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Origin of Cosmic Radiations*

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Based upon recent developments of hydromagnetic turbulence, the interaction of a cosmic-ray particle and a moving interstellar cloud is investigated. It is found that the particle can be accelerated by an inductive mechanism with a comparable efficiency to Fermi's mechanisms of collision. However, the efficiency is the highest when the particle is trapped between two interstellar clouds and is accelerated by the combined action of the two mechanisms in a "push-pull" manner. It is suggested that cosmic radiations are primarily accelerated by such a "push-pull" mechanism in some favorable regions in our galaxy.

I. INTRODUCTION

RECENT measurements of cosmic radiation have shown that the protonic and the nuclear components have very much the same energy spectrum.[1] Consequently the lifetime of a heavy cosmic-ray nucleus in interstellar space cannot exceed five million years. This consequence of the similarity of the energy spectra required therefore a mechanism which is capable of accelerating a charged particle to an extremely high energy in a relatively short time and in such a manner that their age distribution is independent of nuclear charge.

To meet this demand, Fermi proposed that cosmic radiations were accelerated in the trapping magnetic fields along the spiral arm of our galaxy.[2] The general galactic magnetic field is almost parallel to the spiral arm and its intensity is about 6×10^{-6} gauss.[3] Owing to the turbulent motion of the interstellar medium, the lines of force are irregularly pushed back and forth and also the field intensity fluctuates along the spiral arm. Fermi showed that a cosmic-ray particle, spiraling along the lines of force of the general magnetic field H_0 with an angle θ_0 between its momentum vector and the direction of the field (angle of pitch), will reverse its direction of motion when it encounters a region with its maximum field intensity H_m greater than $H_0/\sin^2\theta_0$. In a variable magnetic field, the reversal of the motion of the particle will result in a change of energy. As a rule, the energy will increase or decrease according to whether the turbulent region that causes the reversal moves toward the particle (head-on collision) or away from it (overtaking collision). Let B be the eddy velocity, taken to be positive for a head-on collision and negative for an overtaking collision, and β be the velocity of the particle, both being measured in units of the velocity of light. Fermi showed that the energy gain Δw is given by[4]

$$\Delta w=(2B\beta\cos\theta_0+2B^2)w/(1-B^2)\approx(2B\beta\cos\theta_0)w. \quad (1)$$

This will be called an F-gain for short. Therefore, if a particle is trapped inside two such regions, say, 100 light years apart, with the jaws of the trap moving toward each other at a velocity of 20 km/sec, the particle will have an energy increase of 10% in 3×10^5 years.

With the energy gain, the value of $\sin\theta_0$ decreases proportionally.[2] The acceleration process thus proceeds until θ_0 reaches its minimum value χ, which is defined by

$$\sin^2\chi=H_0/H_m; \quad (2)$$

then the particle will escape the trap.

When this condition is reached, the process of acceleration becomes exceedingly slow. The particle would then keep on spiraling along the magnetic lines of force and be capable of passing without reflection through occasional maxima of field intensity along its path. This condition continues until it encounters a region with a sharp variation in magnetic field where the angle of pitch could be converted back to a larger value and the trap action then be resumed.

It was first suggested by Davis that cosmic radiations could also be accelerated in the spiral arms of our galaxy by the inductive action of varying magnetic fields.[5] The essential difference between Fermi's acceleration mechanism and the inductive mechanism is that the former increases the momentum component parallel to the magnetic field while the later increases the normal component. In other words, an energy gain by Fermi's mechanism results in a decrease of the value of $\sin\theta_0$ reducing the trapping action of two turbulent regions, while the effect of an inductive action is just the reverse. Therefore, if the turbulent motions of the interstellar clouds are accompanied with corresponding time-variations in the galactic magnetic fields, and if the energy gain (which we shall call an I-gain) resulting from the inductive action during an encounter with one of those clouds is greater than the F-gain, then the inductive action will result in θ_0 increasing and the cosmic-ray particle becoming stable in the trap and being very efficiently accelerated. We shall see that this

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[1] Kaplon, Peters, Reynolds, and Ritson, Phys. Rev. **85**, 295 (1952).
[2] E. Fermi, Astrophys. J. **119**, 1 (1954).
[3] S. Chandrasekhar and E. Fermi, Astrophys. J. **118**, 113 (1953).
[4] E. Fermi, Phys. Rev. **75**, 1169 (1949).
[5] L. Davis, Jr., Phys. Rev. **93**, 947 (1954).

will happen whenever $\chi < \theta_0 < \theta_c$, where θ_c is a critical value of θ_0.

