Reich-Kuhn value for electric dipole transitions for ordinary forces, and can be attributed to meson effects. A detailed discussion of the integrated cross section in terms of various sum rules is given by Levinger.[34]

ACKNOWLEDGMENTS

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Total Cross Sections for Scattering and Absorption of Pions by Nuclei*

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The causality conditions of Goldberger for the pion-nucleon scattering have been used to calculate the parameter k_1 of the optical model for scattering of pions from nuclei. These values of k_1 together with values of the absorption coefficient K in nuclear matter were used to obtain the total absorption and diffraction cross sections of pions for carbon, copper, and lead in the range 0–2.5 Bev.

IT has recently been shown by Karplus and Ruderman[1] and by Goldberger[2] that the real part of the forward scattering amplitude for the pion-nucleon scattering can be obtained from a knowledge of the π^+-p and π^--p total cross sections at all energies. This relation has been called the causality condition, and has been used by Anderson, Davidon, and Kruse[3] to calculate the real part of the forward scattering amplitude for the π^+-p and π^--p scattering for energies up to 240 Mev, using the measured total cross sections in the range 0–1.9 Bev. A knowledge of the real part of the forward scattering amplitude would enable one to calculate the total cross section for diffraction scattering of pions by nuclei, if one makes use of the optical model[4] of the nucleus.[5] In the present work, the parameters k_1 and K of the optical model are determined as a function of energy in the range 0–2.5 Bev, and the total pion cross sections are calculated for C, Cu, and Pb. It is assumed that k_1 and K have constant values in the interior of the nucleus and drop sharply to zero at the nuclear radius R which was taken as $1.4\times10^{-13}A^{\frac{1}{3}}$ cm.

The parameter k_1 which measures the change of wave number as the pion enters the nucleus is given by

$$k_1=2\pi\rho[ZD_{\pm}(k)+(A-Z)D_{\mp}(k)]/(kA), \quad (1)$$

where the upper and lower signs pertain to π^+ and π^- scattering, respectively; ρ = density of nucleons; k = wave number; $D_+(k)$ and $D_-(k)$ are the real parts of the forward amplitude for π^+-p and π^--p scattering, respectively. Goldberger, Miyazawa, and Oehme[2] have obtained the following equations for $D_+(k)$ and $D_-(k)$:[6]

$$D_+(k)=\frac{1}{2}\left(1+\frac{\omega}{\mu}\right)D_+(0)+\frac{1}{2}\left(1-\frac{\omega}{\mu}\right)D_-(0)+\frac{k^2}{4\pi^2}\int_\mu^\infty\frac{d\omega'}{k'}\left[\frac{\sigma_+(\omega')}{\omega'-\omega}+\frac{\sigma_-(\omega')}{\omega'+\omega}\right]+\frac{2f^2}{\mu^2}\frac{k^2}{\omega-(\mu^2/2M)}, \quad (2)$$

$$D_-(k)=\frac{1}{2}\left(1+\frac{\omega}{\mu}\right)D_-(0)+\frac{1}{2}\left(1-\frac{\omega}{\mu}\right)D_+(0)+\frac{k^2}{4\pi^2}\int_\mu^\infty\frac{d\omega'}{k'}\left[\frac{\sigma_-(\omega')}{\omega'-\omega}+\frac{\sigma_+(\omega')}{\omega'+\omega}\right]-\frac{2f^2}{\mu^2}\frac{k^2}{\omega+(\mu^2/2M)}, \quad (3)$$

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] R. Karplus and M. A. Ruderman, Phys. Rev. **98**, 771 (1955).
[2] M. L. Goldberger, Phys. Rev. **99**, 979 (1955); Goldberger, Miyazawa, and Oehme, Phys. Rev. **99**, 986 (1955).
[3] Anderson, Davidon, and Kruse, Phys. Rev. **100**, 339 (1955). I would like to thank Professor Anderson for sending me a copy of this paper in advance of publication.
[4] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949); H. A. Bethe and R. R. Wilson, Phys. Rev. **83**, 690 (1951).
[5] I would like to thank Dr. Piccioni for calling my attention to this point.
[6] The units are such that $\hbar=c=1$.

