
&\Lambda'=\tfrac{1}{2}\mu_na(1+\epsilon)s^2[L+K],\\
&\mathrm{X}'=\tfrac{1}{2}\mu_n\mu_pa(1+\epsilon)s^2Y,\\
&K_{-10}{}^{10}=-K_{01}{}^{01}=-K_{10}{}^{10*}=K_{0-1}{}^{01*}=\sqrt{2}\epsilon xe^{i\phi}.
\end{aligned} \quad (6)$$

In Eq. (6) the notation $\mathcal{R}_{m'm}{}^{S'S}$ is used for elements that mix singlet and triplet states.

Since at high energies the Coulomb effects are only significant at small angles, it is often sufficiently accurate to consider in M_c only the terms proportional to θ^{-2} and θ^{-1}. When this is done one obtains the formula of reference 3:

$$\mathcal{R}=-\frac{\eta}{2k\sin^2(\theta/2)}\left[1-\nu i\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\mathbf{n}\sin\theta\right], \quad (7)$$

where

$$\nu=\frac{\epsilon-1}{2\epsilon^2-1}[(2\epsilon+1)+2\epsilon(\epsilon+1)\mu_p]$$
$$\mathbf{n}=(\mathbf{p}_1\times\mathbf{p}_1')/|\mathbf{p}_1\times\mathbf{p}_1'|.$$

The matrix $\frac{1}{2}i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}$ has four nonzero elements; namely, those connecting the triplet $m=0$ with the $m=\pm1$ states. Their value is $(1/\sqrt{2})(m'-m)e^{i(m-m')\phi}$. $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are Pauli spinors acting on the large components only.

This Born approximation formula, (5) or (7), may be multiplied by the nonrelativistic phase factor $\exp[i(-2\eta\ \ln s+2\eta_0)]$, $\eta_0=\arg\Gamma(1+i\eta)$, for this will cause the product to reduce to the exact nonrelativistic Coulomb amplitude at low energies. Of course this would also happen if the phase factor only multiplied the diagonal terms of $\mathcal{R}$, but comparison with the case of electrons scattered by a fixed Coulomb field suggests that it should multiply the spin-dependent terms as well.[4] This treatment of the phase factor is rather arbitrary, but around 200 Mev this nonrelativistic phase factor is close to unity in the Coulomb interference angular region, and hence this approximation might be reasonable. However, this point remains to be investigated further.[5]

[4] In the case of a particle with anomalous moment scattered by a fixed Coulomb field, it may be shown that

$$\mathcal{R}=-(\eta/2k)\exp[2i(-\eta\ \ln s+\eta_0)]s^{-2}[1-\nu_1 i\boldsymbol{\sigma}\cdot\mathbf{m}-2\nu_1s^2+O(\eta^2)],$$

where $\nu_1=[(\epsilon-1)/2\epsilon][1+(\epsilon+1)\mu_p]$, $\epsilon=[1-V^2/c^2]^{-\frac{1}{2}}$, and $\eta=ZZ'\times e^2/\hbar V$.

[5] *Note added in proof.*—Concerning these matters compare G. Breit, Phys. Rev. **99**, 1581 (1955); M. E. Ebel and M. H. Hull, Jr., Phys. Rev. **99**, 1596 (1955); S. Ohnuma and D. Feldman, Bull. Am. Phys. Soc. **30**, 65 (1955).

