(p,d) Pick-Up Deuteron Measurements at 95 Mev*

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Measurements in some detail have been made on carbon and beryllium, and a few measurements on other elements. On the Born approximation, the results can be analyzed to give the momentum distribution of the picked-up neutron, and consequently some information about its wave function. The probable validity of this approximation at 100 Mev is examined, and seems reasonably good. The experimental results for carbon and for the pickup of a "tightly-bound" neutron from beryllium show strong high-momentum components indicating that the nuclear wave function is not strictly of independent-particle nature but that strong two-body interactions are operative inside the nucleus. There is also some indication of alpha-particle substructure in C and Be.

INTRODUCTION

ONE method of investigating the internal structure of the nucleus is by elastic or inelastic scattering of high-energy particles for which the nucleus may be treated as rather transparent. Elastic scattering in this case gives information essentially on the density distribution in the nucleus averaged over all the constituent nucleons. Inelastic scattering, unlike elastic scattering, involves transitions between different nuclear states. If the nuclear wave function can be meaningfully approximated by a product form involving single-nucleon wave functions, then in inelastic scattering we may hope to obtain information on the corresponding single-nucleon states in the nucleus. As an analog in this case, one can observe how inelastic processes involving atomic electrons—e.g., the photoelectric effect—give information essentially on "individual" electronic states—for example on their energies. One might expect that in inelastic scattering the nucleus might show some aspects of independent-particle behavior in view of the rather high degree of success of the shell model and of the optical model, in which a nucleon is pictured as interacting with the rest of the nucleus as with a smooth potential well, imagined as representing an averaged effect of the actual interactions. It has been generally somewhat puzzling to see how well the predictions of such a model agree with experiment; it has in particular been unclear whether a smooth effective potential well could be reconciled with the very strong short-range forces known (from high-energy scattering experiments) to exist in the two-body interactions. One possible explanation would be that inside a complex nucleus—i.e., in dense nuclear matter—the nuclear forces are substantially different than in the case of two isolated nucleons. A more generally believed explanation has been one which invokes the exclusion principle to inhibit "collisions" inside the nucleus; to precisely obtain the detailed quantitative effects of this inhibition requires an essentially correct treatment of the many-particle nuclear problem. Just recently some progress in this direction appears to have been made by Watson, Brueckner, and collaborators, whose results indicate that independent-particle models may be quite successful even though the correct wave function includes strong correlation effects.[1]

One would like to determine, if possible, whether the detailed behavior of a nucleon inside a complex nucleus is very closely that given on an independent-particle model, or whether instead it shows the effects of strong local interactions. As remarked above, one means of investigating this matter is inelastic scattering. Another type of inelastic process which may be used for this purpose and with which this report is concerned, is the "pick-up deuteron" reaction, first observed at high energies by York.[2] It has been found that in the (p,d) reaction at about 100-Mev discrete energy states show up clearly in the energy spectrum of deuterons produced —energy states corresponding to low-lying states of the residual nucleus.[3] On the independent-particle model, and on the impules approximation,[4] the analogy mentioned above with the atomic photoelectric process may be applied. The "snatching" of a nucleon from a particular energy state will then leave a "hole" and a corresponding state of the residual nucleus. The difference between the deuteron energy and the incident proton energy gives the energy level of the target nucleon, and the angular distribution of deuterons of a given energy group gives information on the momentum distribution associated with that particular state—see Fig. 1. If some interaction is present among the nucleons in the target nucleus, then the snatching of a given nucleon may leave the residual nucleus in one of two or more states, rather than in a unique state —this with relative probabilities described by the fractional parentage coefficients.[5]

This report describes pick-up deuteron measurements made with about 3-Mev resolution—measurements made in some detail on carbon and beryllium, and a few measurements on several other elements. The

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[1] See Brueckner, Eden, and Francis, Phys. Rev. **98**, 1445 (1955).
[2] J. Hadley and H. York, Phys. Rev. **80**, 345 (1950).
[3] W. Selove, Phys. Rev. **92**, 1328 (1953).
[4] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[5] A. M. Lane and D. H. Wilkinson, Phys. Rev. **97**, 1199 (1955).

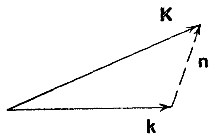

FIG. 1. Momentum diagram of the (p,d) reaction. A proton comes in with momentum $\hbar\mathbf{k}$, a deuteron goes out with momentum $\hbar\mathbf{K}$ at angle θ. (Mass of target nucleus considered infinite.) The momentum difference $\hbar(\mathbf{K}-\mathbf{k})\equiv\hbar\mathbf{n}$ is supplied by the picked-up neutron.

program is continuing, but it seems worthwhile to report the results found so far, which indicate that the nuclear wave function (at least, for carbon) is not strictly of independent-particle nature, but that strong local two-body interactions are operative even inside the nucleus.

APPARATUS AND MEASUREMENTS

A sketch of the experimental arrangement is shown in Fig. 2. Particles were detected in an 8-counter scintillator telescope, and identified by simultaneous measurement of range and specific ionization; this is described further, below. Measurements were made in four range intervals simultaneously, each of range increment about 0.125 g/cm^2 of scintillator plastic, corresponding to about 1 Mev for 95-Mev protons, about 2 Mev for 75-Mev deuterons. The angles θ of the detector and ϕ of the scatterer, the absorber thickness, and the scatterer in-out switching were controlled remotely. The external beam had an energy width of the order of 2 Mev as measured with the telescope. The energy was determined primarily by the field strength in the auxiliary magnet and by the accompanying slit system—this magnetic field was continuously monitored with a proton-resonance magnetometer, and the energy of the beam was also frequently measured with the telescope; the energy remained constant within a fraction of one Mev over periods of days. The beam as normally used had a width of about $\frac{3}{4}$ inch and a height of about $1\frac{1}{4}$ inch at the scatterer, and an intensity of about 10^{-12} ampere. The monitor used was an ion chamber, calibrated by a Faraday cup; the calibration was found to be very stable.

The energy resolution is determined by the energy width of the beam, the inherent resolution (due to finite range interval and staggling) of the telescope, and the energy smearing of the scatterer. The latter smearing can be made a minimum—to first order, zero—by suitable choice of ϕ, the scatterer angle. The angle required is given by

$$(dE/dx)_1 w_1 = (dE/dx)_2 w_2, \qquad (1)$$

where subscripts 1 and 2 refer respectively to incoming and outgoing particles, dE/dx is the energy loss per unit thickness and w is the maximum path length through the scatterer. Since $w_1 = t \sec\phi$ and $w_2 = t \sec\times\phi(-\theta)$, where t is the scatterer thickness, (1) gives

$$\tan\phi = (b-\cos\theta)/\sin\theta, \qquad (2)$$

where $b=(dE/dx)_2/(dE/dx)_1$. For a typical case, for carbon, with 95-Mev protons incident and 75-Mev deuterons outgoing, we have $b=2.07$; thus for $\theta=20°$, e.g., (2) gives $\phi=73.2°$. To give exact first-order compensation, ϕ must be changed with θ. ϕ as given by (2) has typical values of the order of 65° to 75° in this experiment, and even larger values when the scattering angle θ is small. The use of very large values of ϕ has the following disadvantages: (i) Very wide thin targets are required. (ii) The distance from scatterer to telescope varies for different parts of the beam—since the energy distribution of incident particles also varies for different parts of the beam, this leads to a dependence of the effective energy on the angles ϕ and θ. (iii) At large values of ϕ the effective sample thickness, $t \sec\phi$, is a very sensitive function of ϕ—consequently the determination of an accurate absolute cross section requires high precision in the knowledge of ϕ. For these various reasons, measurements with a given scatterer were made with ϕ kept fixed, at a value near the optimum value, over a wide range of detector angle θ; and the maximum value of ϕ used was 75°. The scatterer thicknesses used were such that the fractional contribution of scatterer energy-smearing to the over-all resolution was generally not large; the effective resolution was typically about 3 Mev.

