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Intermediate Coupling in the 1*p*-Shell*

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The region between He^4 and O^{16} is treated for the case of intermediate strength of spin-orbit coupling and central two-body interaction. Energy levels are presented as a function of the relative coupling strength parameter, a/K. Static electromagnetic moments are also computed as functions of a/K. Comparison with experimental results gives a fairly good picture, and determines a definite behavior for a/K as a function of mass number. A possible interpretation of this behavior is suggested.

I. INTRODUCTION

BECAUSE of its relatively simple structure the region of filling of the $1p$-shell between He^4 and O^{16} has been the object of much study, both theoretical and experimental. It has become apparent that one cannot obtain a satisfactory picture of the experimental facts with either the LS-model[1] or the jj-model.[2] This has left the hope that the true picture lies between these two extremes of spin-orbit coupling, in the region of intermediate coupling. Various calculations have been made with this more complicated model, and Inglis[3] has presented an interpolated estimate of level schemes. The amount of computation becomes prohibitive near the center of the shell unless one uses a high-speed electronic computer.

The individual particle wave functions are taken as those in a harmonic oscillator well with individual l and s coupled to give j. These jj-functions are then combined into a many-particle wave function of the Hartree-Fock type with total angular momentum J and isotopic spin T. All possible states for a given number of nucleons in the $1p$-shell are formed in this way. States arising from excitation of one nucleon into the next shells ($2s$, $1d$) would be of opposite parity and would not interact with the former states. Excitation of two nucleons from the $1p$ into $2s$ and $1d$ would again give states of like parity, but these should be high enough in energy so as to have little effect on the low-lying states. They are not included, but for this reason one should view the position of levels of a given isotopic spin, T, with suspicion if they lie more than about 8 Mev above the lowest state of the same T.

The nucleon-nucleon interaction is taken to be a central one with inverted Gaussian radial dependence and an exchange mixture of 0.8 space exchange and 0.2 spin exchange. The rest of the potential energy arises from the one-body spin-orbit term which has only diagonal matrix elements since jj wave functions are being used. For each mass number the energy matrices of spin-orbit energy and central interaction energy arising for each pair of quantum numbers (J,T), are diagonalized on the Argonne automatic digital computer. In this way one obtains energy level schemes for each mass number by using a given set of parameters.

After diagonalization, the resulting wave functions of the ground states are extracted to see how much the jj-configurations are mixed. The ground-state magnetic dipole moments and electric quadrupole moments are also calculated by using the new wave functions.

II. PARAMETERS OF THE CALCULATION

The nuclear parameters which are involved in the calculation come from the radial part of the harmonic oscillator function:

$$R_p(r) \propto r \exp[-(r/r_p)^2], \tag{1}$$

the one-body spin-orbit term:

$$a(\mathbf{l}\cdot\mathbf{s}), \tag{2}$$

and the radial part of the central two-body interaction:

$$A_0 \exp[-(r_{12}/r_0)^2]. \tag{3}$$

The contributions of the central two-body interaction to the energy matrices are usually expressed in terms of two integrals,[4] L and K, which are linear combinations of the two Slater integrals involved. These integrals are functions of the strength of two-body interaction A_0, and the ratio $\rho = r_p/r_0$ which is a measure of the ratio of nuclear radius to range of nuclear forces. The functional dependence of L and K on A_0 and ρ is given graphically in Fig. 1.

The particular combinations of parameters which are useful for interpreting the results are a/K, L/K, and K. The first, a/K, measures the relative spin-orbit and central energy contributions and turns out to be the most important for the computation. The second, L/K, depends only on ρ, and hence the ratio of nuclear size to range of nuclear forces. The diagonalizations are carried out in units of K, which is then left as a parameter to match the experimental energy scale. The magnitude of K is generally about 1 Mev.

The diagonalization has been performed throughout the $1p$ shell with $L/K = 6.8$ and enough values of a/K to cover the range of physically significant values. The

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] E. Feenberg and E. Wigner, Phys. Rev. 51, 95 (1937); E. Feenberg and M. Phillips, Phys. Rev. 51, 597 (1937).
[2] D. Kurath, Phys. Rev. 88, 804 (1952).
[3] D. R. Inglis, Revs. Modern Phys. 25, 390 (1953).
[4] See references 1 and 3 for definition and discussion of L and K.

resulting level schemes for mass numbers 7 through 13 are given in Figs. 2 through 8. The typical effects of varying L/K are given in Figs. 9 through 11. The magnetic dipole moments are given as functions of a/K in Fig. 12; where changing L/K has a sizable effect, results for two values of L/K are plotted. The electric quadrupole moments are given in Fig. 13 in units of $\langle r^2\rangle$, where

$$\langle r^2\rangle=\int_0^\infty r^2R_p^2(r)r^2dr=(5/4)r_p^2.$$

III. DEPENDENCE ON L/K

From Fig. 1 it is clear that the dependence on L/K is a direct measure of the effect of varying the ratio of nuclear size to range of nuclear forces. The magnetic moment curves of Fig. 12 show that with the possible exception of B^{11}, the variation with L/K is not significant.[5] In fact, for Li^7, B^{10}, and Be^9 the difference is not worth plotting. The effect on the level schemes is given in Figs. 9, 10, and 11 for an odd-odd, an even-odd, and an even-even nucleus. Changing L/K is seen to affect states that either lie rather high in energy or have isotopic spin, T, which is greater than that of the ground state. This means that for an even-even nucleus, where excited states are widely spaced, this region is soon reached, so that the effect is rather strong. However, the states affected are so high that interaction with states arising from two-nucleon excitation out of the 1p-shell must be important. Since this has been neglected only the effect on the first $T=1$ states involves states whose position is not suspect.