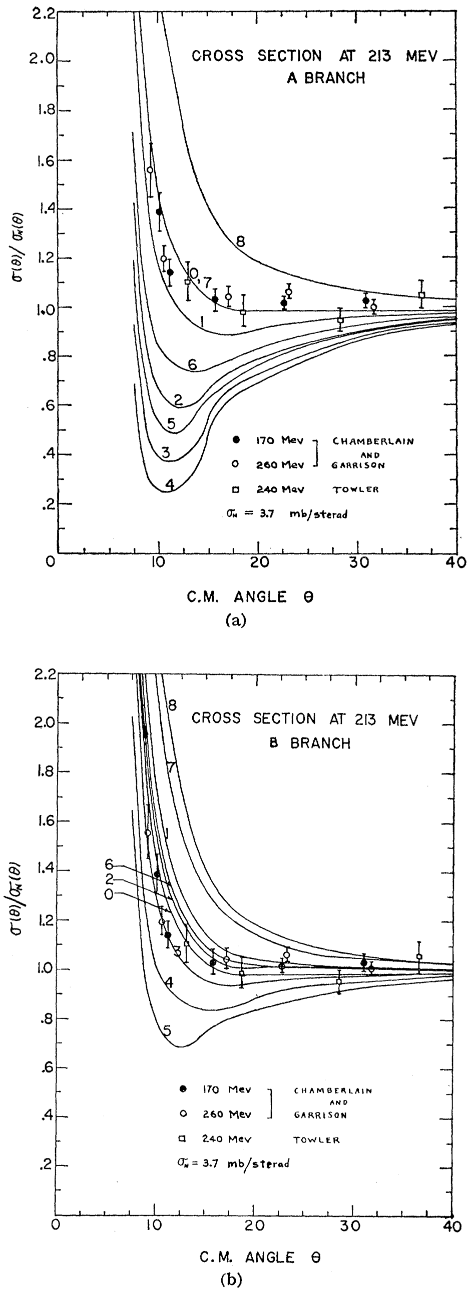

FIG. 2. Curves of differential cross section for the phase shift sets of Table I.

III. COULOMB-NUCLEAR INTERFERENCE

The contribution of the Coulomb interaction to the p-p cross section and polarization is obtained by taking

$$M=\mathcal{R}+\mathcal{N}. \tag{8}$$

In these calculations the expression of $\mathcal{N}$ in terms of phase shifts has been taken to be the same as for the M matrix[6] in the absence of Coulomb forces, except

[6] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).

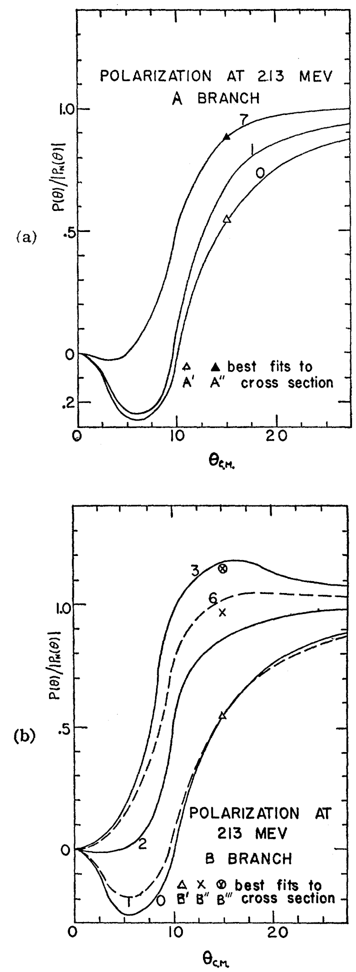

FIG. 3. Curves of polarization for the phase shifts of Table I that best fit the measured small-angle cross sections.

that each term containing $Y_{L,m}$ is multiplied by $e^{2i\eta_L}$, $\eta_L=\arg\Gamma(1+L+i\eta)$. This is no more than the nonrelativistic procedure. $\mathcal{R}$ is given by (5) or (7) multiplied by $\exp 2i(-\eta \ln s+\eta_0)$. Substituting M in the formulas of Wolfenstein and Ashkin[5] for the upolarized cross section σ and polarization P:

$$\sigma(\theta)=\tfrac{1}{4}\,\mathrm{Tr}MM^\dagger=\tfrac{1}{4}\sum_{S,m'm}|M_{m'm}{}^S|^2,$$

$$\sigma(\theta)P(\theta)\mathbf{n}=\tfrac{1}{4}\,\mathrm{Tr}MM^\dagger\boldsymbol{\sigma}_1=\mathbf{n}\frac{1}{2\sqrt{2}}\sum_{m=-1}^{1}\mathrm{Im}\times[M_{0m}{}^1(M_{1m}{}^{1*}e^{-i\phi}-M_{-1m}{}^{1*}e^{i\phi})], \tag{9}$$

we write

$$\begin{aligned}\sigma(\theta)&=\sigma_N(\theta)+\sigma_c(\theta)+\sigma_{\mathrm{int}}(\theta),\\ \sigma(\theta)P(\theta)&=\sigma_N P_N+\sigma_c P_c+(\sigma P)_{\mathrm{int}},\end{aligned} \tag{10}$$

where σ_N arises entirely from $\mathcal{N}$, σ_c entirely from $\mathcal{R}$, and σ_{int} from the cross terms, and likewise with the terms in σP. If we evaluate these terms explicitly and use the small-angle approximation (7) instead of (5),

TABLE I. Representative phase shifts (A branch) and corresponding parameters defined by Eq. 13 fitting the large-angle 200-Mev data: $\sigma(\theta)=3.56$ mb/sterad independent of θ with no Coulomb interference, $P(45°)=0.22$. Other possible sets (B branch) are obtained from those in this table by changing the signs of δ_0, B_0, and leaving other signs unchanged.