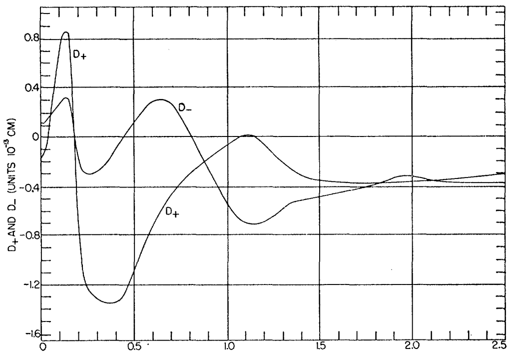

FIG. 1. Values of the forward scattering amplitudes D_+ and D_- for π^+-p and π^--p scattering, calculated using the low-energy phase shifts of Orear.[7] The values for pion energies $T_\pi \leqq 240$ Mev were obtained from the work of Anderson, Davidon, and Kruse.

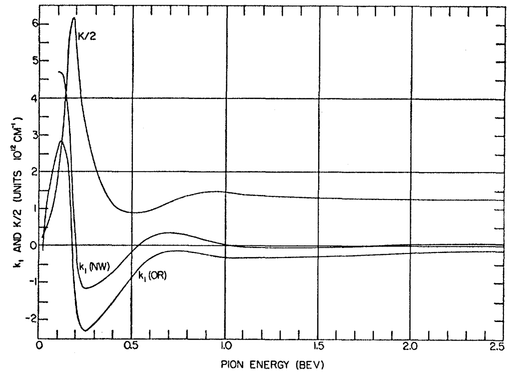

FIG. 2. Calculated values of k_1 and $K/2$ for π^+ or π^- mesons scattered from carbon. The curve of k_1 marked OR was obtained using the low-energy phase shifts of Orear[7]; the curve marked NW was obtained by means of the phase shifts of Noyes and Woodruff.[12]

where $\sigma_+(\omega')$ and $\sigma_-(\omega')$ are the total cross sections for π^+-p and π^--p scattering at the laboratory total energy ω'; ω is the total energy corresponding to the value of k; μ and M are the rest energies of the pion and nucleon, respectively. The principal part of the integrals over $(\omega'-\omega)^{-1}$ is to be taken. $D_+(0)$ and $D_-(0)$ are the forward scattering amplitudes at zero energy. In most of the calculations, we used the values of $D_+(0)$ and $D_-(0)$ obtained from the low-energy phase shifts of Orear,[7] namely $D_+(0)=-0.180\times10^{-13}$ cm and $D_-(0)=+0.114\times10^{-13}$ cm. The last term of (2) and (3) gives the contribution of the bound state; f is the coupling constant; for $2f^2$ the value 0.161 obtained by Chew[8] was used.

$D_+(k)$ and $D_-(k)$ were evaluated for kinetic energies T_π between 240 Mev and 2.5 Bev. Below 240 Mev, the results of Anderson *et al.*[3] were used. In order to evaluate the integrals of (2) and (3), the region from 0 to 2 Bev was divided into eight intervals, in each of which σ_+/k' and σ_-/k' were approximated by quadratic functions of T_π. The references for the experimental work on the measured cross sections are given by Anderson *et al.*[3] The values of σ_+ between 1.2 and 1.9 Bev were obtained from the difference of the π^--d and π^--p cross sections as measured by Clark *et al.*[9] by adding 5 mb[9] to take into account the deuteron shadowing effect.[10] For $T_\pi>1.9$ Bev, it was assumed that σ_+ and σ_- remain constant and equal to their values at 1.9 Bev which were taken as 28 mb and 30 mb, respectively. As discussed below, a slow variation of the cross sections for $T_\pi>1.9$ Bev would have a relatively small effect on the values of D_+ and D_- for $T_\pi \lesssim 2$ Bev. It may be noted that the integrals of (2) and (3) make the most important contributions to k_1. Thus, for carbon, k_1 is given by $\pi\rho[D_+(k)+D_-(k)]/k$. In the sum D_++D_-, the contributions of the bound state approximately cancel each other. Moreover, the term $D_+(0)+D_-(0)$ is quite small for the phase shifts of Orear[7] $(-0.066\times10^{-13}$ cm).

