Comparison of Differential Cross Sections for the Reactions $C^{12}(d,p)C^{13}$ and $C^{12}(d,n)N^{13}$†

R. E. BENENSON,* K. W. JONES,‡ AND M. T. MCELLISTREM§
University of Wisconsin, Madison, Wisconsin

(Received September 19, 1955)

The differential cross sections for the reactions $C^{12}(d,p)C^{13}$ and $C^{12}(d,n)N^{13}$ to the ground states of the mirror nuclei C^{13} and N^{13} are reported for deuteron bombarding energies of 2.68 Mev and 3.26 Mev. The angular distributions at the forward angles are in agreement with those predicted from the formalism of Butler for both reactions. The cross sections are compared at the angle of the first stripping peak, and this comparison indicates that the reduced widths of the mirror nuclei C^{13} and N^{13} are the same. This is in agreement with the assumption of the charge symmetry of nuclear forces.

IT is expected, on the basis of charge symmetry of nuclear forces, that the reduced widths of corresponding levels in mirror nuclei should be equal. A comparison of the relative yields of (d,p) and (d,n) stripping reactions with a self-conjugate target nucleus, using one bombarding energy, furnishes one means for an experimental check of this assumption.

Recently a comparison has been reported by Fujimoto *et al.*[1] for the reactions $Mg^{24}(d,p)Mg^{25}$ and $Mg^{24}(d,n)Al^{25}$. Using the Butler approximation, they extracted reduced widths for the ground states of Mg^{25} and Al^{25} from published cross sections for these reactions. They found that the (d,p) width of Mg^{25} is an order of magnitude larger than the (d,n) width of Al^{25}, and concluded that the (d,p) and (d,n) reactions could not be treated as equivalent nuclear reactions. However, further considerations are pertinent. The $Mg^{24}(d,p)$ reactions to the ground and several excited states were measured by Holt and Marsham,[2] and in a subsequent publication[3] they mention that all cross sections quoted at first and used in the calculations of Fujimoto *et al.* should be reduced by a factor of four. This diminishes the difference between the calculated (d,p) and (d,n) reduced widths by the same factor. In addition, the $Mg^{24}(d,p)$ reactions were studied at a bombarding energy of 8.0 Mev, whereas the $Mg^{24}(d,n)$ reactions were obtained for a bombarding energy of 4.0 Mev. The difference in Coulomb effects for the two reactions would be such as to enhance the (d,p) cross section with respect to the (d,n). Futhermore, such effects as the scattering of the liberated particle by the residual nucleus may be appreciably different for these quite different bombarding energies. In view of these several considerations, it seems difficult to conclude that the experiments discussed above indicate an appreciable difference in the two reactions.

The purpose of this article is to report measurements of differential cross sections for the reactions $C^{12}(d,p)C^{13}$ and $C^{12}(d,n)N^{13}$ at deuteron bombarding energies of 2.68 Mev and 3.26 Mev. The two cross sections were compared at the angle of the forward stripping maximum, in order to estimate the relative sizes of the ground-state reduced widths of C^{13} and N^{13}, respectively.

The $C^{12}(d,p)C^{13}$ cross section was measured absolutely in a differentially pumped gas scattering chamber, using a target thickness of from five to ten kev.[4] For the purpose of comparison with the (d,n) cross section, the (d,p) cross section was averaged over an energy region equivalent to the target thickness in the (d,n) measurement. Absolute measurements of the $C^{12}(d,n)N^{13}$ cross sections were made using a gas recoil neutron spectrometer.[5] Both solid and gas targets were used. Target thicknesses used at these energies were 320 kev of 2.68 Mev and 360 kev at 3.26 Mev. Absolute cross sections were obtained by comparing the neutron yield from the $C^{12}(d,n)N^{13}$ reaction to that from the $D(d,n)He^3$ reaction, whose cross section is known, and also by calculation of the spectrometer efficiency from its parameters.

The uncertainty in the $C^{12}(d,p)C^{13}$ cross sections is 8%, on the average. At forward angles where the cross sections are compared, the uncertainty in these cross sections is about 10%. The $C^{12}(d,n)N^{13}$ cross sections have an estimated uncertainty of about 19% at $E_d=2.68$ Mev, and 13% at $E_d=3.26$ Mev.