Set	δ_0	δ_1^0	δ_1^1	δ_1^2	A_0	B_0	A	B	C	D	E	F
A_0	0	−69.4	4.6	4.4	0	0	0.9249	0.2929	−1.7423	0.8013	1.8129	0.1166
A_1	17.6	−60.0	6.1	5.0	0.0909	0.2882	0.8219	0.3181	−1.4959	0.9832	1.5871	0.0588
A_2	39.4	−35.1	5.7	10.0	0.4022	0.4901	0.5110	0.6811	−0.5410	1.4994	0.9019	0.5527
A_3	38.7	−25.3	2.4	15.0	0.3906	0.4877	0.5228	0.9892	−0.0356	1.8972	0.8395	1.1028
A_4	29.2	−17.1	−2.0	20.0	0.2382	0.4261	0.6750	1.2213	0.4083	2.2737	0.9954	1.5830
A_5	8.2	0.0	−12.0	23.0	0.0202	0.1407	0.8930	1.1882	0.6337	2.4085	1.1984	1.9076
A_6	28.3	13.0	−18.5	15.0	0.2256	0.4181	0.6876	0.5665	−0.0683	1.7143	0.8722	1.2857
A_7	21.6	13.0	−26.0	10.0	0.1353	0.3421	0.7779	−0.1078	−0.5270	1.5987	0.8888	0.4534
A_8	0	0.0	−32.0	7.5	0	0	0.9277	−0.7012	−0.7573	1.9952	0.9617	−0.4423

we obtain

$$\sigma_{\text{int}}=-\eta(4ks^2)^{-1}\,\mathrm{Re}\{e^{2i\eta\ln s}[\mathrm{Tr}\mathfrak{M}-\nu\sqrt{2}\sin\theta\times(\mathfrak{M}_{10}{}^1e^{i\phi}-\mathfrak{M}_{01}{}^1e^{-i\phi})]\}$$

$$(\sigma P)_{\text{int}}=\eta(4ks^2)^{-1}\,\mathrm{Im}\{e^{2i\eta\ln s}[\sqrt{2}(\mathfrak{M}_{10}{}^1e^{i\phi}-\mathfrak{M}_{01}{}^1e^{-i\phi})+\nu\sin\theta(\mathfrak{M}_{11}{}^1+\mathfrak{M}_{00}{}^1-\mathfrak{M}_{1-1}{}^1e^{2i\phi})]\} \quad (11)$$

$$\sigma_c=(\eta/2ks^2)^2,\quad \sigma_cP_c=0.\text{[7]} \quad (12)$$

When σ_{int} and $(\sigma P)_{\text{int}}$ are expanded in terms of phase shifts and only s and p phase shifts are included, one obtains (again ignoring θ^2 terms compared to 1):

$$4k^2\sigma_{\text{int}}=2\eta[(A_0\mathcal{S}_0-B_0\mathcal{C}_0)+(A\mathcal{S}_1-B\mathcal{C}_1)\cos\theta],$$

$$4k^2(\sigma P)_{\text{int}}=\eta\sin\theta[(C+\nu E\cos\theta)\mathcal{C}_1+(D+\nu F\cos\theta)\mathcal{S}_1]. \quad (11')$$