Figure 1 shows D_+ and D_- as calculated from Eqs. (2) and (3) using Orear's low-energy phase shifts. The values of D_+ and D_- for $T_\pi \leqq 240$ Mev were obtained from the work of Anderson *et al.*[3] In support of the extrapolation of $\sigma_\pm$ for $T_\pi>1.9$ Bev, we note that Bandtel *et al.*[11] have obtained a value of 30 mb for σ_- at 4.4 Bev. In order to determine the sensitivity of the results to this assumption, we have calculated the corrections ΔD_+ and ΔD_- to the forward amplitudes which would be introduced if σ_+ had, in fact, a maximum at 3 Bev. The extra term $\Delta\sigma_+$ was assumed to have a maximum value of 15 mb at 3 Bev and to go to zero at $T_\pi=2.5$ and 3.5 Bev. Thus the resulting σ_+ at 3 Bev would be 43 mb. Specifically, the extra term of σ_+/k' was taken as $\Delta\sigma_+/k'=-19.1T_\pi^2+114.6T_\pi-167.1$ for $2.5<T_\pi<3.5$ Bev, with $\Delta\sigma_+$ in millibarns, k' and T_π in Bev. It was found that $\Delta\sigma_+$ would lead to the following corrections ΔD_+ and ΔD_- (in units 10^{-13} cm): $\Delta D_+=0.006$ at $T_\pi=0.5$ Bev, 0.026 at 1 Bev, 0.075 at 1.5 Bev, 0.20 at 2 Bev, and 0.85 at 2.5 Bev. The corresponding values of ΔD_- are: 0.004 at 0.5 Bev, 0.012 at 1 Bev, 0.023 at 1.5 Bev, 0.036 at 2 Bev, and 0.049 at 2.5 Bev. Referring to Fig. 1, it is seen that ΔD_+ becomes important only for $T_\pi>1.5$ Bev, while ΔD_- is negligible throughout the range 0–2.5 Bev. The diffraction cross section for copper was calculated using $D_++\Delta D_+$ and $D_-+\Delta D_-$ and was found to differ from that obtained with D_+ and D_- by less than $0.01\pi R^2$ for the five energies listed above.

Figure 2 shows the resulting values of k_1 for carbon as a function of the energy T_π. The curve marked OR is based on the values of $D_\pm$ which are obtained from the phase shifts of Orear,[7] whereas the curve marked NW pertains to the phase shifts of Noyes and Woodruff[12] which give $D_+(0)=0.244\times10^{-13}$ cm and $D_-(0)=0.440$

[7] J. Orear, Phys. Rev. **96**, 1417 (1954).
[8] G. F. Chew, Proceedings of the Fifth Annual Rochester Conference, 1955 (Interscience Publishers, Inc., New York, 1955).
[9] Clark, Cook, and Piccioni, Proceedings of the Fifth Annual Rochester Conference, 1955 (Interscience Publishers, Inc., New York, 1955) and private communication.
[10] R. J. Glauber, Phys. Rev. **99**, 630 (1955).
[11] Bandtel, Bostick, Moyer, Wallace, and Wikner, Phys. Rev. **99**, 673 (1955).
[12] H. P. Noyes and A. E. Woodruff, Phys. Rev. **94**, 1401 (1954).