In this paper, which is based upon Chandrasekhar's theory of turbulence,[6] a general mechanism of acceleration along the foregoing line is presented. The similarity of the energy spectra of different components can also be explained.

II. POSSIBLE CHARACTER OF INTERSTELLAR MAGNETIC FIELD

Recently Chandrasekhar has developed a statistical theory of hydromagnetic turbulence.[6] In his theory, the process of energy transfer between different Fourier components (specified by the wave number k) of the velocity field and the magnetic field is visualized in terms of two suitably defined eddy coefficients, ν_k, describing the transfer of kinetic energy between the different modes, and λ_k, similarly describing the transfer of energy between the kinetic and the magnetic forms. The essential results of this theory are that for small k, (i.e., for large eddies), there is an equipartition between the kinetic and the magnetic energies, while for large k, (i.e., for small eddies), the solution of the problem allows two possibilities: either there is no magnetic energy in the high Fourier components or the magnetic energy is about 2.5 times the kinetic energy. These two modes are distinguished as the velocity and the magnetic modes respectively. Two schematic representatives of the field intensity in interstellar clouds (of linear dimension L) for these two modes are shown in Fig. 1.

The occurrence of the two modes is interpreted as two extreme cases of the conditions of the turbulent medium. The former case arises when the viscosity provides the principal source of energy dissipation while the latter case arises when joule heating provides the means of energy dissipation. Under the interstellar conditions, the kinematic viscosity ν is so much greater than the resistivity, $1/4\pi\sigma$, that the velocity mode is the one which is most likely to be realized. As will be shown in the following section, this is the basic point upon which the proposed mechanism of acceleration is based.

III. MECHANISM OF ACCELERATION

There are basic reasons for the requirement of a large eddy size. First of all, large eddy size is needed for the trapping action. Also, in large eddies, the motion of a cosmic-ray particle can be described as in a uniform magnetic field, regarding the time and space variation of the fields as a small perturbation. This greatly simplifies the mathematical formulation. A cosmic-ray particle of 10^{15} ev energy describes loops of 1/10 parsec in radius in a magnetic field of 10^{-5} gauss. (1 parsec = 3.26 light years.) Therefore, the size of interstellar clouds, 10–50 parsecs in linear dimension, does satisfy the requirement.

[6] S. Chandrasekhar, Proc. Roy. Soc. (London) (to be published).

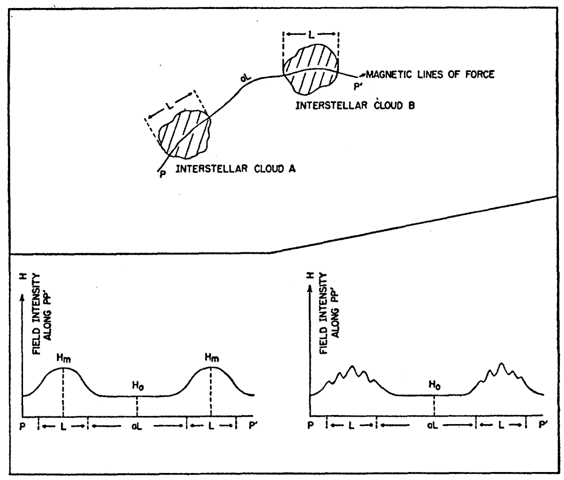

FIG. 1. (a) Velocity mode. (b) Magnetic mode.

