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Interpretation of Some of the Excited States of 4*n* Self-Conjugate Nuclei*

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An attempt is made to give a general account for some of the excited levels of $4n$-type self-conjugate light nuclei, especially for characteristic low-lying 0^+ levels and also for the 2^+ levels which appear not too far above the 0^+ levels in every case except the O^{16} ground state. It is pointed out that these 2^+ levels may arise from the same configuration as the lower lying 0^+ states which are considered to be deformed. The degree of deformation is estimated from the 0^+-2^+ energy separation and found to correspond to quite high deformations in some cases. The 6.06-Mev, 0^+, pair emitting state in O^{16}, which is considered to be a "hole configuration" by Christy and Fowler, may be deformed and is suspected to have a shape like a line of four alpha particles. Also, a rotation-like series of levels with spins and parities, 0^+, 1^-, 2^+, 3^-, and 4^+ among the highly excited states of Mg^{24} which appears to correspond to a series expected for a linear rotator with six alpha particles in a line is discussed. With Ne^{20}, indications of medium-deformed states are found. Analogous discussions on the levels in Be^8 and C^{12} are tried although the experimental evidence is less conclusive.

1. INTRODUCTION

THE energy level structures of $4n$-type light nuclei, like Be^8, C^{12}, O^{16}, Ne^{20}, and Mg^{24}, show some characteristic features which are not quite easy to explain from simple shell-model theories. The alpha-particle model[1] has been considered as a hopeful alternative for describing these levels, and recent re-examination of the alpha-particle model of the O^{16} nucleus[2] seem to show a remarkable agreement with experiment.[3] However, there are still several difficulties with this model, especially in assigning the first dilatational vibration to the 6.06-Mev, 0^+ pair-emitting level.[1,2,4]

Recently Christy and Fowler proposed a "hole configuration," or a configuration where four p particles are raised up to the next shell (s, d orbits) for explaining this state, in analogy to the low-lying $\frac{1}{2}^-$ state in F^{19} and the $\frac{1}{2}^-$ state at around 3-Mev excitation of O^{17} and F^{17}.[5] Schiff also investigated a two-nucleon excitation for the same state,[6] and concluded that in order to account for the observed lifetime of this state a model which is more collective than the independent-particle model with pair interaction and less collective than the conventional alpha-particle model is necessary. Since, however, such 0^+ states have been found in all $4n$ self-conjugate nuclei up to Ne^{20} at around the same energy, it is desirable to try to find a more general argument in connection with other level characterictics. It is the purpose of this note to suggest a possible interpretation of these 0^+ states as rotationless states of strongly deformed configurations.

Sections 2 to 4 deal with O^{16}, Mg^{24}, and Ne^{20} for which the occurrence of rotational series allow the determination of the deformation. In Secs. 5 to 6, similar interpretations are proposed for C^{12} and Be^8.

2. DEFORMATION OF THE O^{16} NUCLEUS

In heavy nuclei it is known that deformation of the nucleus takes place, especially when the nucleus is far from a closed core.[7] Since the O^{16} ground state is a closed-core configuration, it must be spherical; but the "hole configuration", where four p particles are excited to the d orbits, is no longer a closed-core configuration. The d particles will place themselves nonuniformly, and the eight p particles will not make a complete spherical $p_{\frac{3}{2}}$ core, since j does not remain a good quantum number in a deformed potential. The deformation may be quite serious, for after the p shell is deformed, only four s particles are left to form a closed core, if any, which, then is as small as the outer d nucleon group.

If the 0^+ state of O^{16} at 6.06-Mev is really such a deformed state, there should be a 2^+ rotational state not too far above it. Actually there is a 2^+ state at 6.91-Mev, 850-kev higher than the 0^+ state [Fig. 1(c)]. The corresponding moment of inertia is quite high and is appropriate to four alpha particles in a row. The moment of inertia of the system with four alpha particles in a line is obtained by assuming that the assignment of the first rotational state of the tetrahedron ground state to the 6.14-Mev 3^- state in O^{16} is correct and assuming constant-density spherical alpha particles in contact with each other. The corresponding alpha-particle radius is 1.35×10^{-13} cm which is, as has been known for a long time, a reasonable value. The energy of the 2^+ level thus calculated becomes 6.86 Mev, which is quite close to the observed value. So, the "hole configuration" is likely to resemble a structure with four alpha particles in a row.