The angular acceptance of the telescope was determined primarily by the effective beam width at the scatterer and the width of the defining counter, counter C. The angular resolution determined by these was about $2\frac{1}{2}°$ (full width at half-maximum). The other factors contributing to the effective angular resolution—the angular spread in the incident beam (about $\frac{1}{2}°$),

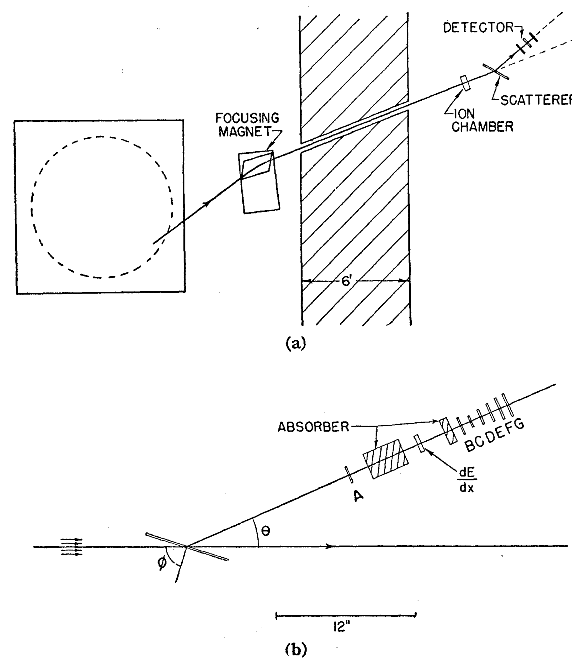

FIG. 2. Arrangement of equipment. (a) Plan of cyclotron setup. (b) Dectector telescope.

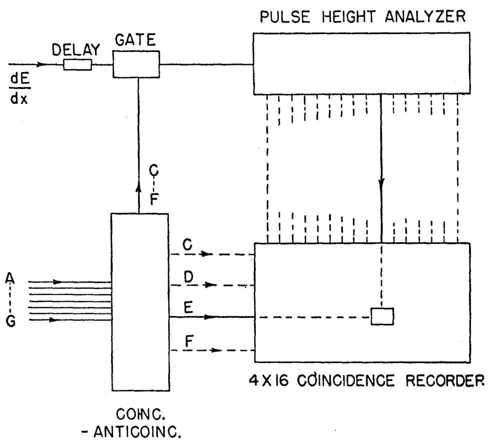

FIG. 3. Block diagram of circuits.

and the multiple Coulomb scattering in the scatterer and absorber (typically about $\frac{1}{2}°$ and 1° respectively)—made the effective resolution typically about 3°.

The detector telescope contained 7 plastic scintillators, *A* through *G*, and a stilbene scintillator to measure specific ionization, dE/dx. The defining scintillator, *C* was 1 in.×$1\frac{1}{2}$ in.; the other scintillators were all large enough to make net scattering losses in the telescope less than a few percent. The plastic scintillators were viewed by 1P21 phototubes, selected for good signal-to-noise ratio under operating conditions. The dE/dx crystal was thick enough (0.5 g/cm^2) to reduce the width of the pulse-size distribution for monoenergetic particles to a satisfactorily small value; this crystal was viewed by a selected 5819 phototube For each charged particle stopping in scintillators *C*, *D*, *E*, or *F*, range was determined by means of a set of coincidence circuits. The 1P21 signal were combined (after pre-amplification with EFP-60 tubes) in twofold coincidence circuits with resolving time about 0.015 μsec, followed by multiple-input coincidence circuits of 1 μsec resolving time. The outputs of the latter were fed into a further coincidence-anti-coincidence circuit, which delivered signals corresponding to particles stopping in scintillators *C*, *D*, *E*, or *F*. For each such output signal, the corresponding dE/dx signal (having meanwhile been passing through a delay line to provide time for making the coincidence tests) was passed through a gate circuit into a 16-channel pulse height analyzer, and the pulse size and range then recorded simultaneously in a set of 4×16 registers, as indicated in Fig. 3. The basic element of the gate circuit is shown in Fig. 4. The gate length used was $\frac{3}{4}$ μsec; the dE/dx signal was delay-line-clipped to $\frac{1}{2}$ μsec.

Deuterons and protons of the same range in the telescope have an energy loss ratio of about 1.37 to 1 in the crystal. The pulse-size resolution obtained in this experiment for a single group of monoenergetic particles was about 10% (full width at half-maximum). Thus even with the additional dE/dx spread due to the finite range intervals, and in spite of a fairly broad tail on the pulse-size distribution, clean proton-deuteron separation was obtained, and it was possible to measure easily a group of deuterons in the presence, for example, of an inelastic proton group of the same range but of tens of times the intensity and of an elastic proton group of different range but of thousands of times the intensity. Some tritons were observed at low range values; the deuteron-triton separation was not as clean as the proton-deuteron separation, but could be made unambiguously enough so as to introduce only a negligible uncertainty in the deuteron intensity.

The results of a typical run on carbon are shown in Fig. 5. These results show a proton intensity (the left-hand peak) approximately uniform in the energy region covered, and a deuteron distribution which is of

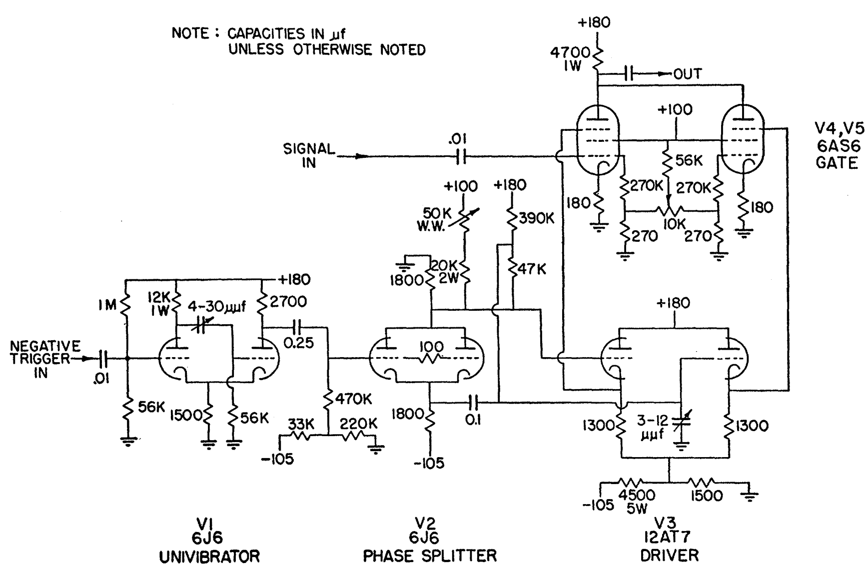

FIG. 4. 1-microsecond gate circuit. *V*4 is normally off; when it is switched on, the plate current surge is balanced by switching *V*5 off. The adjustments in the circuits of *V*2–5 are to balance out switching effects.