Consider a cosmic-ray particle with a charge e spiraling around the magnetic lines of force linking two clouds each of linear dimension L, and approaching one of them. Let θ be the angle between the momentum vector p of the particle and the direction of the magnetic field H (the angle of pitch), and θ_0 the value of θ when the particle is in the general magnetic field H_0 between clouds. The radius of the spiral orbit the particle describes is

$$\rho = pc\sin\theta/eH. \tag{3}$$

During the motion the quantity

$$q = \sin^2\theta/H = \sin^2\theta_0/H_0 \tag{4}$$

is approximately constant.[7] If the maximum field intensity in the clouds, H_m, is greater than $H_0/\sin^2\theta_0$, the particle will reverse its motion where the field intensity is equal to $H_0/\sin^2\theta_0$. The change of the energy caused by this reversal is

$$\Delta w = (2B\beta\cos\theta_0 + 2B^2)w/(1-B^2) \approx (2B\beta\cos\theta_0)w,$$

where $c\beta$ and cB are the velocity of the particle and the velocity of the cloud in the direction of H, respectively, and w is the energy of the particle including its rest mass. This is what we have called the F-gain.

Accompanied with the mass motion of the clouds, there is a net flow of kinetic energy into the magnetic energy. The amount of the flow is given by the equation derived by Chandrasekhar[8] as

$$\frac{1}{2}\frac{d}{dt}(H^2) = 2.5H^2\left(\frac{\partial v}{\partial x}\right), \tag{5}$$

in which $(\partial v/\partial x)$ is the variation of the velocity component of the turbulent medium along the direction of

[7] H. Alfven, *Cosmical Electrodynamics* (Oxford University Press, Oxford, pp. 20, 21).
[8] S. Chandrasekhar, Proc. Roy. Soc. (London) **A204**, 435 (1950).

H per unit distance along H. Since we have assumed that the magnetic fields in the interstellar clouds have no high-frequency components, it follows that

$$dH/dt = 2.5HV/L. \tag{6}$$

where v stands for iB. This time variation of the magnetic field results in an inductive action.

It must be noticed that with a positive value of V (head-on collision), the magnetic field increases and so the induced electromotive force is in the direction of motion of a positive particle. Thus the inductive action and Fermi's mechanism operate to reenforce each other. The same holds true for an overtaking collision.

With the time variation of the magnetic field given by (6), the energy gain by a cosmic-ray particle per loop is

$$(e/c)\pi\rho^2 dH/dt, \tag{7}$$

where ρ is given by (3).

The number of loops the particle describes in traveling a linear distance dx along the direction of H is

$$\frac{dx}{c\beta\cos\theta}\cdot\frac{c\beta\sin\theta}{2\pi\rho}=\frac{dx}{2\pi\rho}\cdot\frac{\sin\theta}{\cos\theta}. \tag{8}$$

Thus the energy gain during the whole course of the encounter is

$$\begin{aligned}\Delta E&=\int\frac{dx}{2\pi\rho}\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{e}{c}\pi\rho^2\frac{dH}{dt}\\&=\int\frac{pc\sin^2\theta}{2c\cos\theta}\left(\frac{1}{H}\frac{dH}{dt}\right)dx,\end{aligned} \tag{9}$$

where the integration is effected over the total linear distance traversed by the particle. This is what we have called the I-gain.

Assuming the kinetic energy of the particle, $E=pc$, to be constant, inserting $2.5V/L$ for $\dot{H}/H$, qH for $\sin^2\theta$, and replacing dx by $(\partial H/\partial x)dH$ in (9), we obtain for the I-gain:

$$\Delta E=\frac{2.5BE}{Lq}\int_{qH_0}^{1}\frac{qH}{(\partial H/\partial x)(1-qH)^{\frac{1}{2}}}d(qH). \tag{10}$$

It can be seen from (10) that the energy gain could be substantial if the particle traverses a relatively long distance in which $\partial H/\partial x$ is small. Therefore, the I-gain depends sensitively on the angle of pitch θ_0 and the spectral distribution of the turbulent magnetic field in the clouds. The smaller the angle of pitch θ_0, the deeper will the particle penetrate into the cloud and the larger will be the I-gain. Our present knowledge of the galactic magnetic filed is still inadequate for a realistic discussion of the process proposed. Nevertheless, as we shall show presently (in Sec. IV), so long as the magnetic field in the clouds has no high-frequency components as indicated in Fig. 1(a) and the size of the clouds is sufficiently large, it is most probable that there exists a range (χ,θ_c) for the angle of pitch θ_0 such that if $\chi<\theta_0<\theta_c$ the I-gain is higher than the F-gain.