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[1] See for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 292.
[2] D. M. Dennison, Phys. Rev. **96**, 378 (1954).
[3] J. W. Bittner and R. D. Moffat, Phys. Rev. **96**, 374 (1954).
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[5] R. F. Christy and W. A. Fowler, Phys. Rev. **96**, 851(A) (1954).
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[7] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Dan. Mat.-fys. Medd. **27**, No. 16 (1953).

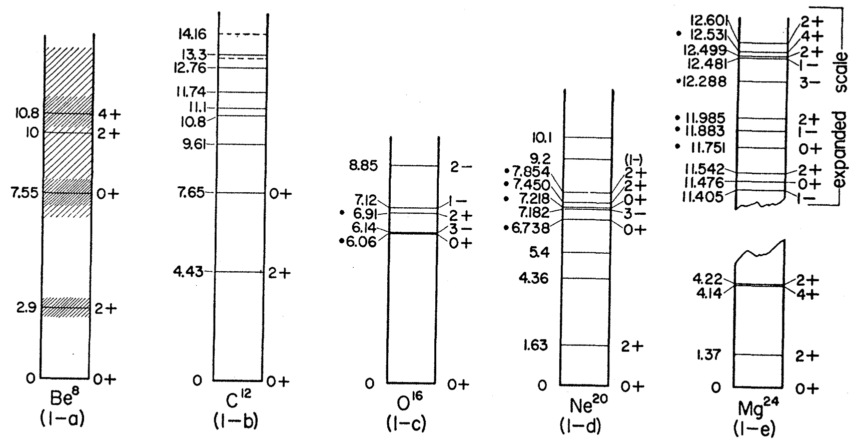

FIG. 1. Lower energy levels of Be^8, C^{12}, O^{16}, Ne^{20}, and Mg^{24}. Dots indicate the states interpreted as deformed states.

Conversely, it is not too unnatural to equate such a long alpha-particle nucleus to a shell state with higher average angular momenta, since on stretching the spherical (tetrahedral) alpha-particle ground state of O^{16} to a line the average angular momentum of the nucleons will increase.

In the alpha-particle model, such a linear configuration may be metastable, since inter-alpha-particle forces are probably made up of a short-range attractive force with a repulsive core and a long-range Coulomb repulsion force. If the binding energy argument of the simple alpha-particle model[1] is accepted, the energy necessary for the structure change may be calculated, provided that all normal frequencies of both structures are known in order to determine the difference in zero point energy. However, as shown by Hafstad and Teller, the zero-point energy per bond is quite constant from one structure to another,[8] and this also holds for four particles in a row.[9] For an order-of-magnitude estimate it will be sufficient just to omit the zero-point energy and to calculate the excitation energy from the change of the number of the bonds and the change in the total Coulomb energy. The energy thus calculated is 9.5-Mev which is about 1.5 times the observed energy of the excited state. Since the bond energy argument in the alpha-particle model does not make too much sense, this comparison should not be taken seriously for or against the linear arrangement.

If the moment of inertia is constant or does not change much by rotation (strong-coupling limit), there should be a 4^+ state at or around 8.91-Mev. It may be rather difficult to find such a 4^+ level from the elastic scattering of alpha-particles on C^{12},[10] since the width is expected to be quite small. It would not have been found in the inelastic scattering of protons on O^{16}, since it would be masked by a proton group leading to a 2^- level at 8.85 Mev.[11,12] Since a 4^+ level can decay by alpha-particle emission, it may be interesting to look for alpha particles with energies around 1.3 Mev following this proton group.[13]