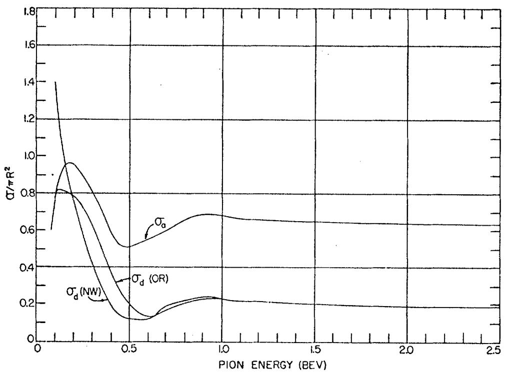

FIG. 3. Total cross sections for absorption σ_a and for diffraction scattering σ_d for π^+ or π^- mesons scattered from carbon. The curve of σ_d marked OR was obtained using the low-energy phase shifts of Orear;[7] the curve marked NW was obtained by means of the phase shifts of Noyes and Woodruff.[12]

$\times 10^{-13}$ cm. It may be noted that the values of D_+ and D_- for $T_\pi \leqq 240$ Mev, as calculated by Anderson *et al.*,[3] are in better agreement with experiment when the calculation is done by means of Orear's[7] values of $D_+(0)$ and $D_-(0)$ than with the values of Noyes and Woodruff.[12] Figure 2 also shows the values of the reciprocal mean free path K, which is given by

$$K = \rho[Z\sigma_\pm + (A-Z)\sigma_\mp]/A, \tag{4}$$

where the upper and lower signs pertain to π^+ and π^- mesons, respectively. For carbon, $K=\rho(\sigma_+ + \sigma_-)/2$. The nuclear radius R was taken as $1.4\times 10^{-13}\,A^{\frac{1}{3}}$ cm so that $\rho = 0.87\times 10^{38}$ cm^{-3}. The large peak of K at 180 Mev and the weak maximum at 900 Mev reflect the maxima of the cross sections at these energies. The change of sign of k_1 at 180 Mev is caused by the change of sign of D_+ and D_- which has been pointed out by Karplus and Ruderman[1] and by Anderson *et al.*[3] In the neighborhood of the maximum of σ_+ and σ_- the integral over $(\omega'-\omega)^{-1}$ predominates and gives a large positive term for $T_\pi < 180$ Mev and a large negative term for $T_\pi > 180$ Mev. The weak maximum of σ_- at 900 Mev has a similar effect. It gives rise to positive values of D_- from 430 to 810 Mev (see Fig. 1) which nearly cancel the negative contribution to k_1 from D_+, so that $|D_+ + D_-|$ has a minimum in this region. The fact that k_1 is negative for $T_\pi > 180$ Mev implies that the nucleus acts effectively as a repulsive potential for pions at these energies. From the values of $k_1(OR)$ one finds that the effective real potential V reaches a minimum of -49 Mev at $T_\pi = 120$ Mev. V becomes positive near $T_\pi = 180$ Mev and attains a maximum of $+42$ Mev at $T_\pi = 240$ Mev. Beyond 600 Mev, V is less than 10 Mev.[13]

Figure 3 shows the total cross sections for absorption σ_a and for diffraction scattering σ_d for carbon, as a function of T_π. The values of σ_a and σ_d were obtained from K and k_1 by means of the expressions given by the optical model.[4] The two curves of σ_d marked OR and NW pertain to the low-energy phase shifts of Orear[7] and of Noyes and Woodruff,[12] respectively. Because of the large value of $D_+(0)+D_-(0)$ for the Noyes-Woodruff phase shifts, $\sigma_d(NW)$ is large at low energies, but as is shown by Fig. 3, the values of σ_d for the two choices of phase shifts are practically the same for $T_\pi > 0.8$ Bev. Figures 4 and 5 show σ_a and σ_d for π^+ and π^- mesons scattered from copper and lead. All values are given in terms of πR^2 which is 324 mb for carbon, 985 mb for copper, and 2159 mb for lead. For copper and lead, k_1 and K are slightly different for π^+ and π^-, and correspondingly σ_a and σ_d show a small difference. In addition, an attempt was made to include the Coulomb effect in obtaining σ_a for copper and lead. According to a well-known formula, the Coulomb effect multiplies the cross sections by a factor $F_c = 1 \mp Ze^2/RT_\pi$ at low energies, where the upper and lower signs refer to π^+ and π^-, respectively. F_c is obtained from a consideration of the impact parameter b of the trajectory which just grazes the edge of the nucleus; b is given by $R(1\mp Ze^2/2RT_\pi)$. Courant[14] has given the appropriate expression for relativistic energies: $F_c = 1\mp 2Ze^2/RE$, where E is the total energy of the pion. In the intermediate region, F_c is given by