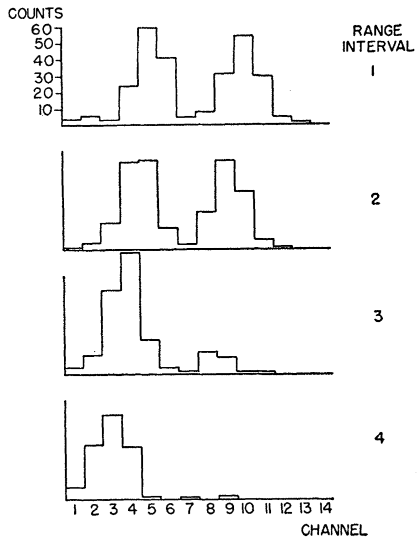

FIG. 5. Pulse-size (dE/dx) distributions in one set of four range intervals. Carbon scatterer, 0.17 g/cm^2, at 70° to beam (ϕ, Fig. 2b). Detector at 24°, $\Delta\Omega = 0.0022$ sterad. Length of run: 7 minutes. The right hand peaks correspond to deuterons of about 75 Mev, the left hand peaks to protons of about 55 Mev.

high intensity in the first two range intervals and falls off to almost zero in the last interval.

The background intensity of deuterons, due almost entirely to deuterons produced in air, was generally no more than a few percent. The energy of the background deuterons changes when the scatterer is present, and the measured background was appropriately energy-corrected before subtraction. In most cases the geometry was such that those background deuterons produced in the air ahead of the scatterer had to traverse the scatterer before reaching the detector. For large detector angles this was not generally true, so that proper energy-correction of background would be more difficult; but there the background was generally less than one percent, so that exact correction was unimportant.

The absolute cross sections given below were obtained by converting the measured range spectrum to an energy spectrum and applying a correction for absorption and net out-scattering in the telescope. The correction, mostly due to nuclear absorption, has been calculated, and well-confirmed experimentally,[6] and varies from about 9% for 55-Mev deuterons to 14% for 80-Mev deuterons. The over-all uncertainty in the absolute cross sections due to all causes other than counting statistics and scatterer angle positioning is believed to be about 5%; the scatterer angle positioning error may have also introduced an uncertainty of about 5%.

EXPERIMENTAL RESULTS

Figure 6 shows the deuteron energy distribution, at several angles, from the $C^{12}(p,d)$ reaction for incident protons of about 95 Mev. The group corresponding to formation of C^{11} in its ground state is seen to be dominant, with some production of excited states present, apparently principally one or both of the pair near 4.5 Mev. The larger width of the main peak at small angles and probably at 48°, is due to greater energy-smearing in the scatterer at these angles.

Figure 7 shows the results for $Be^9(p,d)$. The energy resolution and statistical accuracy were in general not high enough to permit separation of the ground state and 2.9-Mev state[7] groups of Be^8 except in the 16° data, which were taken with an especial view to separation of these two groups. The group corresponding to about 17-Mev excitation of Be^8 does not have a uniquely identifiable Q value, and probably includes contributions from various levels, perhaps of differing angular distribution.

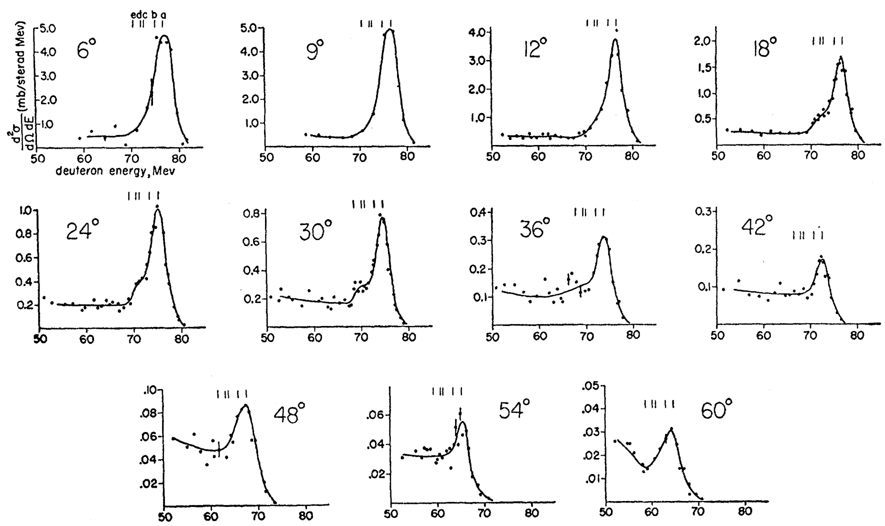

FIG. 6. Deuteron energy distributions for the $C^{12}(p,d)$ reaction for incident protons of about 95 Mev, at a number of different angles. (The laboratory angle is given.) The specific incident energy varies between 94 and 95 Mev for the first 8 angles, and is about 91.5 Mev (thicker target) for the last 3. The expected energies corresponding to production of the first 5 states of C^{11}, at excitation energies of 0, 1.9, 4.23, 4.77, and 6.46 Mev,[7] are indicated by $a-e$. Statistical errors are indicated for a few representative points, in this and the following three figures.