3. ROTATION-LIKE SERIES IN THE EXCITED LEVELS IN Mg^{24}

Among 11 states found in the elastic scattering of alpha particles on Ne^{20}, there is a series of levels with spin and parity 0^+, 1^-, 2^+, 3^-, and 4^+, with a 0^+-2^+ difference of 234 kev and 0^+-4^+ difference of 780 kev.[14] [Fig. 1(e)] The ratio of these energy differences is exactly 3 to 10, which is just to be expected for a rigid rotator. Furthermore, it is interesting to see that the 0^+-2^+ energy difference, 234 kev, is very close to the value expected from the rotation of a linear molecule with six alpha particles in a row. With the same assumption as was used in the case of the O^{16}, namely, taking the alpha-particle radius from the 3^- state in O^{16}, the splitting is calculated to be 237 kev. As long as this series is interpreted as rotational levels, only six particles in a row can give such a high moment of inertia in a simple manner. The height of the 0^+ state is 11.751 Mev which is again a little higher than half of the value calculated for the structure change in the alpha-particle model. If we ignore the zero-point energy change, the latter becomes about 20-Mev.

It is tempting to interpret the intervening 1^- and 3^-

[8] L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938).

[9] The author is grateful to Professor Y. Morino for a discussion on this problem.

[10] R. W. Hill, Phys. Rev. **90**, 845 (1953).

[11] Toppel, Wilkinson, and Alburger, Phys. Rev. **99**, 632(A) (1955).

[12] W. F. Hornyak and R. Sherr, Phys. Rev. **99**, 632(A) (1955).

[13] *Note added in proof.*—In a recent investigation by Wilkinson, Toppel, and Alburger (private communication) it was confirmed that this level is a 2-level which can hardly be assigned to the tetrahedral alpha-particle model. Also in this investigation it was found that the decay of N^{16} to this state has rather low comparative half-life ($\log ft=4.4$). These results are significant since, except for the 0^+ and 2^+ states interpreted as deformed states in above discussion, the lower states of O^{16} far below the lowest $T=1$ state all have negative parity as in the case of Pb^{208}, strongly suggesting that these states are due to single particle excitation. This is also indicated by the rather low comparative half-lives of beta decay from N^{16} to these states.

[14] Goldberg, Haeberli, Galonsky, and Douglas, Phys. Rev. **93**, 799 (1954).

levels also as rotational states. Such levels, however, cannot appear as pure rotational states because of the symmetry requirement in the alpha-particle model. They are possible only if an odd vibration is superposed to make the total wave function symmetric for the interchange of alpha-particle pairs. To be in accord with the small energy of these states, the vibration must have a very small energy, rather smaller than the energy of rotation. It seems plausible that in the linear configuration proposed, a vibration mode corresponding to the shape of an arc can have a rather low frequency. The energies of the 1^- and 3^- states are higher by 54 and 69 kev than the expected energies of (imaginary) pure rotational states. These excess energies, then, would be the energies of excited states of the lowest vibrational mode, if interaction terms are neglected. The higher excess in the energy of the 3^- state may be due to the higher centrifugal force in the 3^- state. The absence of higher vibrational states cannot be interpreted easily. It may mean that only one well-defined vibrational eigenstate exists, since, guessing from the amount of the vibrational energies, the amplitudes of these lowest vibrations are already quite high.

In the case of O^{16}, the linear configuration was considered to correspond to the "hole configuration" with the excitation of four particles in the shell model. For Mg^{24}, however, the length and the thickness of the nucleus makes it inappropriate to give a unique corresponding shell model assignment. It seems, however, that at least the O^{16} core must be broken and some f particles must be considered. So, this 0^+ "ground" state will not correspond to the mere excitation of one group of four particles but should mainly correspond at least eight-particle excitation. This is also suspected from its high energy (11.751-Mev). Hence, one or more 0^+ medium-deformed states may be hidden in the lower energy region where experimental data are rather scarce.