The physical significance of the existance of a critical angle such as θ_c can be seen as follows: For a cosmic-ray particle trapped between two clouds approaching each other, with its initial pitch angle $\theta_0>\theta_c$, the F-gain is greater than the I-gain and an energy gain by the particle results then in a decrease of the pitch angle until θ_0 becomes equal to θ_c. On the other hand, if $\chi<\theta_0<\theta_c$, the I-gain is greater than the F-gain and an energy gain results in an increase of the angle of pitch until again, θ_0 approaches gradually the angle θ_c. Thus θ_c is the "equilibrium" value of the angle of pitch which the particle will tend to have. Therefore, for the case of acceleration, θ_c is the angle of pitch at which the "push-pull" action of Fermi's mechanism and the inductive mechanism is operating.

In the case of two clouds receding from each other, the trapped cosmic-ray particle will be deaccelerated by both Fermi's mechanism and the inductive action. However, there also exists an angle θ_c', similar to θ_c, for which the energy reduction by Fermi's mechanism (the negative F-gain) is equal to that by the inductive action (the negative I-gain). When $\theta_0>\theta_c'$, the particle is stable in the trap. For $\chi<\theta_0<\theta_c'$, the energy reduction will result in a steady decrease of the angle of pitch until θ_0 becomes χ, when the particle will be able to penetrate the maximum magnetic field without reflection and thus escape from the trap.

After the particle has escaped from the trap, it would keep on spiraling along the magnetic lines of force without the reversal of its direction of motion. This condition continues until either it encounters a region with a sharp variation in magnetic field where θ_0 could be converted back to a larger value according to a statistical distribution of directions, or it suffers a "*head-on*" collision with a cloud with an appropriate distribution in magnetic field where θ_0 could be effectively increased by the inductive action, and then the trap action can be resumed.

The angles θ_c and θ_c' are not necessarily equal; in fact, θ_c' is likely to be larger than θ_c. One interesting consequence of this property arises for the cases where clouds are first approaching and then receding from each other. A trapped cosmic-ray particle, which after acceleration between the clouds by the "push-pull" mechanism has attained the angle of pitch $\theta_0(=\theta_c)$ can find itself suddenly in the unstable region (χ,θ_c') when the clouds begin to move away from each other. On the other hand, in those traps where deacceleration has been operating, all the cosmic-ray particles will have their pitch angles greater than θ_c, and thus, they will be ready for acceleration should the "jaws" of the trap reverse their directions of motion. Therefore, a high-energy particle has higher probability of being accelerated than deaccelerated.

Some further remarks which might be made regarding the "push-pull" mechanism are the following. Firstly, it should be noticed that the mechanism operates for positive particles as well as for negative particles. Secondly, the mechanism is very inefficient for nonrelativistic particles as can be seen by putting $E=0$ in Eq. (10). Finally, with the time variation of the magnetic field given by (6), induced currents will be set in the interstellar medium. However, one can readily show that the velocities of the ions and the electrons in the medium and the drift velocities of the guiding centers of the spirals described by the ions and the electrons are much smaller than V. Thus the induced currents will not offset the mass motion of the cloud and consequently the relation (6) will continue to be valid.

IV. ANGLES θ_c AND θ_c'

In order to demonstrate the existance of the equilibrium angles θ_c and θ_c', we shall consider the following two cases:

Case I.—Clouds with $\partial H/\partial x \leqslant 15H_0/L$.

Let us first consider a special case in which $\partial H/\partial x$ can be represented by $2(H_m-H_0)/L$ and then apply the results to more general cases. In this case, (10) reduces to

$$\begin{aligned}\Delta E&=\frac{5BE}{6q(H_m-H_0)}(2+qH_0)(1-qH_0)^{\frac{1}{2}}\\&=2BE\cos\theta_0\frac{H_0(2+\sin^2\theta_0)}{2.4(H_m-H_0)\sin^2\theta_0},\end{aligned}\tag{11}$$

on account of the relation $qH_0=\sin^2\theta_0$.