For even-odd nuclei, the low-lying states are hardly changed and the only important state that is affected is the first $T=3/2$ level. In odd-odd nuclei, the $T=1$

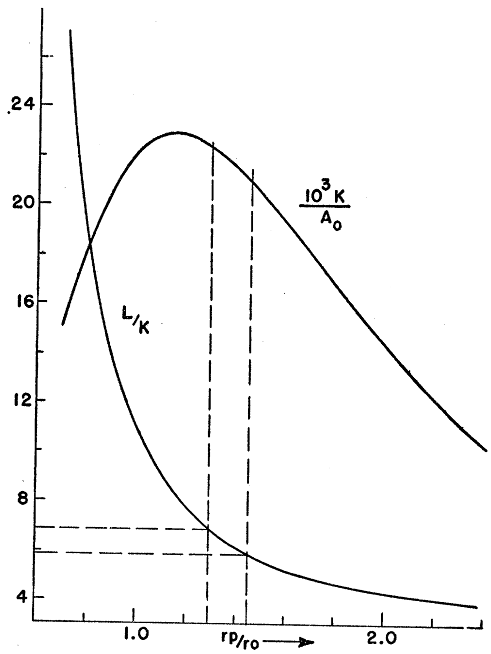

FIG. 1. Functional relationships of the two-body central interaction integrals, L and K, with the strength of interaction, A_0 and ρ. The ratio $\rho=r_p/r_0$ is the range parameter of the wave function divided by that of the interaction. Dotted lines indicate region of this calculation.

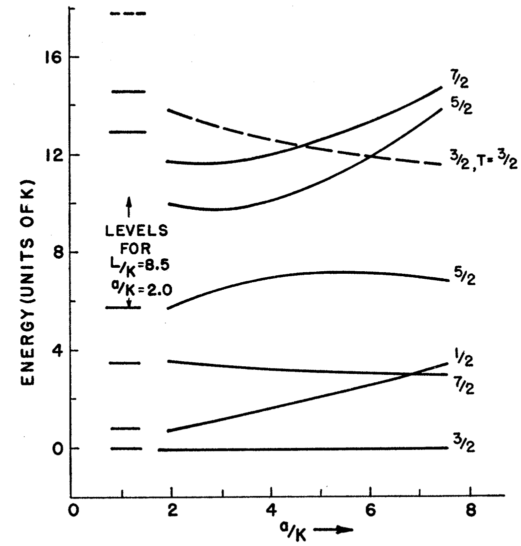

FIG. 2. Level order curves for mass number 7 as a function of a/K for $L/K=6.8$. Numbers are J values. Scheme for $L/K=8.5$, $a/K=2.0$ is on left.

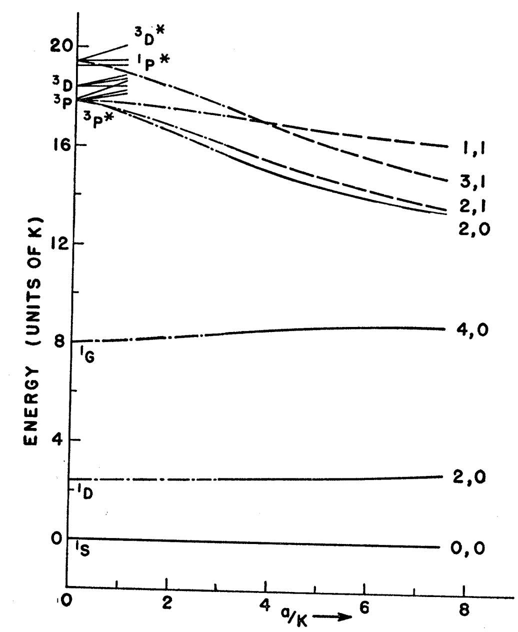

FIG. 3. Level order curves for mass number 8 as a function of a/K for $L/K=6.8$. Numbers are (J,T) values. Supermultiplet identifications on the left; asterisk identifies $T=1$. Values for $0<a/K<3$ are interpolated.

[5] This has been shown by A. M. Lane for the mass 7 and 13 nuclei [A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953); **A68**, 189 (1955); **A68**, 197 (1955); A. M. Lane and L. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954)].

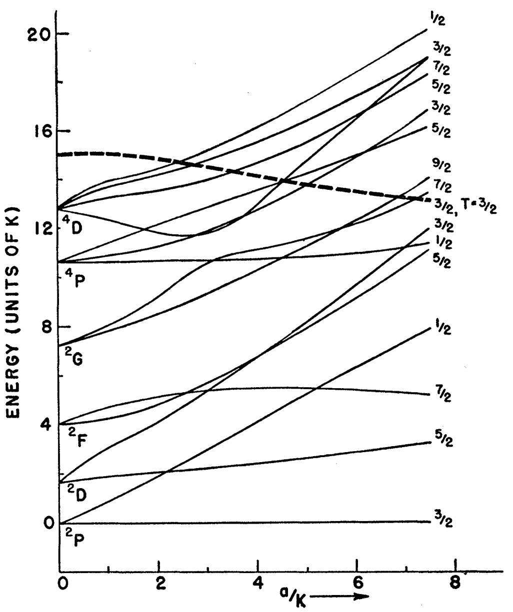

FIG. 4. Level order curves for mass number 9 as a function of a/K for $L/K=6.8$. Numbers are J values. Supermultiplets identified on the left.

states lie much closer to the ground state, so this is the only time that low-lying states are affected. The result is that in B^{10}, the only complicated odd-odd case, there is a wider range of a/K over which one can get good agreement with experiment since one can vary L/K to maintain the match.