4. DOUBLE 0^+-2^+ SERIES IN Ne^{20}

If the interpretation of some of the O^{16} and the Mg^{24} states as due to configurations with several alpha particles in a row is correct, such an arrangement should show up in the excited states of Ne^{20} and in other cases. Among the excited levels of Ne^{20} so far known, there is a group consisting of a 3^-, two 0^+, and two 2^+ levels lying very close together[15] [Fig. 1(d)]. The energies of the latter four are 6.738 (0^+), 7.218 (0^+), 7.450 (2^+), and 7.854 (2^+) Mev. The reduced widths of the lower levels of each spin value are considerably larger than the higher two. It therefore appears that they consist of two pairs of 0^+-2^+ states. From an interpolation between O^{16} and Mg^{24}, the 0^+ state due to the linear alpha-particle molecule should appear somewhere around here; this could account for only one pair, and no matter how the 0^+-2^+ pair is taken the energy difference does not fit with the rotational energy of the linear alpha-particle molecule, about 400 kev. Since the argument about the height of the 0^+ linear state is not a strong one, none of these levels seem to correspond to the linear state.

Since these energy values have the same order of magnitude as the lowest 0^+ states in other nuclei, Be^8, C^{12}, and O^{16}, it is probably more plausible to consider similar four-particle excitations with least energy consumptions. An alternative two-particle configuration theory would seem to meet serious difficulties as follows: Between two-particle configurations with the same angular momentum and parity, especially in the 0^+ states, it is well known that a fairly large amount of configuration mixing takes place.[16–19] The interaction energy is well over 1 Mev. Hence it is rather unlikely that such close-lying 0^+ states (380-kev apart) appear if they have two-particle configurations. Also, the large difference in the reaction widths of these states suggests rather small configuration mixing.

Simple four-particle configurations of Ne^{20} with low energies will be the following: (1) $O^{16}+d^4$ (ground state), (2) $O^{16}+f^4$, (3) $O^{16}+p^{-4}\cdot d^8$. Tentatively, the assumption will be made that (2) and (3) correspond to the two close-lying 0^+ states. Since the lower 0^+-2^+ pair have higher reaction width for elastic scattering, it will be more plausible to assign (2) to the lower pair and the "hole configuration", (3), to the upper states.

Since the first excited state of Ne^{20} is found to be 2^+, at 1.63-Mev, it is interesting to compare the 0^+-2^+ separations on the basis of the above assignment, attributing each 2^+ state to the rotation of the lower-lying 0^+ state. The separations for the lower and higher pairs are 710 kev and 630 kev respectively. For a rough estimate, the sum of l^2 of the particles outside of the core may be considered as proportional to the moment of inertia of the state. Then, the ratios of the 0^+-2^+ separations for the above configurations become $\frac{1}{4}:\frac{1}{9}:\frac{1}{10}$, which gives 740 and 650 kev for the upper two separations, adjusting the constant from the ground-state—first-excited-state separation. Here, in the case of (3) only $s_{\frac{1}{2}}{}^4$ particles were considered to be the core. This procedure should be applicable to the other nuclei. For O^{16}, same constant gives 1090-kev for the separation between the 6.06-Mev 0^+ state and the 2^+ state which actually lies 850 kev above. In the case of Mg^{24} the height of the lowest 2^+ state becomes 0.82 Mev instead of the observed value of 1.38 Mev. This fit is not quite satisfactory. Actually the height of the 4^+ state (4.14-Mev) in Mg^{24} suggests that the strong-coupling formula cannot be applied to the $0^+-2^+-4^+$ series of the lowest states.

The 4^+ states corresponding to the two sets of 0^+-2^+ states in Ne^{20} at 9.1 and 9.4 Mev should appear if the

[15] J. R. Cameron, Phys. Rev. **90**, 839 (1953).

[16] M. G. Redlich, Phys. Rev. **95**, 448 (1954).

[17] D. E. Alburger and M. H. L. Pryce, Phys. Rev. **95**, 1482 (1954).

[18] C. K. Levinson (private communication).

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moment of inertia does not change by rotation. Refined experimental data are not yet available at this energy, but from both the (d,n) reaction[20,21] and the elastic scattering of alpha particles[22] a state is observed at around 9.2 Mev. This level has been suspected to be a 1^- state, but it would be interesting to know the scattering pattern with higher resolution.