[6] J. M. Teem, Ph.D. thesis, Harvard University 1954 (unpublished).
[7] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

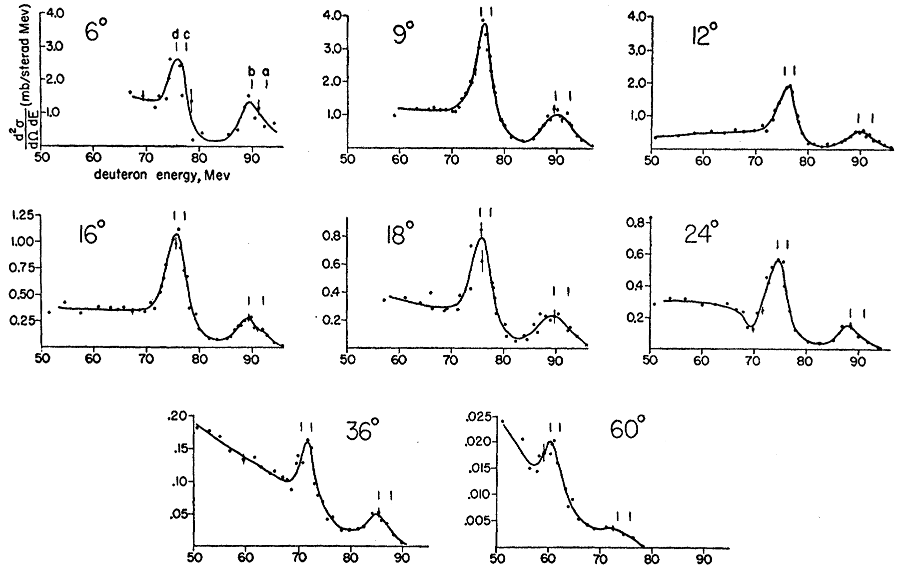

FIG. 7. Deuteron energy distributions for the $Be^9(p,d)$ reaction at about 95 Mev (60°: 91.4 Mev). The laboratory angle is given. The lines a, b, c, d show respectively the expected energies corresponding to production of Be^8 in its ground state, in the 2.90-Mev state,[7] and in states having excitation energies of 16 and 18 Mev.

Figure 8 shows the results for aluminum (100% Al^{27}) and silicon (92% Si^{28}), and Figs. 9(a) and 9(b) for copper and lead, all at 18°.

These results are discussed in the following sections.

REMARKS ON THE THEORY

Theoretical calculations pertaining to the interpretation of pickup deuteron distributions in terms of internal momentum distributions have been made by Chew and Goldberger[8] and by Heidmann.[9] The former appears more readily adapted to an independent-particle model; Heidmann's calculation was principally directed toward explaining the apparent presence of strong high-momentum components, and he suggests that the explanation is to be found in the strong interactions of nucleons with each other inside the nucleus. The present results strongly support Heidmann's idea that these correlation interactions are responsible for relatively strong high-momentum components. However, his calculation used Fermi-gas states for the individual nucleons, and consequently gave rise to a broad energy spectrum of deuterons. The experimental results, on the other hand, show sharp energy groups, indicating that the energy state of the picked up nucleon is more sharply defined than would be true in a Fermi distribution. One is therefore led to consider the nucleons as being in shell-model type states. The reconciliation of this model of independent non-strongly-interacting nucleons with Heidmann's idea of strong interactions will be discussed separately. For the present purpose, the data will be analyzed in terms of the Chew-Goldberger (CG) theory, applied to an independent-particle model.

Derivations of this Born-approximation theory for the pick-up process have been discussed by a number of authors.[10] The Born-approximation theory gives the differential cross section in the center-of-mass system for the pickup of a single nucleon from a state $u(\mathbf{r}_1)$ as

$$\frac{d\sigma}{d\omega}=3\pi|F|^2\frac{A(A-1)}{(A+1)^2}\frac{K}{k}N(\mathbf{n})\times(\alpha^2+q^2)^2\left|\int d\mathbf{r}e^{i\mathbf{q}\cdot\mathbf{r}}D(\mathbf{r})\right|^2, \quad (3)$$

where

$$\mathbf{n}=\mathbf{K}-(A-1)\mathbf{k}/A,$$

$$N(\mathbf{n})=|\phi(\mathbf{n})|^2=\left|\frac{1}{(2\pi)^{\frac{3}{2}}}\int d\mathbf{r}_1e^{-i\mathbf{n}\cdot\mathbf{r}_1}u(\mathbf{r}_1)\right|^2, \quad (4)$$

$$\mathbf{q}=\mathbf{k}-\tfrac{1}{2}\mathbf{K},$$

$$\hbar^2k^2=2m\left(\frac{A}{A+1}\right)^2E_0,$$

$$\hbar^2K^2=\frac{4m(A-1)}{A+1}\left[\frac{A}{A+1}E_0+Q\right]. \quad (5)$$

Here, F is the fractional parentage coefficient,[5] which may be regarded as the matrix element for the $(A-1)$ nucleons which remain in the residual nucleus, A is the atomic weight of the target nucleus, k and K are $\hbar^{-1}$ times the momenta of the incoming nucleon and the outgoing deuteron (see Fig. 1), m is the mass of a nucleon, $\alpha^2=(m/\hbar^2)$ times the binding energy of the deuteron, E_0 is the laboratory energy of the incoming nucleon, Q is the energy released in the reaction, and $D(\mathbf{r})$ is the normalized ground-state deuteron wave function. $\mathbf{r}$ is the neutron-proton separation; $\mathbf{r}_1$ is measured from the center-of-mass of the residual

[8] G. F. Chew and M. L. Goldberger, Phys. Rev. **77**, 470 (1950).
[9] J. Heidmann, Phys. Rev. **87**, 171 (1950).
[10] See, e.g., E. Gerjuoy, Phys. Rev. **91**, 645 (1953); N. C. Francis and K. M. Watson, Phys. Rev. **93**, 313 (1954).

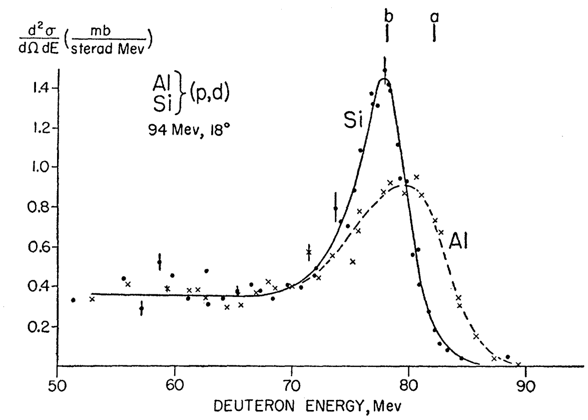

FIG. 8. Deuteron energy distribution for Al and Si. The lines a and b correspond to the ground-state production of Al^{26} and Si^{27} respectively.

nucleus. Coulomb effects are neglected, and the nucleus is treated as transparent. $N(\mathbf{n})$ is the normalized momentum density of the picked-up nucleon. If the orbital angular momentum number l of the state $u(\mathbf{r}_1)$ is not zero, Eq. (3) still holds when $N(\mathbf{n})$ is understood to be averaged over angles: $N(\mathbf{n})=(4\pi)^{-1}\int d\Omega N(\mathbf{n},\theta_n)$. The total normalized momentum-density integral is then given for any l by $\int 4\pi n^2 dn N(\mathbf{n})$.

The angular dependence of the cross section is given by the last three factors of (3). The first of these is the probability of finding the target nucleon with the correct momentum to be picked up—a momentum of approximately $\hbar(\mathbf{K}-\mathbf{k})$ for a heavy nucleus. The other two factors give the probability that a deuteron can be formed with the proper internal momentum $\hbar q$. If one puts into (3) an assumed deuteron wave function, one can then calculate the momentum density $N(\mathbf{n})$ from the experimental cross section.