The underlying reason for this insensitivity to variation of L/K can be found in the supermultiplet theory

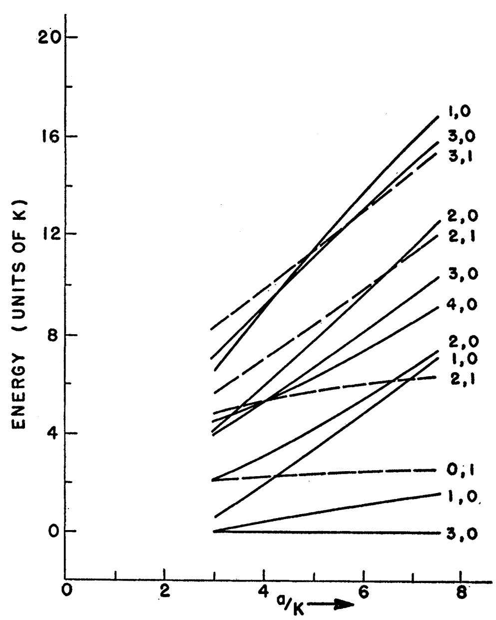

FIG. 5. Level order curves for mass number 10 as a function of a/K for $L/K=6.8$. Numbers are (J,T) values.

at the LS-coupling limit. The states arising from the same partition of n, the number of nucleons in the $1p$-shell, have separations which are independent of L/K as far as the space-dependent part of the two-body interaction is concerned. The spin-dependent part preserves this feature only if the spin quantum number, S, is the same, as one can see on page 600 of reference 1. For even-even and even-odd nuclei the lowest partition gives states all with the same S so that as one departs from LS-coupling there should be a tendency to preserve this independence of L/K for the separation of the low-lying states. Evidently this persists much farther than expected because for $a/K\approx 4$ Figs. 10 and 11 show quite insensitive behavior for those states connected with the lowest partitions. For odd-odd nuclei there are states of different T also arising from

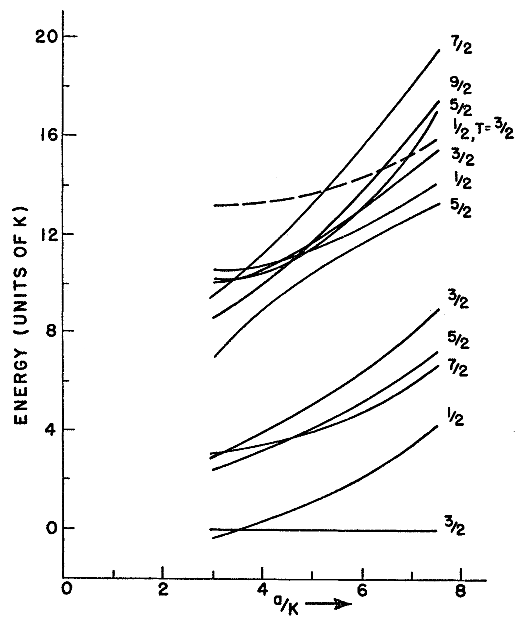

FIG. 6. Level order curves for mass number 11 as a function of a/K for $L/K=6.8$. Numbers are J values.

the lowest partition and while these maintain their separation from each other, their position relative to the group containing the ground state does depend on L/K, as is shown in Fig. 9. The interactions are apparently chiefly between states of the same partition.

Thus, aside from B^{10}, the level order for the important low states is quite insensitive to changes in L/K. Similarly, the magnetic dipole moments and the electric quadrupole moments are not seriously affected. The dependence on the ratio of nuclear size to range of nuclear forces is, therefore, of secondary importance compared to the dependence on a/K, the relative strength of spin-orbit coupling to central interaction.

IV. PARTICULAR CASES

In this section the calculations are compared with the experimental evidence as taken from Ajzenberg and Lauritsen.[6] The particular cases are discussed with the aim of pointing out especially interesting places to check the model.

$A=6$, 7, 13, and 14

These are the cases of two and three nucleons or holes in the 1p-shell which have been treated extensively elsewhere.[7] The results of the present calculation agree closely with previous results and serve chiefly as checks on the method of machine computation. The general indication is that a value of $a/K\sim 2$ is indicated for the Li isotopes while $a/K\sim 5$ or 6 gives best agreement for masses 13 and 14.

In comparing with experiment for $A=13$, the only serious lack of agreement is that the $(5/2, 1/2)^{-}$ level is

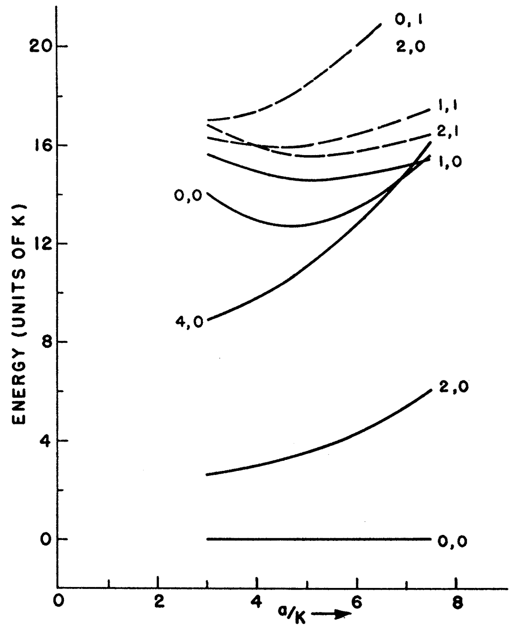

FIG. 7. Level order curves for mass number 12 as a function of a/K for $L/K=6.8$. Numbers are (J,T) values.

not seen experimentally, although the calculation puts it somewhere between 4 and 5 Mev according to Fig. 8.

The only new result in Li^7 concerns the relative splitting of the ^2F and ^2P. Experimentally the ^2P splitting is smaller than that of the ^2F by a factor of 6, while the calculation gives this ratio as nearer 2. The new point is that, as shown in Fig. 2, changing L/K from 6.8 to 8.5 has a negligible effect on this ratio, so there is no improvement of agreement with experiment from this source.

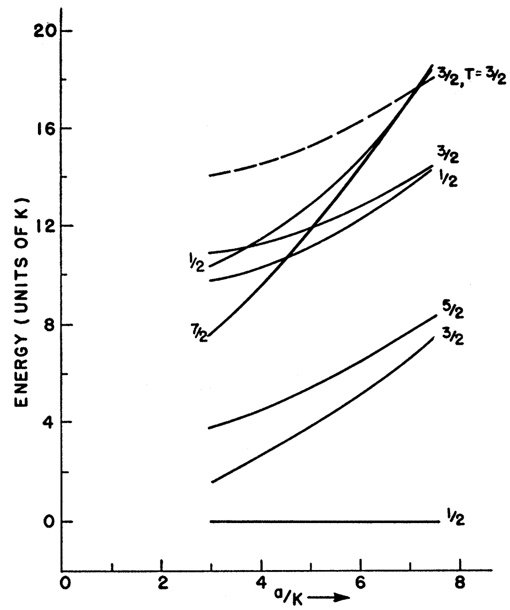

FIG. 8. Level order curves for mass number 13 as a function of a/K for $L/K=6.8$. Numbers are J values.