According to this interpretation there may still be real linear states among higher excited states of Ne^{20}. Among the known levels there is no conclusive evidence, but there is a substantial gap where the level scheme is not very well established.

5. BERYLLIUM-8

For the alpha-particle model this nucleus is the fundamental starting point, but the binding energy argument fails explicitly. The energy level structure seems to be quite well known now and the nature of the levels at 2.9-Mev, 7.55-Mev, and 10.8-Mev with 2^+, 0^+, and 4^+ is quite well established[23] [Fig. 1(a)]. Besides these states it seems to be certain that there is a very broad 2^+ level at around 10-Mev. The D wave phase shift in alpha-alpha scattering goes through 90° at around 10-Mev very slowly.[24] The width of this level is about 7 Mev. Also the energy spectrum of the alpha-particles following the Li^8 beta decay shows some branching to a very broad peak with several Mev half-width and with a maximum at higher energies.[25] After dividing the spectrum by E^5, a broad peak with several Mev half-width and with a maximum at around 10 Mev is seen.[26] The total area under the 10-Mev peak is higher than the total area under the 2.9-Mev peak. Since the log ft value for the latter peak is calculated to be 5.6, the transition to the higher state must be allowed and the state must be 2^+. It is doubtless the same state as that seen by the alpha-alpha scattering.

At first sight it looks as if the 2.9-Mev and 10.8-Mev state represent the rotation of the dumbbell alpha-particle ground state. But this assignment leads to the difficulties. First, the moment of inertia is too high compared with the value calculated from the O^{16} 3^- state. This difficulty might be avoided, however, by assuming that the binding of the two alpha particles is so weak that the nucleus is more stretched (like the deuteron ground state), making the value of the moment of inertia larger. But then the 4^+ state should appear much lower than 10/3 times the energy of the 2^+ state. The experimental value, however, shows the opposite shift. Moreover, both the 2.9-Mev state and, especially, the 10.8-Mev state are quite narrow. It appears, then that the assignment given by Inglis on the basis of the intermediate coupling model[27] describes these two levels better. For the 7.55-Mev 0^+ level, a $p_{\frac{3}{2}}{}^4$ configuration may be assumed in analogy with the 0^+ states in other nuclei. Since this configuration becomes the same state as the ground state in the limit of L-S coupling, namely ^1S, this level should be one of the lowest states. The broad 2^+ state seems to have a large component of a two-alpha particle configuration.

6. CARBON-12

Since elastic scattering experiments to determine energy, spin, and parity of excited states of this nucleus are impossible, not so much is known about the level scheme [Fig. 1(b)]. The characteristic 0^+ level appears at 7.65 Mev.[28] This state is well known for its difficulty of formation in various reactions.[29,30] It is, therefore, tempting to assign the "hole configuration" to this state. Here, the lowest imaginable four-nucleon excitation is $p_{\frac{3}{2}}{}^4 \rightarrow p_{\frac{1}{2}}{}^4$. Although the $p_{\frac{1}{2}}$ configuration outside a spherical core would make a spherical configuration, it will not be able to keep its spherical symmetry in this case since the $p_{\frac{3}{2}}$ core becomes incomplete and an over-all deformation will take place.

If such a deformation corresponds to the state of three alpha particles in a line as was considered in the case of O^{16}, it will give a 2^+ state at around 9.70-Mev. Actually, there is a state at 9.61-Mev which can be 2^+, since it decays by the emission of an alpha particle to the ground state of Be^8. Also, if this is the rigid first rotational state of the 7.65-Mev "ground" state, there should be a 4^+ state at around 14.18 Mev. There is a state known at 14.16 Mev. The spin and parity of these states are not known. The normal reaction width of the 9.61-Mev state contradicts the suggested assignment.