A comparison of (1) and (11) gives the following condition for the I-gain to be larger than the F-gain:

$$\frac{H_0(2+\sin^2\theta_0)}{2.4(H_m-H_0)\sin^2\theta_0}\left(\frac{E}{w}\right)\geqslant 1,$$

or

$$\left[\frac{1.2}{\epsilon}(m-1)-0.5\right]\sin^2\theta_0\leqslant 1,\tag{12}$$

in which m and ϵ stand for H_m/H_0 and E/w respectively. Since $\sin^2\theta_0>H_0/H_m$, we must have

$$\left[\frac{1.2}{\epsilon}(m-1)-0.5\right]\leqslant\frac{1}{\sin^2\theta_0}\leqslant\frac{H_m}{H_0}=m.\tag{13}$$

In other words,

$$m\leqslant m_c,\quad\text{where}\quad m_c=(1.2+0.5\epsilon)/(1.2-\epsilon).\tag{14}$$

When $m<m_c$, it follows from (12) that $\theta_0<\theta_c$, where

$$\sin^2\theta_c=\epsilon/[1.2m-(1.2+0.5\epsilon)].\tag{15}$$

For $\chi<\theta_0<\theta_c$, the I-gain is greater than the F-gain. If the right-hand side of (15) should be greater than 1, the equation should be interpreted as simply $\theta_c=\pi/2$.

The foregoing results can be applied quite generally whenever the value of $\partial H/\partial x$ is less than $H_0(m_c-1)/L$ everywhere in the clouds; for then the integrand in (10) is everywhere larger than that in the special case considered and the value of the integral should also be larger. Two special cases of the foregoing are the following:

$$\text{(a)}\quad H=H_0+(H_m-H_0)\sin(\pi x/L),\quad(0\leqslant x\leqslant L/2),\tag{16}$$

with $H_m/H_0\leqslant(0.64m_c+0.56)=6$ if $\epsilon=1$.

$$\text{(b)}\quad H=H_0+(H_m-H_0)\exp[-(2x-L)^2/L^2],\tag{17}$$

with $H_m/H_0\leqslant(1.2m_c-0.2)=10$ for $\epsilon=1$.

Case II.—Clouds with their field intensity curves very flat near H_m.

Divide the field intensity curve in the clouds into two regions (H_0,H) and (H,H_m), with

$$H=\xi H_m\tag{18}$$

so that, in the region of (H,H_m), $\partial H/\partial x$ can be represented by a constant value $(H_m-H)/l$, when l is the linear distance. This can always be done as long as the magnetic field has no discontinuity in the neighborhood of H_m as indicated in Fig. 1(a). Then (10) becomes

$$\Delta E=(\Delta E)_1+(\Delta E)_2,\tag{19}$$

where

$$(\Delta E)_1=\frac{2.5BE}{Lq}\int_{qH_0}^{qH}\frac{qH}{(\partial H/\partial x)(1-qH)^{\frac{1}{2}}}d(qH),\tag{20}$$

$$\begin{aligned}(\Delta E)_2&=\frac{2.5BE}{Lq}\int_{qH}^{1}\frac{qH}{(\partial H/\partial x)(1-qH)^{\frac{1}{2}}}d(qH)\\&=\frac{5BEl}{3Lq(H_m-H)}(2+qH)(1-qH)^{\frac{1}{2}}.\end{aligned}\tag{21}$$

The sufficient condition that the I-gain be greater than the F-gain is then

$$\frac{\epsilon l(2+\xi m\sin^2\theta_0)(1-\xi m\sin^2\theta_0)^{\frac{1}{2}}}{1.2mL(1-\xi)\cos\theta_0\sin^2\theta_0}\geqslant 1,\tag{22}$$

in which m and ϵ have the same meanings as before. Since $\cos\theta_0\sin^2\theta_0$ has a maximum value $2/3\sqrt{3}$, and $m\sin^2\theta_0>1$, (22) will always be satisfied if

$$\frac{\sqrt{3}\epsilon l(2+\xi)(1-\xi m\sin^2\theta_0)^{\frac{1}{2}}}{0.8mL(1-\xi)}\geqslant 1,\tag{23}$$

or

$$m\sin^2\theta_0\leqslant\frac{1}{\xi}\left[1-\frac{0.64m^2L^2(1-\xi)^2}{3\epsilon^2l^2(2+\xi)^2}\right].\tag{24}$$