$A=8$

The spacing of the first three states of Be^8 is nearly independent of the amount of spin-orbit coupling as is shown in Fig. 3. The supermultiplet levels,[1] have been put on the left so that one can interpolate from the last calculated value of $a/K=3$. There are no calculated levels corresponding to the experimental states at 4.2, 5.4 and 7.55 Mev. For the fairly well established second 0^+ state one must call upon excitation of two nucleons outside the shell or from the 1s-shell to obtain such a state with the independent particle model.

One can obtain an estimate of the appropriate a/K value by going to the $T=1$ states in Be^8 and Li^8. The apparent level order is $J=2$, 1, and 3 with $J=1$ lying a little less than midway between the others. From Fig. 3 this situation appears to occur at about $a/K=2$ in the interpolated region. With $K=1$ Mev, the $T=1$ levels would lie at about the right place, but the $J=4$ level

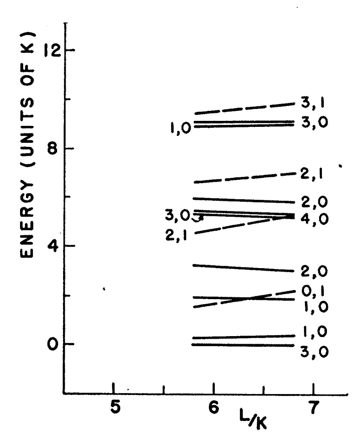

FIG. 9. Level order curves for mass number 10 as a function of L/K for $a/K=4.0$. Numbers are (J,T) values.

[6] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[7] D. R. Inglis, Phys. Rev. **87**, 915 (1952); J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953); E. A. Crosbie, Phys. Rev. **90**, 138 (1953); G. Tauber and T-Y. Wu, Phys. Rev. **93**, 295 (1954); A. M. Lane, reference 5; A. M. Lane and L. Radicati, reference 5.

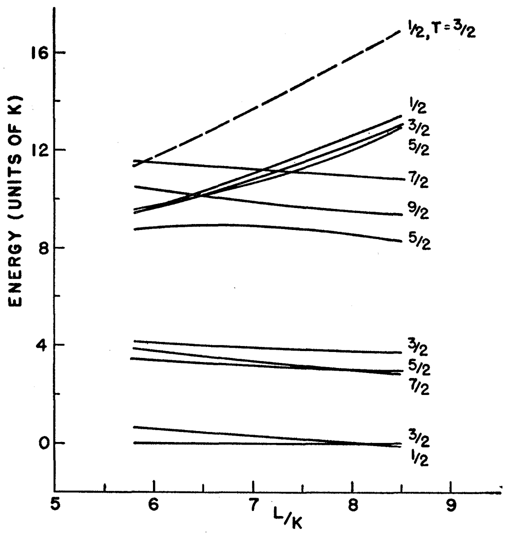

FIG. 10. Level order curves for mass number 11 as a function of L/K for $a/K=4.0$. Numbers are J values.

would be too low. Improvement can be gotten by lowering L/K to 5.8 which drops the $T=1$ levels by about $3K$ with respect to the ground state. Then a value of $K=1.18$ would raise them again and would also improve the position of the two low levels of Be^8. This will give the level scheme of Fig. 14.

$A=12$

This is the other even-even nucleus under consideration, arising from four holes in the shell. The points of interest for comparison with experiment are the $J=2$, the relative position of the $J=4$ and the second $J=0$, and the low $T=1$ doublet. None of the a/K and L/K values give a good fit to the experimental values. With $L/K=6.8$ and $a/K=5$ one gets the level order on the left in Fig. 15. This gives $J=4$ as the second excited state, and the wrong order for the $T=1$ doublet. By going to $a/K\approx 3$, $L/K\approx 6$, $K\approx 1.1$ one can invert the doublet getting about the experimental separations of B^{12} and leave the other states pretty much as on the left of Fig. 15 except that the second $J=0$ is raised about 1 Mev. Another possibility is to use the L/K dependence as shown in Fig. 11 to bring the second $J=0$ below the $J=4$ and invert the doublet while keeping a/K at ≈ 4 to 5. This result, with $L/K=5.5$, $K=1.17$ is shown in the center of Fig. 15, but it has the fault of leaving the 0^+ too high. It would be helpful to learn whether the 9.6-Mev level of C^{12} is 4^+. In any event, one must suppose that the low experimental value of the second 0^+ is due to its interaction with the $J=0^+$ levels arising from exciting two nucleons out of the $1p$ shell, since the present calculation always leaves it too high.

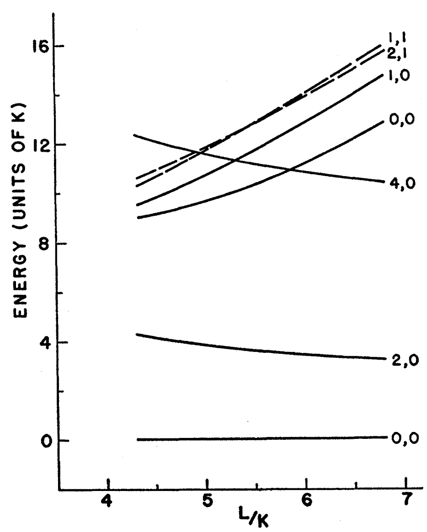

FIG. 11. Level order curves for mass number 12 as a function of L/K for $a/K=4.5$. Numbers are (J,T) values.