7. CONCLUDING REMARKS

Although experimental evidence is not quite conclusive, it seems to be possible to give some general interpretation to the low-lying 0^+ states in connection with the 2^+ states which are found above all 0^+ states, including the ground states of the nuclei discussed except for the O^{16} ground state which is spherical. When the core is not closed, such a 2^+ state is expected to exist either from shell theory or the collective rotational model. The present assignment suggests that extreme collective features show up in some cases. In the case of Mg^{24}, the series of levels mentioned looks like an indication of a rigid rotator. It is not unnatural that the highly collective feature dominates, since in light nuclei the alpha-particle structure is pronounced and L-S coupling is important. But the height of the 4.1-Mev 4^+ state in Mg^{24} shows that the collective feature is not perfect for the ground-state configuration.

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[21] C. F. Powell, Proc. Roy. Soc. (London) **A181**, 344 (1942, 43).
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[24] F. E. Steigert and M. B. Sampson, Phys. Rev. **92**, 660 (1953).
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It is natural to expect more collective behavior when the deformation is appreciable. In the cases of the double 0^+-2^+ series in Ne^{20} the deformations are apparently not complete (not quite like lines). So, the 4^+ levels may be found at lower energies than are expected from the strong coupling-limit theory. In general there will be more different degrees of deformation, when the nucleus gets heavier, between the least deformed ground state and the highest deformed state. The spectrum of such 0^+ states seems to depend on where the nucleus is in the shell.

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Effect of the Tensor Force on the Level Structure of Li^6 and Li^7†*

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The splittings of the $^3S_1$ and $^1S_0$ states of Li^6 and the P doublet states of Li^7 by the tensor force are calculated by a variational method which includes the effect of configuration interaction. Other forces which would contribute to the splitting, such as spin-dependent central forces or vector spin-orbit forces, are not considered. The method of calculation is based on the use of a variational function of the form $\psi=\psi_0+\lambda t'\psi_0$, where t' is essentially the tensor force, treated as a perturbation on a central force oscillator wave function, ψ_0. The effect of the tensor force is shown to be equivalent to a mixture of ordinary and spin-exchange central forces plus a vector type spin-orbit force of rather complicated structure. Using a Hu-Massey Gaussian shape tensor potential, an S-state splitting of 1.4 Mev is found for Li^6 and an inverted P-doublet splitting of 380 kev is found for Li^7. A Yukawa shape potential would give similar results. In view of the approximations made in the analysis, these results are in reasonable agreement with the experimental splittings of 3.5 Mev and 480 kev for Li^6 and Li^7 respectively. The tensor force is found to contribute about 12 Mev to the binding energy of these nuclei and to introduce a 6 percent admixture of excited states into the ground state. The importance of configuration interaction is shown by a second-order perturbation calculation neglecting configuration interaction which gives entirely different results—a negligible S-state splitting for Li^6 and a normal P doublet structure for Li^7. The effect of the tensor force on the P-doublet separation in Be^7 and the F doublet separation in Li^7 is discussed briefly.

I. INTRODUCTION AND SUMMARY

PRIOR to the discovery of the quadrupole moment of the deuteron[1] and its interpretation in terms of a tensor force,[2] many theoretical studies of the level structure of light nuclei were made on the basis of central forces alone.[3–5] To allow for the observed 2.3-Mev singlet-triplet splitting in the deuteron, central spin-exchange forces were used. In view of the fact that the tensor force can account for the entire amount of the deuteron splitting,[6,7] it is clear that the level structure of other nuclei will also be considerably affected by the presence of the tensor force.

In recent years extensive calculations of the level structure of light nuclei have been made by many authors[8–10] using the vector spin-orbit force of the Mayer-Jensen shell model.[11] Qualitative agreement with the observed level structure of the p-shell nuclei can be obtained in this way. For Li^6 and Li^7 practically pure LS coupling seems indicated.[9,12] In the present work we shall neglect the possible presence of vector-type spin-orbit forces, and assume that the nuclear potential consists solely of a mixture of charge- and spin-independent central forces plus a tensor force.

Previous studies of the effect of the tensor force on light nuclei other than the deuteron have been concerned mainly with its effect on their binding

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* Based on the author's Ph.D. thesis, Princeton University, 1952. A preliminary account of some of the results of this work was given at the Washington meeting of the American Physical Society [A. M. Feingold and E. P. Wigner, Phys. Rev. 79, 221(A) (1950)].
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