Before making extensive interpretations on the basis of this theory, one would like to have some idea as to the validity of the theory. A good test is provided by investigating whether the internal momentum distribution obtained by it from experimental data is independent of incident energy. Very few high-energy data are available from which such a test can be made. There is one measurement on C^{12} at 31 Mev,[11] which checks well with the present data when a Hulthén wave function is used for $D(\mathbf{r})$; however, this single point does not constitute a very good test, since nuclear opacity effects could be quite different at 30 and 95 Mev.

Another test consists of applying (3) to the pick-up peak from deuterium itself. Here, the results are moderately encouraging. In this case, $n=q$, and taking $F=1$, one obtains

$$\left(\frac{d\sigma}{d\omega}\right)_{\text{nucleon-deuteron}}=\frac{(2\pi)^4}{3}(\alpha^2+q^2)^2[N_d(\mathbf{q})]^2, \quad (6)$$

[11] R. Britten, Phys. Rev. 88, 283 (1952).

so that one may try to obtain $N_d(\mathbf{q})$ for the deuteron from the experimental data. Unfortunately, the pick-up peak in nucleon-deuteron scattering is not entirely isolated, experimentally or theoretically, from the "direct" scattering which dominates at forward angles, and so this theory does not provide a clean method of determining $N_d(\mathbf{q})$. Thus, e.g., at 95 Mev the p-d cross section at the pick-up peak has a maximum value of 4.0±0.5 mb/sterad,[6] but this may include interference effects from a "direct" contribution. This direct contribution appears to correspond to a cross section (if there were no pick-up contribution) of about 0.5 mb/sterad; this is large enough that, according as there is destructive or constructive interference, the "pure" pick-up term would be about 7 or about 2 mb/sterad, respectively. If one uses a Hulthén function,

$$D(\mathbf{r})=\left[\frac{\alpha/2\pi}{1+(\alpha/\beta)-4\alpha/(\alpha+\beta)}\right]^{\frac{1}{2}}\frac{e^{-\alpha r}-e^{-\beta r}}{r}, \quad (7)$$

$$\beta=6.2\alpha, \quad (7a)$$

the calculated corresponding peak cross section from (6) is 9 mb/sterad. This would then be in moderately good agreement with the experimental value if there were indeed destructive interference present—in fact, perhaps in better agreement than can reasonably be expected from such a simplified theory. Teem[6] has analyzed the shape of the pick-up peak at 95 Mev, and concludes from this and from other evidence that there

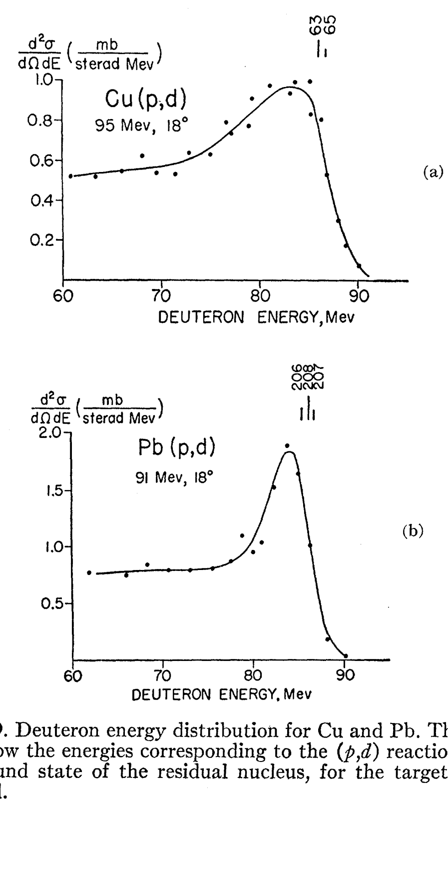

FIG. 9. Deuteron energy distribution for Cu and Pb. The upper lines show the energies corresponding to the (p,d) reaction giving the ground state of the residual nucleus, for the target isotope specified.

is strong indication that destructive interference is present. He finds, moreover, that if this interference between the "pick-up" and "direct" terms is assumed to be destructive, then the momentum distribution obtained from the experimental data is in good agreement with that given from the Hulthén wave function of (7).

There is further evidence giving rough support to the moderately good validity of this theory for the p-d pick-up process at high energies, in the energy dependence of the cross section at the pick-up peak. Using (7), (6) gives

$$\frac{d\sigma}{d\omega}=2700\left(\frac{\alpha^2}{\alpha^2+q^2}\right)^2\left(\frac{\beta^2-\alpha^2}{\beta^2+q^2}\right)^4$$

$$=\frac{2700}{(1+0.1E_0)^2}\left(\frac{37.4}{38.4+0.1E_0}\right)^4 \text{ mb/sterad,}$$

for the pickup contribution at the peak, at an incident nucleon energy E_0 in Mev. Bratenahl[12] has measured $d\sigma/d\omega$ for deuterium for angles near the pick-up peak at 180°, over the energy range 95 to 140 Mev, and finds good agreement with the theoretical predictions. Some measurements made at higher energies, up to 200 Mev, at Rochester, do not agree so well with the theory, but also do not agree well with measurements at Berkeley[12] and Harwell.[13] It may be noted that the internal energies, $\hbar^2q^2/m, =2E_0/9$ at 180°, involved in Bratenahl's work are 20 to 30 Mev—in this range, the behavior of $N_d(\mathbf{q})$ is essentially determined by the binding energy of the deuteron, so that high-momentum components of the deuteron wave function are not really under test here. The theory, however, is under test, except for the uncertainty involving the interference effect mentioned above.

These results offer encouragement that the Born-approximation theory may in fact be reasonably good for light nuclei at 100 Mev or more, although opacity effects, reducing the cross section, cannot easily be estimated very accurately. In applying the theory, the Hulthén function (7) is used, following Chew and Goldberger, and consistently with the p-d pick-up results. (3) then takes the form

$$\frac{d\sigma}{d\omega}=\frac{24\pi^2\alpha}{1+(\alpha/\beta)-4\alpha/(\alpha+\beta)}\frac{A(A-1)}{(A+1)^2}\frac{K}{k}\times|F|^2N(\mathbf{n})\left(\frac{\beta^2-\alpha^2}{\beta^2+q^2}\right)^2. \quad (8)$$

One may use this to calculate the shape of $N(\mathbf{n})$ from the experimental data; and on the independent-particle model, one may take the square root of this result to obtain the form of the momentum wave function $\phi(\mathbf{n})$.