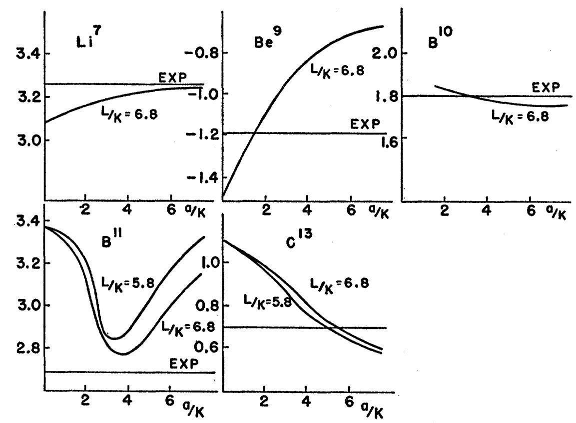

FIG. 12. Magnetic dipole moments as functions of a/K in nuclear magnetons.

$A=9$

The magnetic moment of Be^9 indicates either a quite low value for a/K or else a very high one, since the jj-limit gives a moment quite close to the experimental one. However, the low value of Fig. 12 seems more reasonable, so the level scheme of Fig. 4 has been extended to the LS limit, $a/K=0$. These results are very close to those obtained recently by French, Halbert and Pandya at Rochester,[8] using a Rosenfeld exchange mixture.

If one accepts the magnetic moment value of $a/K=1.5$, then for $L/K=6.8$ a value of $K=1$ Mev gives the level scheme on the left of Fig. 16 which also fits the $T=3/2$ state at about 15 Mev. By going to a lower L/K and higher K one can raise the first seven excited states which all come from the lowest partition (see Section 3) and still fit the $T=3/2$ state. This gives the scheme second from the left in Fig. 16, and is substantially that chosen by the Rochester group.

[8] French, Halbert, and Pandya, Phys. Rev. 99, 1387 (1955).

The low value of a/K represents a sudden large shift from the value that is found desirable in the neighboring $A=10$ nuclei, where there is much more experimental evidence. It is therefore of interest to see what one gets if one increases the magnetic moment disagreement and tries larger values of a/K. With $a/K=2.75$, $L/K=6.8$ and $K=1$ Mev one obtains the results in the third column of Fig. 16. The $J=1/2^-$ is now above the $J=5/2^-$ and the next three states are closely bunched between 5 and 5.5 Mev. It would be of interest to see whether there is any fine-structure to the broad state found experimentally at 4.8 Mev, and to determine the order of the $1/2^-$ and $5/2^-$ levels to decide which a/K value is preferable.

$A=11$

The other complex even-odd case occurs for five holes in the shell at mass number eleven. The magnetic moment curve of Fig. 12 shows reasonable agreement

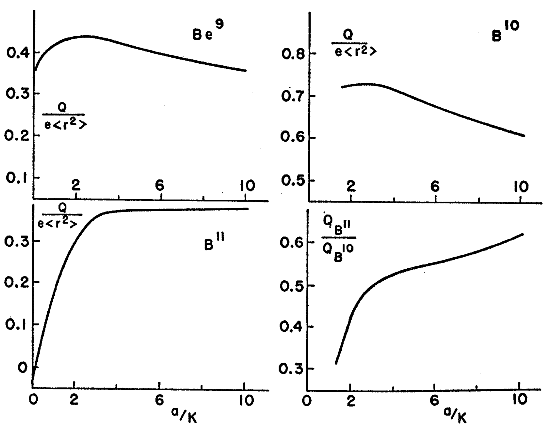

FIG. 13. Electric quadrupole moments as functions of a/K in units of $e\langle r^2\rangle$.

for a/K between 2.5 and either 5 or 6 depending on the L/K value. The level order curves of Figs. 6 and 10 indicate that in order to have the ground state $J=3/2$ and a sizable gap before the next level, a/K should be near 5 or 6. Choosing $a/K=6.0$, $L/K=6.8$, and $K=0.92$ Mev gives the level scheme on the left of Fig. 17. Going to lower a/K, which would be desirable for improving the magnetic moment agreement, lowers the first excited state and also drops the $J=5/2^-$ with respect to $J=7/2^-$. One can keep the first excited state up to some extent by going to lower L/K and at the same time drop the $J=5/2^-$ further as is evident in Fig. 10. With $a/K=5.0$, $L/K=5.8$, and $K=1.15$ Mev one gets the level scheme in the center of Fig. 17 but about the same magnetic moment due to the lower L/K. In order to bring a/K down to a good value for the moment, the $J=1/2^-$ first excited state would have to be put quite low, about 1 Mev. In all these schemes, the first $T=3/2$ state has been kept at about the expected

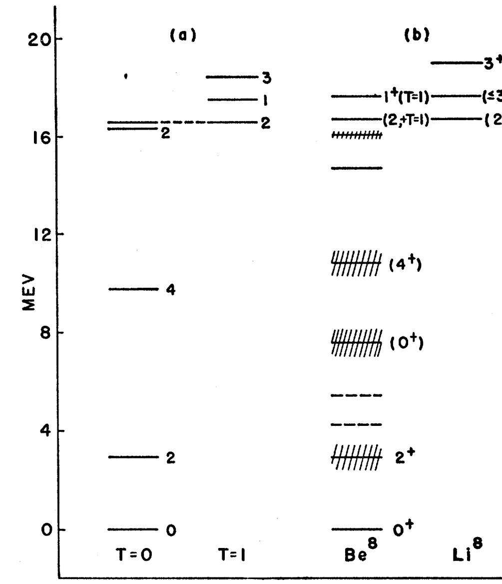

FIG. 14. Comparison of level schemes with experiment for mass number 8. (a) $a/K=2.0$, $L/K=5.8$, $K=1.18$ Mev. (b) Experiment.

13-Mev excitation. It may be that one would have a bigger effect from varying L/K at a larger value of the spin-orbit parameter than the $a/K=4.0$ used for Fig. 10. This would be expected since the states from different partitions would interact more for larger a/K, so one might be able to obtain better agreement with experiment. However, identification of the 2.14- and 5.03-Mev states would be very useful before one tries further calculation.