The angular distributions for the two reactions are shown in Figs. 1 and 2. The solid curves are calculated fits to the data from the formalism of Butler for an angular momentum transfer by the captured particle of one unit. For the purpose of displaying the agreement of the $C^{12}(d,p)C^{13}$ angular distribution with that predicted by Butler, the calculated curve was fitted to the data after an isotropic background cross section had been subtracted from the data. This subtraction is

† Work supported in part by U. S. Atomic Energy Commission and the Wisconsin Alumni Research Foundation.
* Now at Boookhaven National Laboratory, Upton, Long Island, New York.
‡ Now at Columbia University, New York, New York.
§ Now at University of Indiana, Bloomington, Indiana.
[1] Fujimoto, Kikuchi, and Yoshida, Progr. Theoret. Phys. **11**, 264 (1954).
[2] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 258 (1953).
[3] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 467 (1953).
[4] Jones, McEllistrem, Douglas, Herring, and Silverstein, Phys. Rev. **98**, 241(A) (1955). Details of the measurements and measuring equipment will be discussed in a subsequent publication.
[5] Benenson, Shurman, and Haeberli, Phys. Rev. **96**, 836(A) (1954). Details of the spectrometer construction and calibration are contained in R. E. Benenson, Ph.D. thesis, University of Wisconsin, 1954 (unpublished). Spectrometer information will be published subsequently.

shown on the figures by a dashed base line for the Butler curve. These curves are arbitrarily normalized to fit the stripping peak, and do not represent a calculation of the absolute cross sections. As is seen from the figures, the distributions at forward angles agree well with that predicted by the Butler formalism. The agreement for the $C^{12}(d,n)N^{13}$ reaction is in contrast to that observed previously at 8-Mev deuteron bombarding energy by Middleton *et al.*[6]

Several investigations have shown that the theoretical stripping cross section is proportional to the reduced width of the final state. (See, for example, reference 1.) While it is extremely difficult to calculate ***absolute*** values for reduced widths because of the uncertainty in the various factors contained in the formulae for cross section, it is possible to obtain ***relative*** sizes of reduced widths by comparing yields from stripping reactions. In order to compare the reduced widths of the residual mirror nuclei, C^{13} and N^{13}, it is necessary to remove the kinetic factors in the stripping cross section formula, i.e., those factors which depend on the Q-values for the two reactions. To estimate these factors the Butler stripping approximation was used in the form of Eq. (34) of Butler's paper[7] and also in the form expressed in Eqs. (19), (40), and (49) of Huby's paper.[8] The ratio of the kinetic factors for the expressions of Butler and Huby are listed as columns a and b, respectively, in Table I. These two expressions are the same except that Butler's equation contains an additional factor f. For the case of a neutron captured with one unit of angular

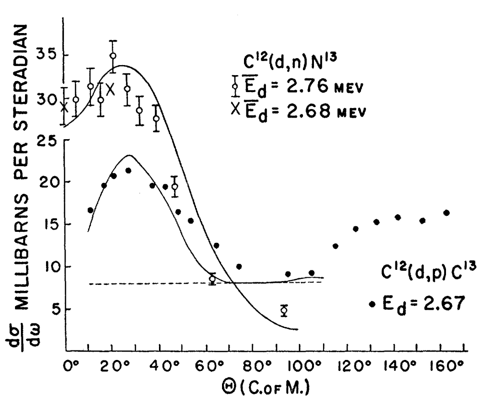

FIG. 1. Angular distributions for $C^{12}(d,n)N^{13}$ and $C^{12}(d,n)C^{13}$ to the ground states of both nuclei. The open circles and the ×-points are the $C^{12}(d,n)$ data. The angular distribution at 2.76 Mev has been arbitrarily normalized to the cross-section measurements at 2.68 Mev. The solid circles are the cross-section measurements for the $C^{12}(d,p)$ reaction. The solid curves are the Butler curves for the two reactions. For the (d,p) reaction, the dashed line indicates the isotropic component of the cross section which was subtracted before fitting the Butler curve. To fit the distributions, a nuclear radius of 4.7×10^{-13} cm was used for the (d,n) curve, and a radius of 6.5×10^{-13} cm for the (d,p) curve. The extent of the vertical bars on the data points indicates the expected statistical fluctuations.