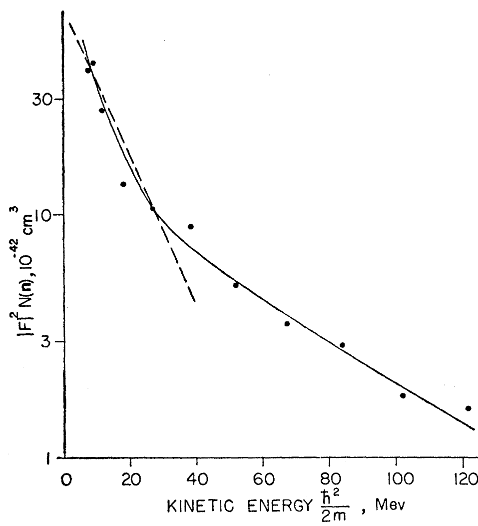

FIG. 10. Momentum density corresponding to the ground-state group, $C^{12}(p,d)C^{11}$. The points are experimental points analyzed according to Eq. (8). The curve is an empirical fit, proportional to $e^{-E/7}+0.15e^{-E/50}$, where E is the kinetic energy $\hbar^2n^2/2m$ in Mev. The dashed line is a 14-Mev gaussian. The abscissa should be labeled $\hbar^2n^2/2m$ rather than $\hbar^2/2m$.

TABLE I. Angular distribution of the main group (presumed to be ground-state group) for $C^{12}(p,d)C^{11}$. Statistical accuracy about ±6%.

θ, lab angle	$d\sigma/d\Omega$, mb/sterad
6°	22.1
9°	22.8
12°	13.9
18°	6.1
24°	4.1
30°	2.89
36°	1.36
42°	0.75
48°	0.50
54°	0.25
60°	0.18

ANALYSIS AND DISCUSSION

The Main Group, $C^{12}(p,d)C^{11}$

The energy spectra in Fig. 6 indicate that the main group is due principally, if not entirely, to deuterons leaving C^{11} in the ground state; the 1.9-Mev state does not appear to be very strongly produced, although the corresponding group could be present with as much as 20% or so of the intensity of the ground-state group. The area under the main peak has been determined on the assumption that the continuum does not extend under the main peak, and the results are given in Table I. Because of the limited energy resolution the proper lower cut-off point is uncertain; consequently the area values involve an uncertainty increasing to perhaps 20% at the largest angles.

The data of Table I were converted to center-of-mass

[12] A. Bratenahl, thesis, Berkeley, 1952 (UCRL report 1842, unpublished).

[13] See D. Klein, University of Rochester report NYO-6450 (unpublished).

cross sections, with relativistic corrections included in the kinematics. The results, analyzed by Eq. (8), give the momentum spectrum of the picked-up neutron. The results are plotted as the points in Fig. 10, with an empirical curve which gives a rough fit to the points. This curve is given by

$$|F|^2N(\mathbf{n})=(e^{-E/7}+0.15e^{-E/50})\times10^{-40}\text{ cm}^3, \qquad (9)$$

where $E=\hbar^2n^2/2m$ is the nucleon kinetic energy in Mev. This expression has a behavior for small E which corresponds roughly to what would be expected from a spherically symmetric state of 18.7-Mev binding energy, the binding energy of a neutron in C^{12}.

The value of $\int d\mathbf{n}N(\mathbf{n})$, which can indicate the number of neutrons effective in producing this energy, group, is quite sensitive to the assumption made about the behavior of $N(\mathbf{n})$ at values of n beyond those reached experimentally. The empirical expression (9) gives the value 0.061 for this integral. On the other hand, the distribution given by Chew and Goldberger,[8] which also fits the present results reasonably well in the range covered experimentally (on the average to about 10 or 15%) but which at higher momenta falls off more slowly than (9), was normalized by them to a value corresponding to 0.76. There is some evidence that their distribution gives incorrectly strong high-momentum components.[14] At the same time, there is no reason to expect the distribution (9) to be especially accurate at high momenta—this distribution can merely be taken as one which gives a reasonable smoothed fit to the present data in the experimental range, and which at extremely high momenta does not disagree with other experimentally derived information. As a matter of fact, there are two reasons for believing that the momentum distribution even for the higher momenta reached in the present data may not be given very accurately by (8). One reason is that (8) is based on an independent-particle model, and further analysis has shown that the present data can probably not be explained properly by a simple shell model; this is discussed further below. Secondly, the fact that the value of $|F|^2\int d\mathbf{n}N(\mathbf{n})$ as calculated from the data is so small suggests that opacity effects may be quite important; and they may have an appreciable effect on the angular distribution.

The dashed line represents a Gaussian momentum density of the form $e^{-E/14}$. Such a distribution has been used to fit the results of quasi-elastic measurements.[15] It can be seen that this distribution gives good agreement with the results of the present analysis in the energy range from about 5 to 30 Mev, but not at higher energies. Other remarks on the interpretation of the present results and on their relation to other information concerning the momentum distribution have been made briefly in a previous note,[14] and a more detailed discussion will be given separately.

[14] W. Selove, Phys. Rev. **98**, 208 (1955).
[15] See footnotes 11–13 of preceding reference.

An important conclusion can be drawn from the results shown in Fig. 10—namely, the high-momentum components are too strong to be accounted for by a purely independent-particle model, as one finds out by calculating the momentum spectrum for carbon nucleons on such a model. Consequently, one infers that the potential acting on a nucleon in carbon includes strong short-range fluctuations and does not consist purely of a smoothed well.[14]

The existence of strong high-momentum components has been strongly indicated by previous experiments but perhaps not as cleanly as by the present results. Brueckner, Eden, and Francis have independently considered this aspect of the nuclear internal momentum distribution, and have discussed it and its bearing on the nuclear model in a recent paper.[1] Their calculation of the high-momentum distribution making use of nucleon-nucleon scattering cross sections gives results closely resembling those of Fig. 10, although that calculation is not intended to apply for low internal momenta. Other calculations of the correlation effect on the momentum distributions, and of other corrections to the first-order calculation, are being pursued here, in an attempt to obtain a better idea as to just how precisely one can interpret the results of the high-energy deuteron pick-up process in terms of a momentum distribution and a nuclear model.

Other States, $C^{12}(p,d)C^{11}$; the Question of α-Particle Structure

As suggested by Chew and Goldberger on the basis of York's early results, it appears that the pick-up reaction in C^{12} at energies around 100 Mev involves predominantly a single final state in C^{11}, and thus by implication a single initial state for the picked-up nucleon. This is what one would expect on the α-particle model. On the independent-particle shell model, on the other hand, one might expect for C^{12} to find two different types of initial nucleon states, $1s$ and $1p$, and correspondingly two distinct energy groups of deuterons. What energy spacing is to be expected for these two groups?