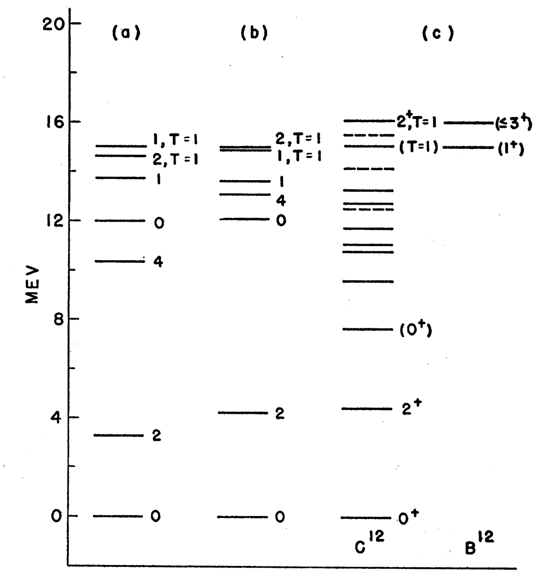

FIG. 15. Comparison of level schemes with experiment for mass number 12. (a) $a/K=5.0$, $L/K=6.8$, $K=0.94$ Mev. (b) $a/K=4.5$, $L/K=5.5$, $K=1.17$ Mev. (c) Experiment.

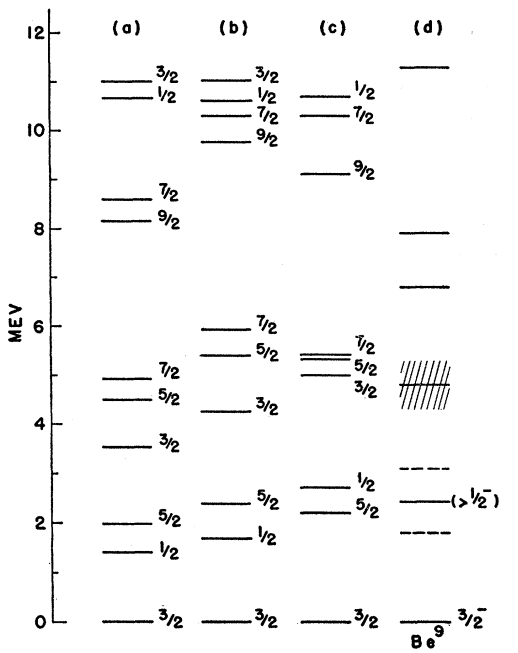

FIG. 16. Comparison of level schemes with experiment for mass number 9. (a) $a/K=1.5$, $L/K=6.8$, $K=1.00$ Mev. (b) $a/K=1.5$, $L/K=5.8$, $K=1.20$ Mev. (c) $a/K=2.75$, $L/K=6.8$, $K=1.00$ Mev. (d) Experiment.

$A=10$

Mass number ten is the middle of the $1p$-shell and has therefore the most complex spectrum of energy levels. It also has the largest number of positively identified spins, so it is encouraging, though somewhat surprising that it provides the best agreement of all the cases. The magnetic moment of Fig. 12 appears to give reasonable values for all a/K. It had been shown previously[9] that $J=3$ is the ground state for a large region of a/K, as one sees in the level order curves of Fig. 5. In comparing with the many experimental levels, one finds very good agreement in the region near $a/K=5$. The level scheme on the left of Fig. 18 comes from $a/K=4.75$, $L/K=6.8$ and $K=0.9$ Mev. From Fig. 9 it is apparent that one can go to lower a/K by also decreasing L/K, and the central scheme of Fig. 18 arises from $a/K=4.0$, $L/K=5.8$ and $K=1.13$ Mev.

Both of these schemes give the positions of the first five levels of B^{10} as well as the first two states of Be^{10} almost quantitatively. One reason may be that there are so many states for each spin in the center of the shell that most of the states which interact with the low-lying levels arise from these $1p$-shell configurations. One will, however, have to wait for the identification of more levels in the neighboring $A=9$ and 11 nuclei where similar conditions exist before believing such a reason. One point of interest in B^{10} is the level at 4.7 Mev, experimentally identified as probably $J=1^+$. If this is correct, one would have to call upon two-nucleon excitation since there are no 1^+ states nearby in the present calculation. Low-lying states of this type would make the good agreement for the other states puzzling since one might expect sizable interactions of the kind which has been neglected.

V. WAVE FUNCTIONS AND STATIC MOMENTS

The ground-state wave functions for a/K near 5 have considerable mixing of the zero-order jj-configurations. For such a degree of mixing, analysis in terms of LS functions would lead to equally strong mixing, as has been illustrated elsewhere[10] for the mass 19 nuclei. On the other hand, for $a/K=1.5$ in Be^9 one is still near LS coupling and the ground state is 92% zero-order LS-state, ${}^2P_{3/2}$. By $a/K=4.5$, this percentage would be reduced to 69%, about the same percentage as that of the lowest zero-order jj-state. An analysis is given for the ground states of Be^9 through C^{12} in Table I in terms of jj wave functions.