[6] Middleton, El Bedewi, and Tai, Proc. Phys. Soc. (London) **A66**, 95 (1953).
[7] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).
[8] R. Huby, Progr. Nuc. Phys. **3**, 177 (1953).

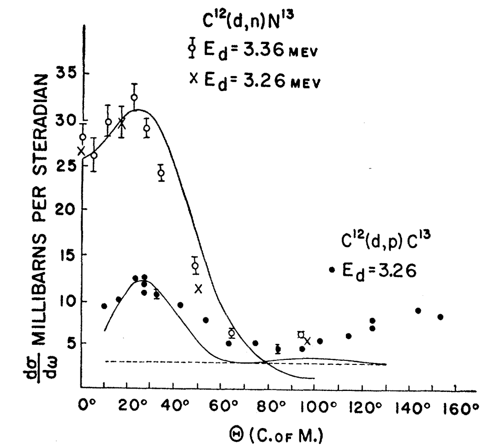

FIG. 2. Angular distributions for $C^{12}(d,n)N^{13}$ and $C^{12}(d,p)C^{13}$ to the ground states of both nuclei. The open circles and the ×-points are the $C^{12}(d,n)$ data. The angular distribution at 3.36 Mev has been arbitrarily normalized to the cross-section measurements at 3.26 Mev. The solid circles are the cross-section measurements for the $C^{12}(d,p)$ reaction. The solid curves are the Butler curves for the two reactions. For the (d,p) reaction, the dashed line indicates the isotropic component of the cross section which was subtracted before fitting the Butler curve. To fit the distributions, a nuclear radius of 4.7×10^{-13} cm was used for the (d,n) curve, and a radius of 6.5×10^{-13} cm for the (d,p) curve. The extent of the vertical bars on the data points indicates the expected statistical fluctuations.

momentum, $f=[(1+K_sr_0)/K_sr_0]^2$, using the notation of Butler's paper.

In calculating these kinetic factors from the expressions of Butler and Huby, it is necessary to assume a nuclear radius. Many of the recent stripping angular distributions for light nuclei have been fitted with nuclear radii in the vicinity of 5×10^{-13} cm. For this reason, and because the $C^{12}(d,n)N^{13}$ distribution was fitted with a radius of 4.7×10^{-13} cm, the kinetic factors were calculated using a radius of 4.7×10^{-13} cm for both reactions. These factors provide the ratios quoted in columns a and b of Table I.

It is to be noted that a somewhat different radius was used to fit the angular distributions of the (d,p) reaction. A momentum analysis of the Butler amplitudes for the (d,p) reaction has been carried out for the two radii involved, and it is found that most of the difference in the results of this analysis for the two different radii is in the low momentum components of the distribution. In fact, the relative contributions of those momenta for $l>3$ are the same for either choice of radius. The high-momentum components, where pure stripping is expected to predominate, are insensitive to the choice of radius. The low-momentum components, which do depend on the choice of radius, are seriously affected by compound nucleus formation. It is believed that the larger radius (6.5×10^{-13} cm) needed to fit the angular distribution of the (d,p) reaction is a result of the interference of the compound nucleus and stripping amplitudes.

Several effects believed present in the reactions have

TABLE I. Differential cross section and kinetic factor ratios.

E_d Mev	$\left(\frac{d\sigma}{d\omega}\right)_p(25°)$ mb/sterad	$\left(\frac{d\sigma}{d\omega}\right)_n(25°)$ mb/sterad	$\left(\frac{d\sigma}{d\omega}\right)_n / \left(\frac{d\sigma}{d\omega}\right)_p$ Ratio of cross sections	Ratio of calculated kinetic factors a	b
2.68	19±2	32±5	1.7±0.3	2.79	2.01
3.26	13±0.6	31±4	2.4±0.4	2.89	2.08

a Butler formalism, $r_0=4.7\times10^{-13}$ cm.
b Huby formalism, $r_0=4.7\times10^{-13}$ cm.