On a simple oscillator or square-well model, one can estimate that the $1s$ state should be 10 to 15 Mev below the $1p$, if spin-orbit coupling and pairing energy are not included. In estimating the deuteron energy difference to be expected when a nucleon is snatched from a $1s$ or $1p$ state in C^{12}, however, the pairing energy —more accurately, the grouping energy—enters in a very important way. The $1s$ shell represents the most tightly bound 4-group in the nucleus, and may be expected to retain some degree of the resistance to breakup shown in the elemental nucleus containing this group, He^4. This suggests that states with a $1s$ hole will have energies far above the ground state. At the same time, there is an especially strong counteracting tendency in the case of C^{11}. The ground-state configuration is $(1s_{\frac{1}{2}})^4(1p_{\frac{3}{2}})^7$. If we ask for the excitation energy corresponding to the configuration $(1s_{\frac{1}{2}})^3(1p_{\frac{3}{2}})^8$, then indeed the $1s$ shell has been broken up, but at the same time the $1p_{\frac{3}{2}}$ subshell has been closed, which tends to reduce the excitation energy. The result may be to make the net excitation energy very small. For an estimate we may observe that to remove a $1s$ neutron from C^{12} might be expected, in analogy with He^4, to require of the order of 20 Mev. This is to be compared with the energy required to remove a $1p$ neutron—this presumably leads to the ground state of C^{11}, and this process requires 18.7 Mev. As suggested by Lane, therefore,[16] one might expect to find the $(1s)^3(1p)^8$ configuration, $\frac{1}{2}^+$, within a few Mev of the ground state (presumably, like B^{11}, $\frac{3}{2}^-$). The spin and parity assignments for the low-lying states of C^{11} are very incompletely known. However, in the charge-symmetric nucleus B^{11} there is some evidence suggesting that the first excited state, at 2.14 Mev, may have $J=\frac{1}{2}$ and may be of opposite parity to the ground state.[7]

It may be concluded, then, that the extraction of a $1s$ neutron from C^{12} might lead to one of the rather low-lying states in C^{11}, perhaps to the first-excited state. The intensity to be expected from the corresponding deuteron group, relative to the intensity of the ground state group, contains a factor $\frac{1}{2}$ because there are two $1s$ neutrons but four $1p$ neutrons in C^{12}, and an angle-variable factor involving the momentum distribution. It seems somewhat surprising that if a group reflecting these properties were present it would not have appeared more prominently. However, the energy resolution is not quite adequate to make firm statements about this matter. A more definite conclusion will have to await measurements with forthcoming higher resolution although the evidence does appear to be against the existence of a "$1s$" group of proper intensity.

A convincing demonstration of the existence of shell-model type states would exist if one were to find groups having appropriate momentum distributions (i.e., as determined through angular distributions) to display the value of l involved. The determination of the l value involved is similar in principle to the procedure used in low-energy stripping or pick-up reactions, but unfortunately the kinematics of the reaction are such that at the present energy the l value cannot easily be identified. This may be understood from the example of a pickup involving an $l=1$ state. At low energies (say, below 10 or 15 Mev) the forward minimum which is characteristic of such a process reflects the fact that at forward angles the momentum n of the picked-up nucleon is small, and in a state with $l>0$ the momentum density $N(\mathbf{n})$ is zero for $n=0$. At high energies, on the other hand, even at forward angles the momentum n may not be small enough to be below the peak of the $N(\mathbf{n})$ distribution. As has been suggested,[14] it would be very valuable in this connection to explore the forward angles using an initial energy (center-of-mass) about twice the magnitude of the Q of the reaction. For $C^{12}(p,d)C^{11}$ this means an initial energy of 30 to 40 Mev. Even at that low an energy the Born-approximation calculation should still be good enough for the determination of l.

It remains to consider the possibility that the extraction of a $1s$ neutron from C^{12} might require much more energy than is estimated above, and so might lead to a highly excited state of C^{11}. As Lane and Wilkinson have suggested in another case,[5] such a highly excited state would be likely to have a very short lifetime, and so might have so great an energy width that the corresponding deuterons would not be clearly identifiable as a sharp group. In this case, then, as in the case of the α-particle model, only a single strong sharp group would be observed from C^{12}.

In trying to evaluate this possibility, it would be helpful if the absolute values of the observed cross sections gave some evidence as to the number of neutrons contributing to a particular group. For example, if the ground-state group corresponds to pickup of one of the four $(1p)$ neutrons present on the shell model, and if the ground state of C^{11} were the sole parent state for these neutrons, then one would expect $|F|^2\int d\mathbf{n}N(\mathbf{n})$ to be equal to 4. This is to be compared with the experimentally indicated value, which is probably something between the 0.061 given by (9) above, and the 0.76 given by the Chew-Goldberger distribution. This is rather small compared with what might be expected, and presumably indicates rather large effects from nuclear opacity and from the parentage overlap matrix element. One may reasonably expect that the explanation involves these effects rather than more general failings of the theory, in view of the respectable quantitative agreement in the p-d case. In any event, the quantitative experimental results are evidently not of much help in deciding between possible models for C^{12}. A better selection between models may result when measurements are made with better energy resolution, and also at lower energies in order to give better information on the l value involved.

The interpretation of the C^{12} results in terms of alpha-particle structure is discussed further, in the next section; in comparison with the results for He^4 and Be^9, there is some suggestion that C^{12} and Be^9 do indeed show evidence of alpha-particle structure.

Prominent Groups, $Be^9(p,d)Be^8$

As can be seen from Fig. 7, the results on beryllium do not permit clean separation of individual states, either from each other or, in the case of the "17-Mev group," from the continuum. However, the "loosely-bound" and "tightly-bound" groups are well separated, and some useful analysis is possible on these even though each may consist of more than one state. In terms of the fractional parentage coefficients F, we

[16] A. M. Lane (private communication).

TABLE II. Angular distribution of the two prominent groups, $Be^9(p,d)Be^8$. Statistical accuracy about ±8%. See text for further discussion of uncertainty.

	$d\sigma/d\Omega$, mb/sterad "tightly-bound" group		
θ, lab angle	(a) continuum subtracted	(b) continuum not subtracted	"loosely-bound" group
6°	7.1	13.6	8.8
9°	12.0	18.2	7.5
12°	6.6	9.4	3.6
16°	4.0	6.1	1.73
18°	3.0	4.7	1.86
24°	2.15	3.2	0.86
36°	0.28	0.75	0.30
60°	0.036	0.12	0.035

may say that each group may be treated as an entity with $\sum|F|^2=1$. This is equivalent to the closure argument of Chew and Goldberger.[8] (This method breaks down, of course, if there are groups with unresolvable extraction energies but coming from different initial states of the picked-up nucleon, a possibility mentioned above for the ground—and 1.9-Mev—groups of C^{11}.) On this basis, the total area under the peaks may be simply analyzed. The angular distributions of the two prominent groups are given in Table II. At the two largest angles there is a quite large uncertainty in separation of the 17-Mev group from the continuum, and even at smaller angles this uncertainty is considerable—the absolute magnitude of the integrated cross-section associated with the peak is uncertain to perhaps ±20% even at small angles, although the *shape* of the angular distribution involves less uncertainty, especially at the smaller angles.