The magnetic dipole moments which one computes with the resultant ground state wave functions have a general tendency to give much better agreement with experiment in the region of intermediate coupling than at the extremes. This tendency has been shown previously[11] by using approximate forms for the wave functions which include the principle configurations. That calculation used a two-body spin-orbit interaction and seemed to stay much nearer the LS-limit in ob-

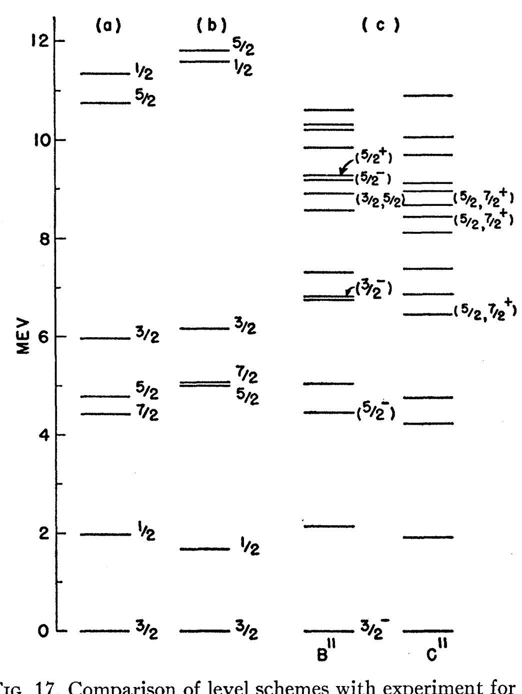

FIG. 17. Comparison of level schemes with experiment for mass number 11. (a) $a/K=6.0$, $L/K=6.8$, $K=0.92$ Mev. (b) $a/K=5.0$, $L/K=5.8$, $K=1.15$ Mev. (c) Experiment.

[9] N. Zeldes, Phys. Rev. **90**, 416 (1953).

[10] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).

[11] R. Schulten, Z. Naturforsch. **8a**, 759 (1953).

taining the moments, which would not be possible for B^{11} and C^{13} in the present calculation. The special value of the magnetic moment calculations is that they provide a relatively easy way to find the region of a/K which should be carefully investigated for level order. This is because one can extract the wave function, and hence get the moment, without carrying out the diagonalization.

The electric quadrupole moments are given in Fig. 13. For Be^9 and B^{10} the results are the same as those of the Rochester group.[8] The magnitudes are in rough agreement with those of experiment for $\langle r^2\rangle \approx 6\times 10^{-26}$ cm^2. In the ratio of B^{11} to B^{10} we may assume the nuclear radius factor to be about the same, which gives the lower right hand curve of Fig. 13. The experimental value[12] of 0.48 gives an a/K of 2.6, considerably lower than desired for the level scheme of B^{10}. However, a 10% change in the ratio would give agreement, and cooperative effects of the Bohr-Mottelson type might bring about sufficient changes in the electric quadrupole moments to remedy things without seriously affecting the level schemes.

VI. DISCUSSION

A. Central Force Integrals

A survey of the results from comparison with experiment throughout the 1p-shell shows that while a/K seems to vary considerably, satisfactory results are obtained by holding the central interaction integrals, L and K, nearly constant. With $L/K=6.8$ and K between 0.9 and 1.0 Mev, one gets a reasonable set of fits by varying a/K. On the other hand, $L/K=5.8$ and K between 1.1 and 1.2 Mev gives equally good agreement with lower a/K values for each particular case. The reason for the simultaneous decrease of a/K, L/K, and increase of K is that lowering L/K lowers the high-lying states, particularly those of higher T while leaving the states of the ground-state partition relatively unchanged as discussed in Sec. III. Lowering a/K drops the low-lying states more than the upper ones so that the net result is to have a compressed energy scale compared to the original values. Then one can compensate for this and get back to a scheme not very much different from the original one by increasing K. There is therefore a range of parameters which provides essentially the same picture.

TABLE I. Contributions of the various zero-order jj-configurations to the final ground states. Configurations are $(p_{\frac{3}{2}})^{n-x}(p_{\frac{1}{2}})^x$, where n=number of nucleons in 1p-shell. $a/K=1.5$ for Be^9, 4.5 for the rest.

Nucleus	$(p_{\frac{1}{2}})^0$	$(p_{\frac{1}{2}})^1$	$(p_{\frac{1}{2}})^2$	$(p_{\frac{1}{2}})^3$	$(p_{\frac{1}{2}})^4$
$B^{10}(J=3)$	0.654	0.225	0.108	0.011	0.001
$B^{11}(J=\frac{3}{2})$	0.641	0.052	0.274	0.027	0.006
$C^{12}(J=0)$	0.487	0	0.402	0.072	0.039
$Be^9(J=\frac{3}{2})$	0.399	0.250	0.313	0.018	0.021

[12] H. G. Dehmelt, Z. Physik **133**, 528 (1952).

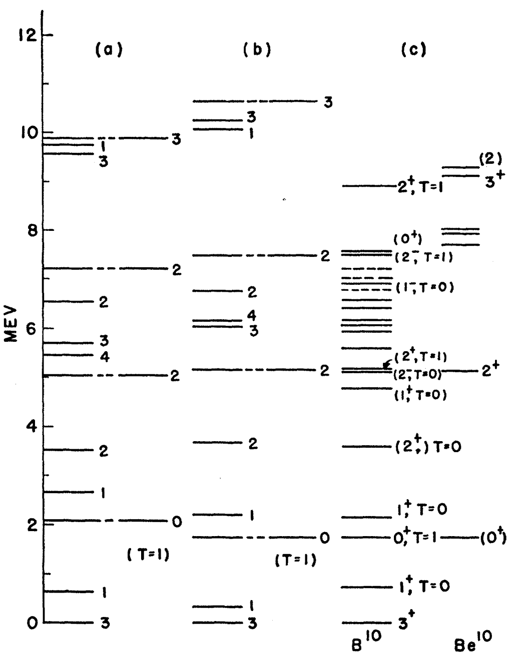

FIG. 18. Comparison of level schemes with experiment for mass number 10. (a) $a/K=4.75$, $L/K=6.8$, $K=0.90$ Mev. (b) $a/K=4.0$, $L/K=5.8$, $K=1.13$ Mev. (c) Experiment.

The fact that constant values for the L- and K-integrals throughout the shell seem adequate means that no effect of nuclear size is apparent in them. Such an effect would manifest itself as a change in the harmonic oscillator parameter r_p. From Fig. 1 this would mean a simultaneous decrease or increase in L/K and K in the region of values used. While the effect is not large, particularly for K, there is no evidence that such variation is desirable as regards agreement with experiment.