$$F_c = 1 \mp 2Ze^2E/(Rc^2p^2), \tag{5}$$

where p is the pion momentum. For this correction to be valid, it is necessary that the pion wavelength $\lambda\!\!\!^{-}$ be small compared to R, so that classical considerations can be applied. The same condition, $\lambda\!\!\!^{-} \ll R$, determines the validity of the WKB method[4] used in calculating σ_a and σ_d. For 100-Mev pions, $\lambda\!\!\!^{-}$ is 1.01×10^{-13} cm, which is appreciably smaller than the radius R for carbon. However, below 100 Mev, the results of Figs. 3–5 are expected to be only qualitatively correct. In addition, it should be noted that the present values of

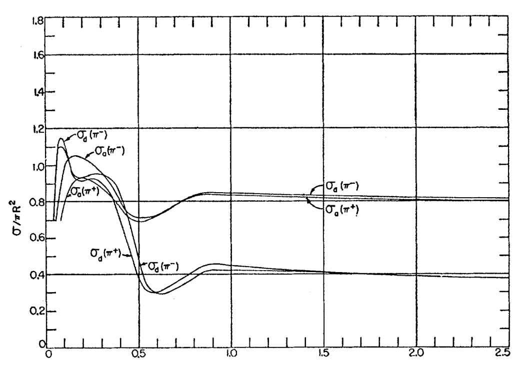

FIG. 4. Total cross sections for absorption σ_a and for diffraction scattering σ_d for π^+ and π^- mesons scattered from copper. The values of σ_d were obtained by means of the phase shifts of Orear.[7]

[13] It should be noted that the wave number k to be used in Eq. (1) should properly be the wave number of a pion inside the nucleus, i.e., of a pion having kinetic energy $T_\pi - V$. Since V is very small compared to T_π except in the resonance region, this substitution would have a negligible effect at high energies ($T_\pi \gtrsim 500$ Mev).

[14] E. D. Courant, Phys. Rev. 94, 1081 (1954).

σ_a do not include the absorption of the pion by a pair of nucleons in the nucleus (with star formation) which becomes important below 100 Mev.[15] This process predominates for very low pion energies ($\lesssim 40$ Mev) but is expected to become relatively small above 150 Mev. In connection with the difference between $\sigma_a(\pi^+)$ and $\sigma_a(\pi^-)$ in Figs. 4 and 5, Courant[14] has shown that if the neutron distribution is more extended than the proton distribution, as has been suggested by Johnson and Teller,[16] there will be an additional correction to the cross sections which is of the same order as the Coulomb effect.