The tightly-bound group involves an excitation energy for the residual nucleus of a little more than 17 Mev, and thus corresponds to a neutron extraction energy of about 19 Mev. This is very close to the extraction energy of 18.7 Mev leading to the ground state in $C^{12}(p,d)C^{11}$, and both of these are quite similar to the neutron extraction energy of 20.6 Mev in the reaction $He^4(p,d)He^3$. This is suggestive of alpha-particle structure in Be^9 and C^{12}—a model for which there is of course much other evidence.[17] Lane and Wilkinson have pointed out[5] that even on the shell model one can identify a group of excited states in Be^8 near 17 Mev as states of [3,1] symmetry, such as correspond for example to low states of Li^8, and they suggest that spatial grouping, characteristic of the alpha-particle model, is not implied by the experimental data. This identification, however, seems only to involve the charge independence of nuclear forces—it would remain curious, on the shell model, that the breakup of the symmetrical 4-group $(p)^4$ corresponding to the ground state of Be^8, to produce [3,1] symmetry (of either the excited Be^{8*} or the Li^8 ground state), should require so nearly the same energy as the extraction of one nucleon from the closed shell (on *jj* coupling) $(p)^8$ configuration of C^{12} or from the $(s)^4$ configuration of He^4. The alpha-particle model, on the other hand, offers a very simple explanation of the binding-energy similarities observed between the "17-Mev" group from Be^9 and the ground-state groups from He^4 and C^{12}.

[17] See e.g., D. R. Inglis, Revs. Modern Phys. 25, 390 (1953).

Comparison of Results from He^4, Be^9, C^{12}

Although this similarity of binding energies gives some support to the alpha-model for Be and C, these nuclei cannot be described purely on that model. This is evident from a study of the energy levels.[17] Measurements have recently been made on He^4 at 95 Mev,[18] and in further investigation as to similarity between the structure of Be and C, and of He, the (p,d)—derived momentum distributions are compared in Fig. 11. (For the tightly-bound group from Be^9, the data of column (b), Table II, are used.) One sees that the tightly-bound group from Be^9 has a momentum distribution very similar in shape to that from C^{12} (the "ground-state" group). The loosely-bound Be^9 group has relatively weaker high-momentum components, as one would expect from the very low binding energy of a neutron (the "outer" one) in Be^9.

It appears that not a great deal can be said, from Fig. 11, about the possibility of alpha substructure in Be^9 and C^{12}. There are two features of the data which might be thought to suggest such structure. One is the similarity in shape of the C^{12} and the "tight" Be^9 group, and the fact that the relative intensities are probably of the order of 33:2. The second is that the C and Be curves show a faint trace of the peaking at about 35-Mev internal kinetic energy which appears

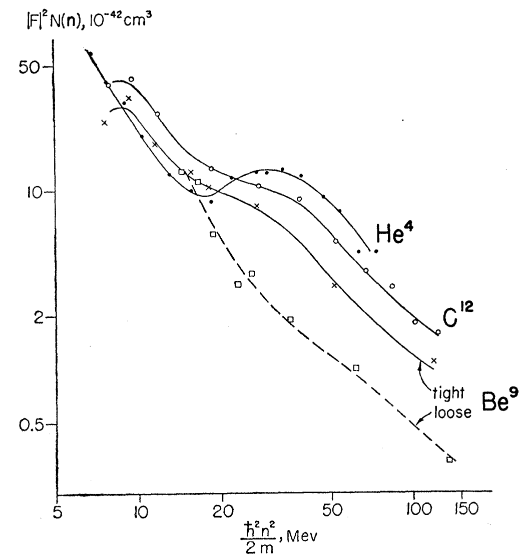

FIG. 11. Momentum densities derived from the 95-Mev (p,d) data on He^4, C^{12}, and Be^9. The Be curve for the "tightly-bound" group has an uncertainty of (+0, −30%) in absolute scale—see text.

[18] Kruse, Selove, and Teem (unpublished).

so prominently in the He data. On the other hand, both of these items could easily be explained away: The Be intensity, and therefore the C:Be intensity ratio, is uncertain, as mentioned above, and instead of 3:2 the C:Be ratio might be 2:1, a value more appropriate to the shell model. And any shape similarity of the high-momentum parts of the distributions might be explained by the fact that this region is dominated by correlation effects, as discussed above, and these correlation effects could produce this similarity even if alpha substructure were not present, although this counter-explanation is somewhat weakened by the considerable difference in the results for the two different groups from Be. It may further be noted that the C and Be curves, unlike that for He, show a suggestion of $l=1$ nature in a flattening and possible downward turning at low momenta. It would be interesting to see if a calculation could be made of the momentum distribution to be expected from say C^{12} on the alpha-model, making use of the experimental data from He^4.

Aluminum and Silicon

For these elements, and for copper and lead, measurements were made at only one angle, and hence only restricted conclusions can be drawn.

The results in Fig. 8 show that the $Si^{28}(p,d)$ reaction leads to a much more sharply defined final state or group of final states than does $Al^{27}(p,d)$. Such a result at very low energies, where compound nucleus formation can be effective, could be simply understood as arising from the fact that Si^{27} on the shell model could have the configuration $(d_{5/2})^{-1}$, closed-shell minus one, with correspondingly few closely-spaced states as compared to the $(d_{5/2})^{-2}$ configuration of Al^{26}. At higher energies, however, one may expect that the (p,d) reaction will involve more importantly the nature of the ground state of the *target* nucleus. That is to say: on the impulse approximation, at high energies one nucleon is snatched out in a time short compared to the rearrangement time of the rest of the nucleus. The results would then be influenced less by which states of the residual nucleus *can* occur at *all*, and more by which states of the residual nucleus are represented as parents in the wave function of the *initial* nucleus.

On this interpretation, the results show that the ground state of Al^{27} contains more varied parent states than the ground state of Si^{28}. This result is understandable on the alpha model (again perhaps mildly suggested by the 17.2-Mev extraction energy for a neutron from Si^{28}, compared to 20.6 for He^4, ~19 for the tight group from Be^9, 18.7 for C^{12}, and to 15.6 for O^{16}, 16.9 for Ne^{20}, 16.6 for Mg^{24}), and also, as Lane and Wilkinson have emphasized,[5] on the shell model, taking closed-shell structure—$(1d_{5/2})^{12}$—for Si^{28}, and nonclosed structure for Al^{27}. On the latter interpretation, the wider distribution of final states found for Al^{27} shows that the six neutrons of the filled $d_{5/2}$ subgroup in Al^{27} interact with the corresponding but unfilled proton subgroup—this would not be in agreement with a strictly jj-coupling model, but is not in contradiction to an intermediate coupling model. It will be of interest to make (p,d) measurements on K^{39} and Ca^{40}, where a clearer choice between the results expected according to the alpha or shell model may be obtained. Unfortunately, for these larger nuclei, both the "transparency" approximation and the "instantaneous" impulse approximation will be less valid.

Copper and Lead

Only the meager data of Fig. 9 are available on these. Copper, like the lighter elements, still shows a peak—but a broad, weak one, as one might expect from the increased number of neutron states filled in Cu. Lead, on the other hand, shows a rather sharp peak. This is probably due at least partly to the closed-shell nature of Pb. It may also be due in part to an opacity effect. The Pb nucleus is large, and not very transparent to outgoing deuterons. The observed peaked group of deuterons probably comes largely from neutron states having a large probability of giving a neutron near the nuclear surface—i.e., a large reduced width. These states could constitute a relatively narrow energy group.

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