Consequently, the sufficient condition of the existence of the equilibrium angle $\theta_c(>\chi)$ is

$$\frac{(1-\xi)H_m}{l^2}\leqslant 5\epsilon^2(2+\xi)^2\frac{H_m}{(mL)^2}\approx 45\epsilon^2\frac{H_m}{(mL)^2}. \quad (25)$$

The quantity of the left hand side of (25) represents the curvature of the magnetic field at H_m.

The foregoing arguments can be applied equally well to overtaking collisions to show the existance of θ_c'. To the first order of approximation, θ_c and θ_c' are equal. However, to a higher order of approximation, θ_c' is likely to be larger than θ_c. For when terms of the second order in B are taken into account, the absolute value of the negative F-gain is less than the positive F-gain by the amount $4B^2w$. The same should be true of the I-gains. This proves $\theta_c'>\theta_c$.

V. TIME SCALE OF THE ACCELERATION

It was first suggested by Kaplon, Peters, *et al.* that[1] the integral spectra of different components of cosmic radiations can be represented by a single formula

$$n(>E)\sim 1/(1+E)^s, \quad (26)$$

where n is the number of particles whose energy per nucleon is greater than E and s is about 1 for the protonic component and is about 1.3 for the nucleonic components. This formula holds for $E<50$ Bev/nucleon and has been confirmed by other groups of cosmic-ray workers.

Now let us consider the "push-pull" mechanism discussed in Sec. III. A cosmic-ray particle which is trapped between two clouds approaching each other, will spend part of its time in the clouds and the rest of its time in traveling between clouds. Let T_1 be the time taken for an encounter with a cloud, and let T_2 be the time taken in traversing the distance between two clouds. T_2 will be simply

$$T_2=aL/(c\cos\theta_c), \quad (27)$$

where aL is the distance between the clouds. The value of T_1 can be estimated as follows:

At the equilibrium angle θ_c, the F-gain is equal to the I-gain. Since the energy gain by the inductive action per unit time is, according to (7).

$$\left(\frac{dE}{dt}\right)_i=\left(\frac{e}{c}\pi\rho^2\dot{H}\right)\left(\frac{\beta c\sin\theta}{2\pi\rho}\right)$$
$$=1.25E(V/L)\sin^2\theta, \quad (28)$$

where we have put $\beta=1$ and $(1/H)(dH/dt)$ equal to $2.5V/L$. Thus

$$2Bw\cos\theta_c=\int_0^{T_1}1.25E\frac{V}{L}\sin^2\theta dt$$
$$\approx 1.25(V/L)T_1, \quad (29)$$

as the particle spends most of its time in the region of the cloud where $\theta\approx\pi/2$. For the acceleration of a relativistic particle $(E\sim w)$,

$$T_1\approx 1.6L\cos\theta_c/c. \quad (30)$$

The sum of T_1 and T_2 gives the time interval between two successive encounters as

$$T=T_1+T_2=\frac{(1.6\cos^2\theta_c+a)L}{c\cos\theta_c}, \quad (31)$$

with an energy gain of $4Bw\cos\theta_c$. Hence the energy gain per encounter is

$$dw/dN=4Bw\cos\theta_c, \quad (32)$$

and the energy gain per unit time is

$$dw/dt=4Vw\cos^2\theta_c/(1.6L\cos^2\theta_c+aL). \quad (33)$$

It is clear that the process of acceleration cannot last forever since the distance between the clouds, aL, is limited. The maximum time allowed for the acceleration is of the order of $al/2V$. After that the particle has to undergo another cycle of acceleration as well as a cylce of de-acceleration although the former has a higher probability. Consequently an efficiency factor η should be introduced in (33), giving the average rate of the energy gain of the cosmic-ray particle as

$$(dw/dt)_{\mathrm{Av}}=w/A, \quad (34)$$

where A, given by

$$A=(1.6+a\sec^2\theta_c)L/(4\eta V), \quad (35)$$

determines the time scale of the acceleration.