Figures 3–5 show that σ_a has maxima near 180 Mev and 900 Mev, which are due to the maxima of the elementary cross sections σ_+ and σ_-. At 180 Mev the values of σ_a uncorrected for the Coulomb effect are close to πR^2 for all three elements. However, the maximum at 900 Mev is higher for lead than for carbon ($0.91\pi R^2$ as compared to $0.69\pi R^2$) because of the larger radius for lead which is responsible for more effective absorption. In a similar manner, σ_a at high energies ($T_\pi > 2$ Bev) increases with increasing A from $0.63\pi R^2$ for carbon to $0.89\pi R^2$ for lead. It would be of interest to observe the maximum of σ_a in the 180-Mev region, as well as the minimum at 500 Mev, which is quite pronounced for carbon. Concerning σ_d, we note that for copper and lead there are two maxima in the low-energy region ($T_\pi < 500$ Mev) which are separated by a minimum at 180 Mev. The peak near 70 Mev is very noticeable both for copper and lead, while the second maximum at 300–400 Mev is conspicuous only for π^- mesons on lead. It is also seen that the average values of σ_d in the low-energy region are appreciably larger than for $T_\pi > 500$ Mev. The double maximum of σ_d can be attributed to the behavior of k_1, which becomes zero at 180 Mev. Since σ_d depends only on the absolute value of k_1 and increases with increasing $|k_1|$, the cross section increases on each side of the zero of k_1. Moreover, σ_d increases with increasing K, which explains why the values are higher in the resonance region than at larger energies. The weak maximum of σ_d at 900 Mev is also due to this effect. It would be of interest to observe the double peak of σ_d for the heavy elements.[17] Moreover,

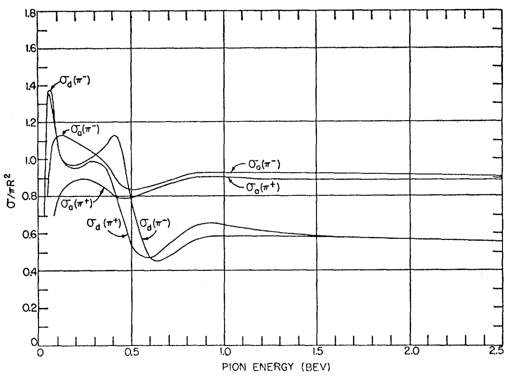

FIG. 5. Total cross sections for absorption σ_a and for diffraction scattering σ_d for π^+ and π^- mesons scattered from lead. The values of σ_d were obtained by means of the phase shifts of Orear.[7]

the decrease of σ_d above the resonance region is quite pronounced and should be easily observable. Experimentally, one would probably obtain values of σ_d by a separate measurement of the total cross section σ_t in "good" geometry and of the absorption cross section σ_a in "bad" geometry, i.e., such that diffraction scattering is not detected. Thus $\sigma_\alpha = \sigma_t - \sigma_a$.

Kessler and Lederman[18] have measured the absorption cross section for 125-Mev negative pions on carbon and lead. With $R = 1.4\times 10^{-13}\, A^{\frac{1}{3}}$ cm, their data give $\sigma_a/\pi R^2 = 0.95\pm 0.13$ for carbon and 1.15 ± 0.18 for lead. Within the experimental errors, the calculated values agree with these results. In comparing the calculations with experiment, it should be noted that the present values are slightly affected by the uncertainty about the nuclear radius and by the fact that one should use a rounded edge for the nucleus in an exact calculation. However, these effects would probably introduce only minor corrections.

I would like to thank Dr. O. Piccioni for suggesting this calculation and for helpful discussions. I am also indebted to Dr. R. Jastrow, Dr. L. M. Lederman, and Dr. R. Serber for valuable comments.

[15] Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).

[16] M. H. Johnson and E. Teller, Phys. Rev. **93**, 357 (1954).

[17] The fact that the double peak of σ_d is present for copper and lead, but not for carbon, can be explained as follows. σ_d depends essentially on[4] $\sigma_a/\pi R^2$ and on $|k_1|R$. For copper and lead, σ_a remains approximately constant and very high ($>0.95\pi R^2$) over a considerable range of energies, and the minimum at 500 Mev is not very deep, while for carbon the region of large σ_a is limited, and σ_a drops to a rather small value at 500 Mev. Thus for carbon the decrease of σ_a on each side of the maximum at 180 Mev counteracts the increase of $|k_1|R$, whereas for the medium and heavy elements, the variation of $|k_1|R$ predominates and gives rise to the double peak.

[18] J. O. Kessler and L. M. Lederman, Phys. Rev. **94**, 689 (1954).