Here $\mathcal{C}_L$ and $\mathcal{S}_L$ are respectively the real and imaginary parts of

$$s^{-2}\exp(2i\eta\ln s+i\eta_L-i\eta_0),$$

and the other parameters in (11′) are defined by Eq. (13) below:

$$\begin{aligned}
&A_0=\sin^2\delta_0,\\
&B_0=\tfrac{1}{2}\sin(2\delta_0),\\
&A=\sin^2\delta_1{}^0+3\sin^2\delta_1{}^1+5\sin^2\delta_1{}^2,\\
&B=\tfrac{1}{2}[\sin2\delta_1{}^0+3\sin2\delta_1{}^1+5\sin2\delta_1{}^2],\\
&C=-2\sin^2\delta_1{}^0-3\sin^2\delta_1{}^1+5\sin^2\delta_1{}^2, \qquad (13)\\
&D=\tfrac{1}{2}[-2\sin2\delta_1{}^0-3\sin2\delta_1{}^1+5\sin2\delta_1{}^2],\\
&E=2\sin^2\delta_1{}^0+3\sin^2\delta_1{}^1+7\sin^2\delta_1{}^2,\\
&F=\tfrac{1}{2}[2\sin2\delta_1{}^0+3\sin2\delta_1{}^2+7\sin2\delta_1{}^2],\\
&\delta_0=\delta({}^1S_0),\quad \delta_1{}^J=\delta({}^3P_J).
\end{aligned}$$

TABLE II. Phase shifts for s and p waves at 200 Mev that fit the large-angle cross section and polarization, and the small-angle cross section.

Set	δ_0	δ_1^0	δ_1^1	δ_1^2	$P(15°)$
$A'=B'(=A_0)$	0	−69	5	4	0.065
$A''(=A_7)$	37	−37	6	10	0.103
B''	−38	−31	4	12	0.112
B'''	−30	10	−16	17	0.132

[7] It is interesting to note that σ_cP_c does not vanish if we include higher order terms in θ, but because of the exchange terms in M_c one gets $\sigma_cP_c=8\nu(\eta/2k)^2\csc\theta\sin[2\eta\ln\tan(\theta/2)]$.

IV. APPLICATION TO THE ANALYSIS OF THE 200-Mev DATA

An example of the effect of Coulomb interference in a particular case is provided by the analysis in terms of s-and p-wave phase shifts of the 200-Mev data.[2,8] The large-angle cross section and polarization measurements limit the possible phase shifts to a one-parameter family with two branches.[2,9] Table I shows representative phase shifts on one of these branches, called A, arranged so as to permit interpolation between the entries, as well as corresponding values of the parameters defined by Eq. (13).

Using (10), (11′), (12) and the formulas of reference 7 for σ_N, $\sigma(\theta)/\sigma_N(\theta)$ has been calculated for the phase shifts of the two branches, A and B. These are shown in Fig. 2. The curves show that Coulomb interference discriminates rather well between the phase-shift sets of Table I. To obtain the best fit to the data, we have interpolated between the curves. Thus for the A branch sets A_0 and A_7 are quite good, for the B branch three sets are obtained: B_0, one obtained by interpolating between B_2 and B_3, the third by interpolating between B_5 and B_6. However A_0 and B_0 are identical (since $\delta_0=0$ for them), so there are four distinct phase shift sets that fit both the large-angle data and the cross section in the region of Coulomb interference. These are shown in Table II, which was also given in reference 2.

Referring to (11′), $\mathcal{S}_0$ and $\mathcal{S}_1$ are about 1/15 as large as $\mathcal{C}_0$ and $\mathcal{C}_1$, while $\mathcal{C}_0\sim\mathcal{C}_1$. Hence a measurement of Coulomb interference at this energy is mainly a measurement of B_0+B, i.e., of $\mathrm{Re}[\mathrm{Tr}\mathfrak{M}(0)]$. Spin-dependent terms in $\mathcal{R}$ do not appreciably affect $\sigma(\theta)$.

On the other hand, the spin-dependent terms (proportional to ν) give a large contribution to the interference term in the polarization. This can be seen by comparing C to νE in Table I, since these are respectively proportional to the leading spin-independent and spin-

[8] A. Garren, Phys. Rev. **92**, 213, 1587 (1953).

[9] An experiment of L. Marshall and J. Marshall at Chicago [Phys. Rev. **98**, 1398 (1955)], indicates that the sign of the polarization is positive. Hence two of the four branches explained in reference 2 are eliminated, and the remaining two, which we call A and B, differ only in the sign of δ_0.

dependent contributions to $(\sigma P)_{\text{int}}$ [see Eq. (11′) noting that at this energy $\nu \simeq 0.5$]. Figure 3 shows curves of P/P_N for phase shifts of Table I that best fit the small-angle cross sections, and show that good small-angle measurements of the polarization would also help in determining the phase shifts.