The present knowledge on the interstellar clouds suggests $L=10$ parsec, $aL=30$ parsec, $V=10$ km/sec for the clouds in the neighborhood of the sun. Taking $\eta=10\%$ and $\theta_c=30°$, we have $A=4.2\times10^{14}$ sec $=14$ million years, which is too long unless a higher value for η (say 30%) is used. On the other hand, the recent measures[9] of the 21-cm radiation from the innermost part of our galaxy indicates the velocities of clouds in the region of 1.5 to 2.5 kiloparsecs from the center are as high as 50–70 km/sec, and even higher towards the center. If the high values of V are taken, A becomes of the order of several million years. In conclusion, there seems to be regions in our galaxy where the "push-pull" mechanism does provide enough efficiency for the acceleration of cosmic radiation.

VI. ENERGY SPECTRA

One of the essential points of Fermi's theory of the origin of cosmic radiation is that the energy spectra are determined by the rate at which the cosmic-ray particles are accelerated and the rate at which the particles are eliminated. We have seen from (34) that the energy

[9] Kwee, Muller, and Westerhout, Bulletin of the Astronomical Institutes of the Netherlands No. 458 (1954).

of a particle at time t is

$$w(t)=w_0 \exp(t/A). \tag{36}$$

Assuming the lifetime of the particle to be governed by the statistical law

$$P(t)dt=(dt/B)\exp(-t/B), \tag{37}$$

we can readily show that the energy spectrum is of the following form

$$dn \sim s w_0{}^s dw/w^{1+s}, \tag{38}$$

with

$$s=A/B. \tag{39}$$

In the original form of his theory,[4] Fermi assumed that the nuclear collisions effectively destroy the particle. Under this assumption, B will be different for different species of nuclei and so will the energy spectra. This is contradicted by the experimental results.

In order to overcome this difficulty, Morrison *et al.*[10] and Fermi[2] independently suggested that cosmic-ray particles are eliminated from our galaxy by leakage and thus B would be the same for all components of the cosmic radiations. This assumption can be slightly generalized by saying that the cosmic-ray particles are eliminated by leakage from the region or regions where the acceleration is operating. This will also give an inverse power-law spectrum which is the same for all the components. The advantage of the modified assumption is that there may be in the galaxy only a few regions where the physical condition is suitable for the acceleration of cosmic radiation. Then the leakage from a smaller region seems to be more natural than the leakage from the galaxy.

Assume for instance that the nucleus of the galaxy is the only region where cosmic radiation could be efficiently accelerated. The motion of a cosmic-ray particle inside of the nucleus can be described as a random walk, with length of steps $(a+1)L$, and duration τ. The average number of steps a particle takes before it leaves the nucleus is $3R^2/2(a+1)^2L^2$, where R is the radius of the nucleus. Therefore

$$B=\frac{3R^2}{2(a+1)^2L^2}\tau. \tag{40}$$

Putting $R=3000$ parsec, $a=3$, $L=10$ parsec, and $B=A/1.2$ as required by the shape of the energy spectrum, and $A=3$ million years, we have

$$\tau=10^{10}\text{ sec}, \tag{41}$$

which is about twice longer than the average time duration of each step without trapping action.

As far as the present knowledge goes, the center of our galaxy seems to have a physical condition favorable for the acceleration of cosmic radiation. However, before one can definitely assume the central region of the galaxy to be the principal source of cosmic radiation, various questions related to the isotropy of the cosmic radiation should be answered.[11]

Two remarks should be added in conclusion. The first is on the mechanism of injection. Recent discovery of the solar component of cosmic radiation indicates that stars or stars of particular types are probably the injectors. The second is on the cutoff in the energy spectrum. Since the "push-pull" mechanism is inefficient for accelerating nonrelativistic particles, the low efficiency of the acceleration mechanism serves as another possible explanation of the cutoff.

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[10] Morrison, Olbert, and Rossi, Phys. Rev. **94**, 440 (1954).

[11] L. Davis, Jr., Phys. Rev. **96**, 743 (1954).