B. Spin-Orbit Strength

The relative spin-orbit strength parameter, a/K, does change as one fills the 1p-shell as was evident in the work of Inglis,[3] Lane,[5] and others who found $a/K\sim 2$ near the beginning of the shell and $a/K\sim 5$ near the end. This effect is due primarily to a, for the cases of a single particle or hole in the shell where the $j=l+1/2$ and $j=l-1/2$ are split only by the spin-orbit interaction give $a\simeq 2$ Mev for $A=5$ and $a=4.2$ Mev for $A=15$.

The results of the present calculation seem to indicate that the change in a/K as one crosses the shell is not smooth. From Li^6 to Be^9 a slow increase in a/K from 1 to 2 seems satisfactory, while from B^{10} to N^{14} values near 5 seem desirable. The change may not be so abrupt if experiment shows that a/K nearer 3 fits Be^9 despite the magnetic moment, since B^{10} can be stretched to $a/K=4$.

However, it is interesting to speculate about the explanation of the abrupt change between $A=9$ and $A=10$ if it turns out to be real. There is also an abrupt

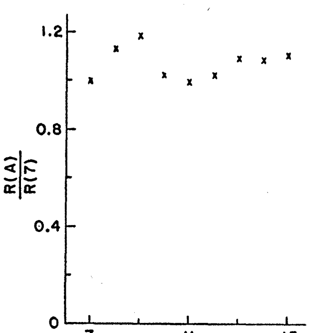

FIG. 19. Ratio of nuclear radius at mass number A to nuclear radius at mass number 7. Radii are computed from the Coulomb energy difference assuming a simple charged sphere model.

change at this point when one plots the nuclear radius determined by the Coulomb energy difference of mirror isobars, and it is tempting to relate these phenomena. The Coulomb energy differences have been calculated with the independent-particle model,[1,2] but the results are not seriously different from those obtained by using the simple picture of a uniformly charged sphere.[13] The difference in binding energy for spherical charge distributions of radius R is

$$B_Z - B_{Z+1} = 1.2Ze^2/R. \tag{4}$$

Using the experimental binding energy differences from (p,n) thresholds and beta decays taken from Ajzenberg and Lauritsen,[6] one can apply (4) to obtain the ratios of radii at mass number A to the radius at mass number 7 in Fig. 19. An independent-particle calculation gives a somewhat smoother curve, but the same size of jump from $A=9$ to $A=10$.

The correlation with the behavior of a/K as a function of mass number can be seen in the following qualitative way. Lane and Elliott[14] have shown that with a two-body spin-orbit force of the form

$$\{(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[(\mathbf{r}_1-\mathbf{r}_2)\times(\mathbf{p}_1-\mathbf{p}_2)]\}V(r_{12}), \tag{5}$$

one can obtain the fact that the splitting of $1p_{3/2}-1p_{1/2}$ changes from about 3 Mev for a single $1p$ nucleon in the shell to 6.3 Mev for a single $1p$ hole. They state further that the effect of the two-body force as the $1p$ shell is filled is to give an effective single-body force, $a(\mathbf{l}\cdot\mathbf{s})$, with a increasing steadily. The modifications of this steady increase by changing nuclear size according to Fig. 19 are the following. The fact that the nucleus gets bigger from $A=7$ to $A=9$ would counteract the steady increase of a brought about by adding more nucleons because the average separation of the nucleons would be bigger. Then at $A=10$ there is a sudden decrease in nuclear size, so one would expect a large increase in a from the value at $A=9$. From $A=10$ to 15 there is a more gradual increase in nuclear size and therefore a tendency to lower the slope of the original steady increase in a.

The qualitative behavior agrees with what one derives from the magnetic moments and level schemes of the present calculation. In order to get a quantitative idea of the dependence of a on nuclear radius, R, one can see what power of R is needed to give the discontinuity between $A=9$ and 10. The ratio of a/K for $A=10$ to a/K for $A=9$ is about 2.0 to 2.5. If one neglects the relatively slight change in the integral K, the fact that the ratio of nuclear radii is about 1.2 means that

$$a(R) \propto R^{-4} \text{ to } R^{-5}. \tag{6}$$

This is a higher inverse power of R than that gotten by Lane and Elliott, but not unreasonable from another point of view. If one starts with the Hamiltonian for particles in a potential well, $V(r)$, one can show that for orbital angular momentum, l, not equal to zero, the expectation value of dV/dr is given by

$$\langle dV/dr\rangle = (\hbar^2/m)l(l+1)\langle r^{-3}\rangle. \tag{7}$$

The only necessary condition for obtaining this result is that the radial part of the individual particle wave function, $R_{nl}(r)$, is a continuous function which vanishes at the origin. Therefore, if $a(R)$ has the form of the Thomas precession term, $(1/r)(dV/dr)$, the dependence given by Eq. (6) is not unreasonable.

VII. CONCLUSIONS

There is then the possibility that the variation of spin-orbit strength with mass number is strongly affected by nuclear size as well as by the number of nucleons in the shell. However, the fact that at the same time the central force integrals require no size dependence is disturbing. There will have to be more theoretical work and correlation with experiment before this picture is cleared up.

The over-all picture for the $1p$-shell shows that the intermediate coupling model gives considerable improvement over the models of extremely weak or strong spin-orbit coupling. One can even begin to make quantitative comparisons with experiment using what is conceptually a very simple model. While the agreement with the complicated B^{10} nucleus is very encouraging, one will have to wait for more experimental identifications in the neighboring Be^9 and B^{11} nuclei to test the model further. Other information which can throw more light on the a/K dependence is also desirable. For this reason computation of some gamma transition probabilities is being planned, particularly for the many $M1$ transitions that have been observed.

VIII. ACKNOWLEDGMENTS

The author wishes to thank the members of the AVIDAC group, particularly Joseph M. Cook, for programming and carrying out the machine computations. Thanks are also due to Maria G. Mayer and David R. Inglis for many helpful discussions.

[13] See, for example, J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 216 ff.

[14] A. M. Lane and J. P. Elliott, Phys. Rev. **96**, 1160 (1954).