It should be remarked that it now seems unlikely that s and p waves suffice to describe the scattering at this energy, but this example shows the value of Coulomb interference for distinguishing between phase shifts that fit the large-angle data equally well.[10]

The writer takes pleasure in thanking Professor L. Wolfenstein who suggested these investigations for his very helpful advice and guidance, and Professor G. C. Wick and Professor J. Ashkin for their interest and suggestions.

[10] C. A. Klein [Nuovo cimento **1**, 581 (1955)] has carried out a somewhat similar phase-shift analysis.

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Effect of the Finite Size of the Nucleus on μ-Pair Production by Gamma Rays*

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The cross section for μ-pair production by gamma rays is calculated by using the Bethe-Heitler formula, taking into account the finite nuclear size. The cross section is shown to be considerably smaller than that for a point-charge nucleus.

INTRODUCTION

In recent years, experiments on the pair production of μ mesons by gamma rays have been performed[1] with the hope of further establishing the nature of the μ meson. The pair cross section is so small, however, that to date it has been possible to determine only the upper limits for its value. These seem to indicate that nuclear forces do not play a signficant role in the interaction of μ mesons with nuclei. Estimates given by Hough[2] and based on purely electromagnetic interaction give a value for the cross section that is about 20 times smaller than the most recently determined upper limit.[1]

Experiments now in progress at Stanford[3] are bringing the upper limit of the pair cross section close to Hough's estimate.[4] These experiments attempt to measure the cross section

$$d^2\sigma/d\Omega dE, \tag{1}$$

for obtaining one of the μ mesons of the pair in a given solid angle $d\Omega$ and with an energy between E and $E+dE$.

It is the purpose of this paper to present the results of a calculation (in Born approximation) of the cross section (1) on the basis of a purely electromagnetic interaction of the μ meson with the nucleus. This calculation consists in treating the μ meson as a heavy, spin-$\frac{1}{2}$, Dirac particle, and the nucleus as a static distribution of protons. The incoherent effects of the individual protons, the excitation of higher nuclear energy states, and nuclear recoil energies are neglected.[5]

Under these assumptions, the differential cross section for pair production is given, in Born approximation, by the Bethe-Heitler formula.[6] Because of the larger rest mass μ of the μ meson, the μ-pair cross section differs from the electron result by a mass scaling factor $\sim(1/207)^2$, and by a nuclear form factor. The nuclear form factor arises because, for μ-pair production, the recoil momentum q transferred to the nucleus can be so large that the associated wavelength,

$$\lambda\!\!\!^{-}_q = \frac{\hbar}{q} = \frac{1.865\times10^{-13}\text{ cm}}{(q/\mu c)},$$

becomes smaller than the nuclear radius. In this case the matrix element describing the interaction of the pair with the electric field of the extended nucleus becomes smaller than the point-charge matrix element because the regions of space that most contribute to it have dimensions smaller than $\lambda\!\!\!^{-}_q$.

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† C.B.P.F., Rio de Janeiro, Brazil, at Stanford under a Fellowship of the Brazilian National Research Council.

[1] Feld, Julian, Odian, Osborne, and Wattenberg, Phys. Rev. **96**, 1386 (1954); further references are given in this paper.

[2] P. V. C. Hough, Phys. Rev. **74**, 80 (1948); the approximations used in this paper are not applicable for the energy region near threshold for which the present calculations are performed.

[3] Masek, Lazarus, and Panofsky, Phys. Rev. **98**, 650(A) (1955); G. E. Masek and W. K. H. Panofsky, Phys. Rev. (to be published).

[4] As suggested in reference 2, an estimate of the effect of the nuclear form factor can be obtained by multiplying the integrated point—charge result by f^2 taken at the most probable value. For the ranges of the variable considered in this paper, this procedure gives a result too large by a factor of ~ 2.

[5] For example, the recoil energies occurring for a 500-Mev photon vary between 0.1 and 16 Mev for beryllium, and between 0.05 and 5 Mev for aluminum; the most probable values are approximately 0.6 and 0.2 Mev, respectively. In the case of a single proton, the minimum recoil energy is 1.2 Mev; the most probable values occur near 6 Mev.

[6] See Eq. (8) in the Appendix.