not been considered in these calculations. Coulomb effects have not been considered, because, although they certainly strongly influence the magnitude of the individual cross sections, the effect on the ratio of the two cross sections cannot be very large. Effects involving the incident deuteron appear in both reactions and do not affect the ratio. Therefore, the Coulomb effect involving the outgoing proton will be the only one which would change the cross section ratio. Since the outgoing proton has an energy greater than 5 Mev, well above the Coulomb barrier, this would not be a large correction. Nuclear effects not considered in these calculations which strongly influence the stripping reaction[9] are the elastic scattering of the incident deuteron wave by the target and the scattering of the liberated particle by the residual nucleus. The deuteron energy is the same for both reactions and hence also the deuteron scattering effect. The scattering of the liberated particle is expected to be most important for low values of angular momentum. For these reactions the estimate of Tobocman and Kalos[9] suggests that both the protons from the $C^{12}(d,p)C^{13}$ and neutrons from the $C^{12}(d,n)N^{13}$ are strongly affected only for l-values <2. The penetration probabilities for the outgoing neutrons and the outgoing protons of both S and P wave momenta are nearly the same. This means, if we assume hard sphere scattering, that the potential scattering amplitudes for the outgoing neutrons in the $C^{12}(d,n)$ reaction will be about the same as the scattering amplitudes for the protons from the $C^{12}(d,p)$ reaction.

To be more specific, we may compare phase shifts for the systems $C^{13}+n$ and $N^{13}+p$. The analysis of the elastic scattering of neutrons[10] by C^{12} and the analysis of the elastic scattering of neutrons[11] by N^{14} provide empirical S and P wave potential phases for the neutron scattering which are very similar. If we average the S and P wave potential phases from these two analyses for the same neutron energy as encountered from the $C^{12}(d,n)N^{13}$ reaction at $E_d=2.68$ Mev, we obtain S and P wave phases of $-80°$ and $-15°$, respectively. These phases provide scattering amplitude magnitudes of 0.98 and 0.26 for these neutrons. Fowler and Johnson, using a nuclear radius of 3.7×10^{-13} cm, calculate hard-sphere phases of $-62°$ and $-15°$ for S and P waves. If we use this same nuclear radius to calculate hard-sphere phases for the scattering of the $C^{12}(d,p)$ protons (5 Mev protons) by C^{13}, the residual nucleus, we obtain S and P wave phases of $-56°$ and $-21°$, respectively. These phases provide scattering amplitude magnitudes of 0.83 and 0.36 for the protons. These magnitudes are quite similar to the empirical neutron amplitudes quoted above.

Resonant scattering for the two cases will of course be the same assuming charge symmetry of nuclear forces, since by the conservation of energy, the compound nucleus (N^{14}) for the two scattering processes will be at the same excitation energy. Therefore we expect that effects due to the nuclear scattering of the liberated particles by the residual nucleus will be quite similar for the two reactions, $C^{12}(d,p)$ and $C^{12}(d,n)$, since they are compared at the same deuteron bombarding energy.

Another source of possible discrepancy in the calculated ratios arises from the presence of compound nuclear amplitudes in the (d,p) and (d,n) reactions, since the kinetic factors for such amplitudes depend upon the Q-value of the reactions in quite a different manner than do those for stripping. An analysis of the data in terms of stripping and compound nucleus amplitudes is not presently available, so that an accurate correction to the kinetic factors cannot be made. In order to obtain some idea of the possible size of this effect, we calculate that if the compound nucleus amplitude were one-third that of the stripping amplitude in the (d,p) reaction, and if this amplitude was equally divided between S and P wave outgoing particles, the tabulated kinetic factor ratios would be reduced about 10%. If the compound nucleus amplitude equalled the stripping amplitude in magnitude, the ratios could be reduced by almost a factor of two.

It is seen in Table I that the differences in (d,p) and (d,n) cross sections can be explained entirely from the kinetic factors, to within the accuracy to which they are calculated. There seems to be no need to conclude that the reduced widths differ by an appreciable amount. This is in agreement with the assumption of charge symmetry.

The authors wish to express their gratitude to Professor H. T. Richards for suggesting this comparison and for valuable advice concerning the interpretation of the measurements reported.

[9] J. Horowitz and A. M. L. Messiah, J. phys. radium **14**, 12, 695 (1953); W. Tobocman, Phys. Rev. **94**, 1655 (1954). W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955).
[10] R. Budde and P. Huber, Helv. Phys. Acta **28**, 49 (1955).
[11] J. L. Fowler and C. H. Johnson, Phys. Rev. **98**, 728 (1955).